## Brackets, Equations & Inequalities

# Year (8)

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#MathsEveryoneCan
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	Week 1	Week 2	Week 3	Week 4	Week 5	Week 6	Week 7	Week 8	Week 9	Week 10	Week 11	Week 12
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	Algebraic Techniques					Developing Number						
Spring	Brackets, equations and inequalities		Sequences	Indices	! 	ns and ntages	Standard index form					
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## Spring 1: Algebraic Techniques

#### Weeks 1 to 4: Brackets, Equations & Inequalities

Building on their understanding of equivalence from Year 7, students will explore expanding over a single bracket and factorising by taking out common factors. The higher strand will also explore expanding two binomials. All students will revisit and extend their knowledge of solving equations, now to include those with brackets and for the higher strand, with unknowns on both sides. Bar models will be recommended as a tool to help students make sense of the maths. Students will also learn to solve formal inequalities for the first time, learning the meaning of a solution set and exploring the similarities and differences compared to solving equations. Emphasis is placed on both forming and solving equations rather than just looking at procedural methods of finding solutions.

National curriculum content covered:

- identify variables and express relationships between variables algebraically
- begin to model situations mathematically and express the results using a range of formal mathematical representations
- substitute numerical values into formulae and expressions, including scientific formulae
- understand and use the concepts and vocabulary of expressions, equations, inequalities, terms and factors
- simplify and manipulate algebraic expressions to maintain equivalence by:
  - collecting like terms
  - multiplying a single term over a bracket
  - taking out common factors
  - expanding products of two or more binomials
- · understand and use standard mathematical formulae
- use algebraic methods to solve linear equations in one variable

#### Week 5: Sequences

This short block reinforces students' learning from the start of Year 7, extending this to look at sequences with more complex algebraic rules now that students are more familiar with a wider range of notation. The higher strand includes finding a rule for the n<sup>th</sup> term for a linear sequence, using objects and images to understand the meaning of the rule.

National curriculum content covered:

- generate terms of a sequence from either a term-to-term or a position-toterm rule
- recognise arithmetic sequences and find the n<sup>th</sup> term
- recognise geometric sequences and appreciate other sequences that arise

#### Week 6: Indices

Before exploring the ideas behind the addition and subtraction laws of indices (which will be revisited when standard form is studied next term), the groundwork is laid by making sure students are comfortable with expressions involving powers, simplifying e.g.  $3x^2y \times 5xy^3$ . The higher strand also looks at finding powers of powers.

National curriculum content covered:

- use and interpret algebraic notation, including  $a^3$  in place of  $a \times a \times a$ ;  $a^2b$  in place of  $a \times a \times b$
- use language and properties precisely to analyse algebraic expressions
- begin to model situations mathematically and express the results using a range of formal mathematical representations
- substitute values in expressions, rearrange and simplify expressions, and solve equations



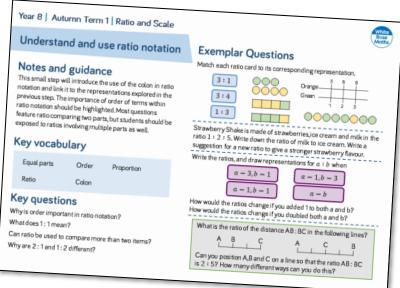
## Why Small Steps?

We know that breaking the curriculum down into small manageable steps should help students to understand concepts better. Too often, we have noticed that teachers will try and cover too many concepts at once and this can lead to cognitive overload. We believe it is better to follow a "small steps" approach.

As a result, for each block of content in the scheme of learning we will provide a "small step" breakdown. It is not the intention that each small step should last a lesson – some will be a short step within a lesson, some will take longer than a lesson. We would encourage teachers to spend the appropriate amount of time on each step for their group, and to teach some of the steps alongside each other if necessary.

#### What We Provide

- Some *brief guidance* notes to help identify key teaching and learning points
- A list of *key vocabulary* that we would expect teachers to draw to students' attention when teaching the small step,
- A series of **key questions** to incorporate in lessons to aid mathematical thinking.
- A set of questions to help *exemplify* the small step concept that needs to be focussed on.

