Expanding & factorising

Year 11

#MathsEveryoneCan

White Rose Maths

	Week 1	Week 2	Week 3	Week 4	Week 5	Week 6	Week 7	Week 8	Week 9	Week 10	Week 11	Week 12
Autumn	**Graphs**						**Algebra**					
	Gradients & lines		Non-linear graphs		Using graphs		Expanding & factorising		Changing the subject		Functions	
Spring	**Reasoning**						**Revision and Communication**					
	Multiplicative		Geometric		Algebraic		Transforming & constructing		Listing & describing		Show that…	
Summer	**Revision**						**Examinations**					

Autumn 2: Algebra

Weeks 1 and 2: Expanding and factorising

This block reviews expanding and factorising with a single bracket before moving on to quadratics. The use of algebra tiles to develop conceptual understanding is encouraged throughout. Context questions are included to revisit e.g. area and Pythagoras' theorem.

National Curriculum content covered includes:

- know the difference between an equation and an identity; argue mathematically to show algebraic expressions are equivalent, and use algebra to support and construct arguments **{and proofs}**
- simplify and manipulate algebraic expressions by: factorising quadratic expressions of the form $x^2 + bx + c$, including the difference of two squares; **{factorising quadratic expressions of the form $ax^2 + bx + c$}**
- know the difference between an equation and an identity; solve quadratic equations **{including those that require rearrangement}** algebraically by factorising, **{by completing the square and by using the quadratic formula}**
- identify and interpret roots; deduce roots algebraically **{and turning points by completing the square}**
- solve two simultaneous equations in two variables (linear/linear **{or linear/quadratic}**) algebraically; find approximate solutions using a graph

Weeks 3 and 4: Changing the subject

Students consolidate and build on their study of changing the subject in Year 9. The block begins with a review of solving equations and inequalities before moving on to rearrangement of both familiar and unfamiliar formulae. Checking by substitution is encouraged throughout. Higher tier students also study solving equations by iteration.

National Curriculum content covered includes:

- solve linear inequalities in one variable
- know the difference between an equation and an identity; argue mathematically to show algebraic expressions are equivalent, and use algebra to support and construct arguments **{and proofs}**
- translate simple situations or procedures into algebraic expressions or formulae; derive an equation (or two simultaneous equations), solve the equation(s) and interpret the solution
- **{find approximate solutions to equations numerically using iteration}**

Weeks 5 and 6: Functions

As well as introducing formal function notation, this block brings together and builds on recent study of quadratic functions and graphs. This is also an opportunity to revisit trigonometric functions, first studied at the start of Year 10.

National Curriculum content covered includes:

- where appropriate, interpret simple expressions as functions with inputs and outputs; **{interpret the reverse process as the 'inverse function'; interpret the succession of two functions as a 'composite function'}**
- solve two simultaneous equations in two variables (linear/linear **{or linear/quadratic}**) algebraically; find approximate solutions using a graph
- identify and interpret roots; deduce roots algebraically **{and turning points by completing the square}**
- solve linear inequalities in one **{or two}** variable**{s}**, **{and quadratic inequalities in one variable}**; represent the solution set on a number line, **{using set notation and on a graph}**
- recognise, sketch and interpret graphs of quadratic functions
- apply Pythagoras' Theorem and trigonometric ratios to find angles and lengths in right-angled triangles **{and, where possible, general triangles}** in two **{and three}** dimensional figures