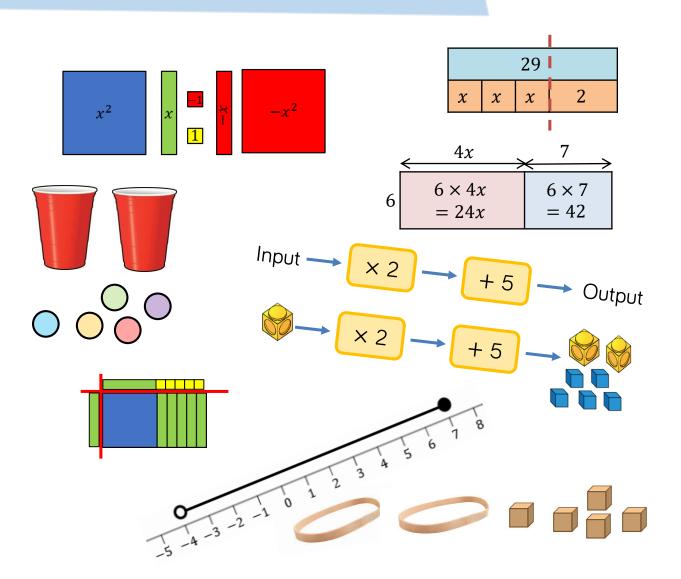


- These include reasoning and problem-solving questions that are fully integrated into the scheme of learning. Depending on the attainment of your students, you many wish to use some or all of these exemplars, which are in approximate order of difficulty. Particularly challenging questions are indicated with the symbol
- For each block, we also provide ideas for key representations that will be useful for all students.

In many of the blocks of material, some of the small steps are in **bold**. These are content aimed at higher attaining students, but we would encourage teachers to use these with as many students as possible – if you feel your class can access any particular small step, then please include it in your planning.



#### **Key Representations**



Here are a few ideas on how you might represent algebraic expressions and the solutions of equations and inequalities.

Cups, cubes and elastic bands lend themselves well to representing an unknown, whereas ones (from Base 10) and counters work well to represent a known number. Be careful to ensure that when representing an unknown, students use equipment that does not have an assigned value – such as Base 10 equipment and dice.

Bar models are useful to support the forming of equations and also help students to make sense of the approach to a solution. Algebra tiles are also very powerful for this and help to make sense of multiplying brackets, as is the area model that also provides a good link to multiplication of two-digit numbers by 1-digit numbers.



# Brackets, Equations & Inequalities

### Small Steps

- Form algebraic expressions
- Use directed number with algebra
- Multiply out a single bracket
- Factorise into a single bracket
- Expand multiple single brackets and simplify
- Expand a pair of binomials
  - Solve equations, including with brackets
- Form and solve equations with brackets
- Understand and solve simple inequalities
  - H
- denotes higher strand and not necessarily content for Higher Tier GCSE



# Brackets, Equations & Inequalities

#### Small Steps

- Form and solve inequalities
- Solve equations and inequalities with unknowns on both sides

Form and solve equations and inequalities with unknowns on both sides

H

Identify and use formulae, expressions, identities and equations



denotes higher strand and not necessarily content for Higher Tier GCSE



#### Form algebraic expressions

#### Notes and guidance

This step revises the basic algebraic notation students have met in Year 7. Students may need reminding that  $\times$  and  $\div$  signs should not appear in algebraic expressions, numbers are written before letters and that e.g. aa is written  $a^2$ . Students could revisit the use of function machines and explore the use of algebraic expressions within any other area that needs revising e.g. probability.

#### Key vocabulary

Expression Simplify Term

Substitute Coefficient Equivalent

#### **Key questions**

What is the difference between a term and an expression? When can/can't an expression be simplified? Spot the mistake(s) in this expression e.g. 6ff, 3a4b. Why are e.g. q-4 and 4-q not equivalent?

#### **Exemplar Questions**

k is a number. Write an expression for the number that is,

 $\blacksquare$  Five more than k

 $\blacksquare$  One third of k

 $\blacksquare$  Four multiplied by k

 $\blacksquare$  Seven less than k

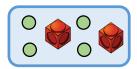
 $\blacksquare$  The difference between k and 10

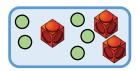
Give your answers in correct algebraic notation.

Compare your answers with a partner's, do you have different but equivalent expressions? Check by substituting in values for k and comparing your answers.

On each card, a cube represents 3p and a counter represents 2q Write an expression for the total of each card, giving your answers as simply as possible.

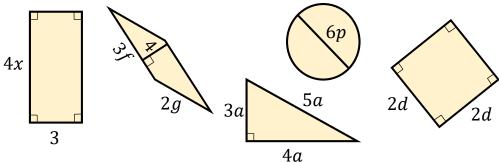






Repeat if the cube now represents 5p + 2 and the counter 3q - 1

Write simplified expressions for the perimeter and area of each shape.





#### Use directed number with algebra

#### Notes and guidance

This step revisits the use of directed number and substitution into algebraic expressions, both of which were covered in Year 7, in preparation for the more complex expressions coming up later in this block. Double-sided counters are very helpful to support understanding of the four operations with directed numbers. Entering negative numbers on a calculator is not always obvious and may need modelling by the teacher.

#### Key vocabulary

Positive	Negative	Directed
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Substitute Solve Simplify

#### **Key questions**

Why is it not true that 'two minuses make a plus'?

How do we enter...on a calculator?

Which order do we perform operations when substituting numbers into an expression? Why?

Is e.g.  $2x^2$  always, sometimes or never the same value as  $(2x)^2$ ?

#### **Exemplar Questions**

Work out the value of these expressions when a=2 and b=-4

Now find the values again this time using a=-2 and b=-4 Which expressions give the same answer as before? Why?

Mo is answering the question on the card.

Find the value of  $x^2$  when x = -2.5

Mo enters  $-2.5^2$  into his calculator and is surprised to get the answer -6.25 as he thinks the answer should be positive. Discuss why the calculator shows a negative answer.

Solve the equations.

$$x + 1.7 = 6.8$$
  $x - 1.7 = -6.8$   $6.8 = 1.7x$   $\frac{x}{1.7x - 5.1 = -6.8}$   $6.8 = 2x - 1.7$ 

Simplify the expressions on the cards.

$$3p + 4p - 8p \qquad -3p + 4p - 8p \qquad -3p - 4p - 8p$$

$$3 \times -4p \qquad \boxed{-3 \times -4p} \qquad \boxed{-3 \times 4p} \qquad \boxed{-3 \times -4p \times -2}$$



#### Multiply out a single bracket

#### Notes and guidance

It is useful to represent the expansion of brackets in many forms making links to number work in particular through the use of the area model. As well as including all combinations of + and - signs, examples should include those where the multiplier is a constant e.g. just 5, a variable e.g. just x or more complex e.g. 3a. Examples involving more than two terms inside the bracket are also useful to include.

#### Key vocabulary

Expand	Multiply out	Coefficient
Bracket	Identity	Product

#### **Key questions**

What does expand mean when we are working with brackets?

What's the link between multiplication and repeated addition?

Is it possible to have three or more terms inside a bracket? How would this look as a diagram?

#### **Exemplar Questions**

Annie is working out  $6 \times 82$  and reasons she can do the same with any number x

$$6 \times 82$$
=  $6 \times (80 + 2)$ 
=  $6 \times 80 + 6 \times 2$ 
=  $480 + 12$ 
=  $491$ 

$$6 \times (a+2)$$

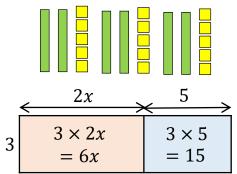
$$= 6 \times a + 6 \times 2$$

$$= 6a + 12$$

Compare the value of  $6 \times (a + 2)$  with the value of 6a + 12 for different values of a (positive, negative, fractions, decimals) Do they always have the same value?