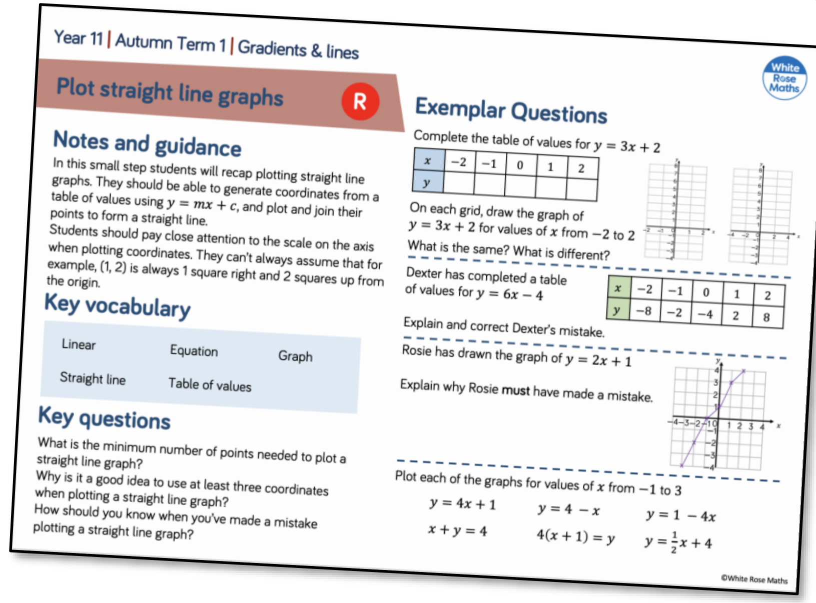
Why Small Steps?

We know that breaking the curriculum down into small manageable steps should help students to understand concepts better. Too often, we have noticed that teachers will try and cover too many concepts at once and this can lead to cognitive overload. We believe it is better to follow a "small steps" approach.

As a result, for each block of content in the scheme of learning we will provide a "small step" breakdown. ***It is not the intention that each small step should last a lesson – some will be a short step within a lesson, some will take longer than a lesson.*** We would encourage teachers to spend the appropriate amount of time on each step for their group, and to teach some of the steps alongside each other if necessary.

What We Provide

- Some ***brief guidance*** notes to help identify key teaching and learning points
- A list of ***key vocabulary*** that we would expect teachers to draw to students' attention when teaching the small step,
- A series of ***key questions*** to incorporate in lessons to aid mathematical thinking.
- A set of questions to help ***exemplify*** the small step concept that needs to be focussed on.



Year 11 | Autumn Term 1 | Gradients & lines

Plot straight line graphs R

Notes and guidance

In this small step students will recap plotting straight line graphs. They should be able to generate coordinates from a table of values using $y = mx + c$, and plot and join their points to form a straight line.
Students should pay close attention to the scale on the axis when plotting coordinates. They can't always assume that for example, (1, 2) is always 1 square right and 2 squares up from the origin.

Key vocabulary

Linear, Equation, Graph, Straight line, Table of values

Key questions

What is the minimum number of points needed to plot a straight line graph?
Why is it a good idea to use at least three coordinates when plotting a straight line graph?
How should you know when you've made a mistake plotting a straight line graph?

Exemplar Questions

Complete the table of values for $y = 3x + 2$

x	-2	-1	0	1	2
y					

On each grid, draw the graph of $y = 3x + 2$ for values of x from -2 to 2
What is the same? What is different?

Dexter has completed a table of values for $y = 6x - 4$

x	-2	-1	0	1	2
y	-8	-2	-4	2	8

Explain and correct Dexter's mistake.

Rosie has drawn the graph of $y = 2x + 1$

Explain why Rosie **must** have made a mistake.

Plot each of the graphs for values of x from -1 to 3

$y = 4x + 1$ $\quad y = 4 - x$ $\quad y = 1 - 4x$

$x + y = 4$ $\quad 4(x + 1) = y$ $\quad y = \frac{1}{2}x + 4$

- These include reasoning and problem-solving questions that are fully integrated into the scheme of learning. Depending on the attainment of your students, you may wish to use some or all of these exemplars, which are in approximate order of difficulty. Particularly challenging questions are indicated with the symbol .
- For each block, we also provide ideas for key representations that will be useful for all students.

Some of the small steps are in **bold** and labelled with H to indicate this is Higher tier GCSE content. We would encourage teachers to use these with as many students as possible – if you feel your class can access any particular small step, then please include it in your planning. Steps that review content covered at Key Stage 3 are labelled R .