Explain how these representations show that 3(2x + 5) = 6x + 15

2	2 <i>x</i> -	+ 5	2	2 <i>x</i> -	+ 5	2	2 <i>x</i> -	+ 5
x	x	5	x	x	5	х	x	5
6x + 15								





Expand these brackets.

$$3(x + 5)$$

$$3(x - 5)$$

$$-3(x+5)$$

$$-3(x-5)$$

$$3(5 + x)$$

$$3(5 - x)$$

$$x(x+5)$$

$$2x(5-x+y)$$



#### Factorise into a single bracket

#### Notes and guidance

Students do not always link factorising expressions with looking for factors of numbers, so it is useful to be explicit about the similarities. This helps to reinforce the language of common factor and highest common factor, and these topics could be revisited during starters. When factorising into a single bracket, again using a variety of signs and types of terms (numerical, algebraic) is useful.

#### Key vocabulary

Factor Factorise Factorise fully

Common Common factor HCF

#### **Key questions**

Is it useful to have 1 as a common factor? Why/Why not? Can 0 ever be a factor of an expression?

What do you look for to find the highest common factor of a set of terms?

Is it always true that if you can't halve an expression then the expression doesn't factorise?

#### **Exemplar Questions**

List the factors of the numbers or expressions on each card.

10

18

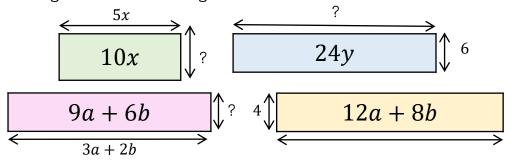
3*x* 

 $\chi^2$ 

 $2x^2$ 

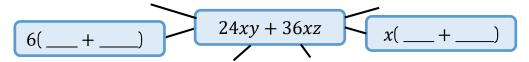
6*xy* 

The area and the length of one of the sides is given for each of the rectangles. Find the missing sides.



Complete the factorisations.

$$6x + 9y \equiv 3( ___ + ___ )$$
  $4x - 6y \equiv 2( ___ + ___ )$   
 $xy + 7x \equiv x( ___ + ___ )$   $a^2 + ab + 6a \equiv __ (a + b + 6)$   
 $12pq - 15qt = __ (4p - 5t)$   $20d^2 + 5d \equiv __ ( __ + 1)$ 



- How many ways can you find to factorise the expression?
- Fully factorise the expression.



#### Simplify multiple single brackets

#### Notes and guidance

Students often only expand and simplify expressions of the form  $3(x\pm 4)\pm 4(x\pm 5)$  and make errors with shorter expressions like  $3\pm 4(x\pm 5)$ . Using concrete manipulatives to 'build' the expressions is a useful way of developing understanding of the difference between similar looking expressions. Careful choice of numbers in examples and exercises, and varying numbers and signs is also helpful.

#### Key vocabulary

Expression Simplify Like terms

Unlike terms Expand Equivalent

#### **Key questions**

How do we write 1x'?

Is it possible to simplify an expression and end up with the answer O?

Does the order in which we work out expansions matter?

#### **Exemplar Questions**

Without expanding the brackets, decide whether you think these expressions will be equivalent or not.

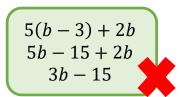
$$3(x+4)+2$$
  
 $2+3(x+4)$ 

$$2x + 3(x + 4)$$
  
 $3(x + 4) + 2$ 

Checking by building the expressions using algebra tiles. Simplify the expressions and compare with your concrete versions.

Ron has made mistakes in both these simplifications.

$$5 + 3(a + 6) \\
8(a + 6) \\
8a + 48$$



Explain Ron's errors and work out the correct answers.

Expand and simplify the expressions.

$$3(5a+2)+4(2a+3)$$

$$3(5a+2)-4(2a+3)$$

$$3(5a+2)+4(2a-3)$$

$$3(5a-2)-4(2a+3)$$

$$3(5a-2)+4(2a-3)$$

$$3(5a-2)-4(2a-3)$$

$$3(5a-2)-5(3a-2)$$

$$3(4a-2)-2(6a-3)$$



#### Expand a pair of binomials



#### Notes and guidance

The vocabulary binomial (the sum or difference of two terms) and quadratic (an expression where the highest power of the variable is 2) will be new to most students. Concrete and pictorial ways of finding the expansions will support written methods which will be developed over the coming years. Students need to be confident with simplification and dealing with negative numbers for this higher strand step.

#### Key vocabulary

Binomial Simplify Like terms

Unlike terms Expand Quadratic

#### **Key questions**

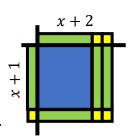
Why do you get four terms when you multiply two binomials?

Why can you simplify some quadratic expressions to three or fewer terms, but not others?

Do simplified quadratics always have three terms?

#### **Exemplar Questions**

Explain how the algebra tiles show that  $(x + 1)(x + 2) \equiv x^2 + 3x + 2$ 



Use algebra tiles to expand the expressions.

$$(x+4)(x+3)$$

$$(x+3)(x+4)$$

$$(x + 4)(x - 3)$$

$$(x+3)(x-4)$$

$$(x+3)^2$$

Here is Tommy's method for working out  $62 \times 43$  by thinking of the calculation as  $(60 + 2) \times (40 + 3)$ 

×	60	2
40	2400	80
3	180	6

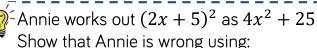
$$2400 + 180 + 80 + 6 = 2666$$

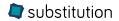
Complete this adaptation of Tommy's method to work out (a + 3)(b + 4)

×	а	3
b	ab	3 <i>b</i>
4		

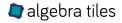
$$ab + 3b + ...$$

What would be different if Tommy was working out (a + 3)(a + 4)?











#### Solve equations with brackets

#### Notes and guidance

Solving one-step and two-step equations should be secure before moving on to working with equations with brackets. 'Think of a number' problems are a good introduction (see next step), but students should also deal with equations with non-integer solutions (using a calculator when necessary) to avoid reliance on 'spotting' solutions. Alternate methods should be explored as exemplified in the final question.

#### Key vocabulary

Solve Equation Unknown Coefficient Expand Solution

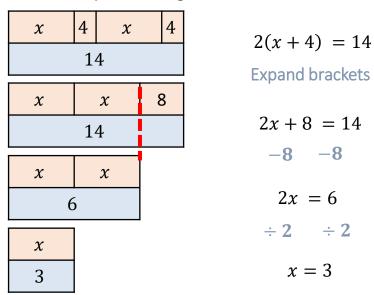
#### Key questions

Do you have to expand the brackets to solve the equation? Which order do we need to carry out the steps when solving an equation?

How many solutions will the equation have?

#### **Exemplar Questions**

Whitney uses bar models to solve 2(x + 4) = 14She explains her steps on the right hand side.



$$3(x+2.7)=4.5$$

$$3(x+2.7) = 4.5$$
  $12 = 2(x-3)$   $6(e-1) + 2e = 10$ 

$$\bigcirc$$
 6( $e - 1$ ) + 2 $e = 10$ 

Compare these solutions of the equation 3(x + 5) = 12Explain the steps in each method. Which approach do you prefer?

$$3(x + 5) = 12$$

$$3x + 15 = 12$$

$$3x = -3$$

$$x = -1$$

$$3(x+5) = 12$$
$$x+5=4$$
$$x=-1$$



#### Form/solve equation with brackets

#### Notes and guidance

'Think of a number' problems and flowcharts are good models to support students to distinguish between e.g. 2x+3 and 2(x+3). It is worth investing class time developing students' skills in forming equations as the mechanics of solving can be more easily practised as homework. It is also useful to interleave other topics here e.g. forming equations to find missing angles on a straight line, missing probabilities etc.

#### Key vocabulary

Equation	Side	Form
Solve	Unknown	Check

#### Key questions

What is different about 2x + 3 and 2(x + 3)? What is the first step you need to think about when forming an equation from a worded problem? How can we check if the answer to the equation is correct?

#### **Exemplar Questions**

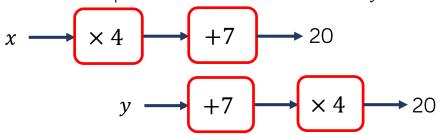
Compare the two number puzzles.