Expanding and factorising

Small Steps

- Expand and factorise with a single bracket (R)
- Expand binomials (R)
- Factorise quadratic expressions
- **Factorise complex quadratic expressions** (H)
- Solve equations equal to 0
- Solve quadratic equations by factorisation
- **Solve complex quadratic expressions by factorisation** (H)
- **Complete the square** (H)

H denotes Higher Tier GCSE content

R denotes 'review step' – content should have been covered at KS3

Expanding and factorising

Small Steps

- Solve quadratic equations using the quadratic formula (H)

(H) denotes Higher Tier GCSE content

(R) denotes ‘review step’ – content should have been covered at KS3

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Single bracket

R

Notes and guidance

This reviews concepts covered in Key Stage 3. Illustrate expanding a single bracket using the area model (e.g. rectangle, length of 5 and width of $x + 3$) or by using algebra tiles. Factorise numbers (e.g. $24 = 12 \times 2$) before algebraic expressions to make the link between factors and factorising. Students need to be careful to find the highest common factor of the terms in an expression in order to factorise fully.

Key vocabulary

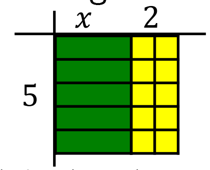
Expand	Factorise	Multiply out
Coefficient	Bracket	Identity
HCF	Factorise fully	

Key questions

What is the link between multiplication and repeated addition?
Is it possible to have three or more terms inside a bracket?
What do you look for to find the HCF of a set of terms?
Is it always true that if you can't halve an expression then the expression doesn't factorise?

Exemplar Questions

Aisha uses algebra tiles to expand $5(x + 2)$.



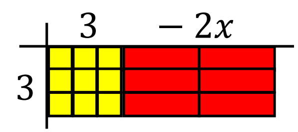
$$5(x + 2) \equiv 5x + 10$$

Expand the brackets.

- $5(2 - x)$
- $-5(2 + x)$
- $-5(2 - x)$
- $-(2 - x)$
- $x(x + 2)$
- $x^2(2 + x)$

Show that $5(2 - x) - (2 - x) \equiv 8 - 4x$.

Dani factorises $9 - 6x$ using algebra tiles.



$$9 - 6x \equiv 3(3 - 2x)$$

Factorise.

- $9 + 6x$
- $12x + 4$
- $-15 - 10x$
- $x^2 + 6x$

Find the highest common factor of each pair.

- 3 and 15 $3a$ and $15b$
- 24 and 36 $2rs$ and $3sr$
- 4 and 16 x and x^2
- 5×5×3 and 5×3×3 r^2s and rs^2

Factorise.

- $3a + 15b$
- $x^2 - x$
- $2rs - 3sr$
- $r^2s + rs^2$

$$12a^2 + 18ab - 24a \equiv 2(6a^2 + 9ab - 12a)$$

Why doesn't this show the full factorisation of $12a^2 + 18ab - 24a$?
Fully factorise the expression.

Expand binomials

R

Notes and guidance

Here we revisit the meaning of binomial and quadratic, and use the area model as a visual prompt for discussion on how to expand binomials. Concrete resources such as algebra tiles are useful in supporting student confidence in this step. Students need to be confident with simplification and dealing with negative numbers. Where appropriate, extend to contexts where students generate the binomials and then manipulate them.