I think of a number.
I add on 6
I double my answer.
Now I'm thinking of 18
What was my original number?

I think of a number.
I double it.
I add on 6 to my answer.
Now I'm thinking of 18
What was my original number?

What's the same and what's different? What were the numbers? Represent the puzzles using concrete materials, pictures and equations to check your answers.

Write and solve equations to find the values of x and y



The area of the rectangle is 72 cm<sup>2</sup>

Work out the value of x and hence find the perimeter of the rectangle.



#### Simple inequalities

#### Notes and guidance

Students will be familiar with the inequality signs from earlier work on comparison, but solving inequalities and the idea of a solution set (as opposed to a single value) will be new to most. It is worth discussing that e.g. x > 7 and 7 < x mean the same; reading the inequalities aloud is helpful in determining meaning. Students sometimes replace the given sign with an equals sign; this is error-prone and should be discouraged.

#### Key vocabulary

Inequality Satisfy Solution set

Solve Greater/less than (or equal)

#### **Key questions**

What's the same and what's different about solving an equation or an inequality?

How many solutions does an inequality have?

How can we check our solution to an inequality is correct?
What values would be useful to test with?

#### **Exemplar Questions**

Which of the inequalities does the number 7.5 satisfy?

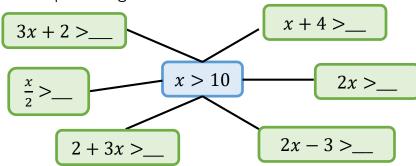
$$x > 7 \qquad \boxed{7 < x \qquad \boxed{7 \le x \qquad x < 8 \qquad x \ge 8}}$$

What's the same and what's different about the inequalities?

Mo says "If x > 10, then x + 1 > 11"

Explain why Mo is right.

Complete the spider diagram.



If x > 10, which of the cards are true and which are false?

$$\boxed{10 < x} \qquad \boxed{40 < 4x} \qquad \boxed{-x > -10}$$

Solve the inequalities.

$$x + 2 > 7$$
  $x + 2 > -7$   $x - 2 > 7$   $x - 2 > -7$   $x + 2 < 7$   $x + 2 < -7$   $x - 2 \le 7$   $x - 2 \ge -7$ 

$$2x + 2 < 7$$
  $4x + 2 \ge -7$   $3 + 5x \le 7$ 



#### Form and solve inequalities

#### Notes and guidance

Teacher modelling is again important here, as students often find forming equations/inequalities from given information difficult; class time can be spent just forming the inequalities with the solving left to later in the lesson and/or homework. Consideration needs to be given as to whether the full solution set or only particular integers are required. It is also worth discussing which values could be chosen to test whether the solution set is correct.

#### Key vocabulary

Solution Inequality Form

Solve Unknown Check

#### **Key questions**

Which way round will the inequality sign point in this question? Why?

What does integer mean? How does this change the question?

How can we check our solution to an inequality is correct? What values would be useful to test with?

#### **Exemplar Questions**



Three more than double my number is greater than 10

Write an inequality and solve it to find the possible range of values for Whitney's number.

What is the smallest integer Whitney could be thinking of?

Annie has £100

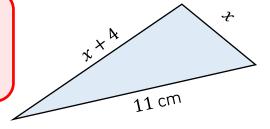
She wants to buy three T-shirts and a jumper.

The jumper costs £45, and she doesn't have enough money to buy everything she wants.

What can be worked out about the price of the T-shirts?

Explain why you cannot make a triangle with three sides of lengths 4 cm, 5 cm and 12cm.

This triangle had sides x, x + 4 and 11 cm. Work out the range of possible values of x.





#### Unknowns on both sides

#### H

#### **Exemplar Questions**

#### Notes and guidance

Students should be familiar with the 'balance' method of solving equations that lends very well to equations of this form. It is important that students consider three-term as well as four-term equations, and as ever deal with a variety of letters and positions. As well as the bar model illustrated, cups and counters is a good model for unknowns on both sides, alongside the abstract method so students make sense of the method for more difficult equations.

### Key vocabulary

Equation	Balance	Side

Solve Unknown Check

#### **Key questions**

How can we check our solution to an equation is correct? When solving a four-term equation, why is it better to deal with the letters before the numbers? Do we always start solving equations by subtracting something from both sides? Why or why not?

x	х	х	х	12
x	x	18		

Write down the equation shown by the bar model.

x	x	x	x	12
x	х		1	8

Write down the new equation if the two left-most xs are removed from the bars. Work out the value of x.

Use the bar model to help you complete the workings to find the value of y.

$$5y = 3y + 15$$

$$-3y$$
  $-3y$ 

$$2y = 15$$
 etc.

у	y	y	у	y
у	y	у	1	5

Solve the equations.

$$5x + 1 = 71$$

$$5x + 1 = 7x$$

$$5x + 1 = 2x + 7$$

$$17 = 4x - 3$$

$$2x = 4x - 3$$

$$2x + 1 = 4x - 3$$

What's the same what's different about how you approach them?



### Complex equations/inequalities •

#### Notes and guidance

Building on the earlier steps of this block, students can explore equations and inequalities with brackets and unknowns on both sides. It is important that students only move on to this step when they have a good understanding of the earlier material. With complex equations and inequalities it is more important than ever to check the answer by substituting back into the original problem.

#### Key vocabulary

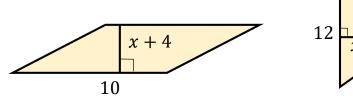
Equation	Inequality	Form
Solve	Unknown	Check

#### **Key questions**

Can you think of an equation with more than one solution?
Can an inequality have more than one solution?
Describe the steps you need to take to solve.....
Does the order of the steps matter? Why?
How do you form an equation from a worded problem?
What do you need to decide first?

#### **Exemplar Questions**

The area of the parallelogram is twice the area of the trapezium. Work out the total area of the two shapes.



12 x 6

Dora and Amir are both given the same starting number.



I triple the number and add on seven I add two to the number and then multiply by 4



Dora's answer is less than Amir's.

Is it possible that the starting number was negative? If so, give an example.

Verify, by substitution, that x=3 is the solution to the equation.

$$7x + 3(2x - 4) = 4(2x + 4) - 2(3x - 8)$$

Now solve the equation algebraically.

Esther adds together three consecutive even numbers. Her total is less than 80. Use an algebraic method to work out the greatest of Esther's three numbers.



#### Identify algebraic constructs

#### Notes and guidance

In this step, students have the opportunity to practise distinguishing between expressions, equations, formulas (or formulae) and identities. It is particularly important that students know that an identity is true for all values of the variable(s) and includes the symbol  $\equiv$ , whereas an equation can be solved to find particular values. A formula is distinguished from an equation as it can be used to find particular values of the subject.

#### Key vocabulary

Expression	Identity	Formula
Equation	Equivalent	Variable
Subject	Substitute	

#### Key questions

What do we mean by an expression?
How could you change an expression into a formula or an equation? What symbol do we use to show an identity?
Can an equation have more than one variable?
What formulas do you use in other subjects?

#### **Exemplar Questions**

What's the same and what's different about the cards?

$$2(a+b)$$

$$P = 2(a+b)$$

$$2(a+b) \equiv 2a + 2b$$

Which of these formulas do you recognise? Explain the meaning of all the variables in the formulas.

$$C = \pi d$$

$$P = 2l + 2w$$

$$A = \frac{1}{2}bh$$

$$P = 4s$$

$$S = \frac{D}{T}$$

$$A = lw$$

Investigate how the value of the subject of the formula changes as the other variables change.

An equation is anything with an equals sign.



Rosie

Explain why Rosie is wrong.

Which of these cards show equations? What do the other cards show?

$$3a + 2(a + 5) = 25$$

$$v = u + at$$

$$5x + 6x \equiv 11x$$

$$b \times b \times b \equiv b^3$$

$$10 = \frac{p}{2} - 3$$

$$6 - m = 2$$

$$\frac{1}{2}(a+b)h$$

$$a^2 + b^2 = c^2$$

$$6 - m = F$$