Key vocabulary

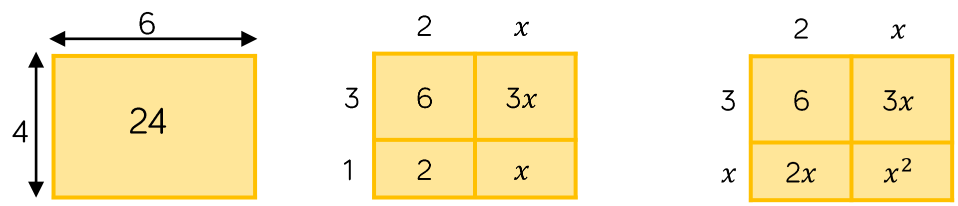
Binomial	Simplify	Like/unlike terms
Expand	Quadratic	Difference of two squares

Key questions

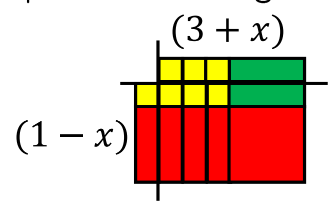
Why do you get four terms when you multiply two binomials?
Why can you simplify some quadratic expressions to three or fewer terms, but not others?
Do simplified quadratics always have three terms?
What happens when a single bracket is squared?

Exemplar Questions

Compare the diagrams. What is the same and what is different?



Explain how the algebra tiles show that.



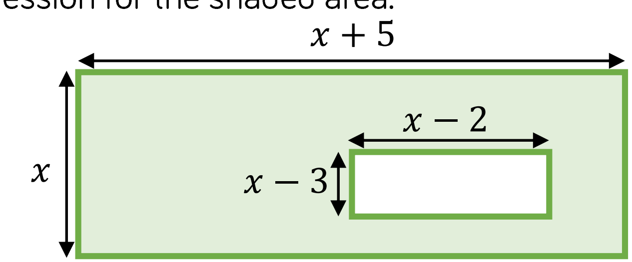
$$(3+x)(1-x) \equiv 3 + x - 3x - x^2$$
$$(3+x)(1-x) \equiv 3 - 2x - x^2$$

Use algebra tiles to expand the expressions.

- $(x+2)(x+3)$
- $(2x+2)(x+3)$
- $(x+2)(x-3)$
- $(x-2)(x-3)$
- $(x+2)^2$
- $(x-3)^2$

Expand and simplify $(x+3)(x-3)$ and $(y-5)(y+5)$.
What do you notice? Can you generalise?

Find an expression for the shaded area.



Factorise quadratic expressions

Notes and guidance

High attaining students may have covered this step in Year 10. Here students need to link finding factors with factorisation. Students should understand that a quadratic expression has a maximum of two binomial factors. Students consider how the factors of the constant terms relate to the coefficient of the x term. Again, algebra tiles can be used. Finally, students should factorise quadratics with negative x terms or a negative constant.

Key vocabulary

Expression	Quadratic	Term
Coefficient	Factor	Factorise

Key questions

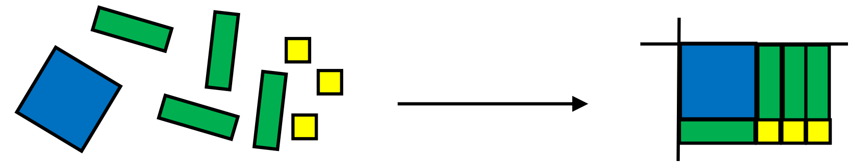
How can algebra tiles be used to show that a quadratic expression only has two factors?

How do the factors of the constant term relate to the coefficient of x?

Why is factorising e.g. $x^2 + 4x + 3$ different from factorising $x^2 + 4x$?

Exemplar Questions

Annie factorises $x^2 + 4x + 3$ using algebra tiles. What is her answer?

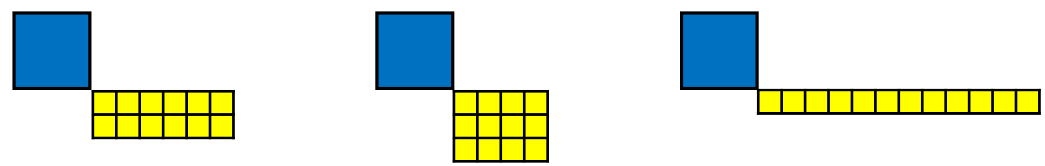


Can you make any different rectangles using Annie's algebra tiles?

List the factors of $x^2 + 4x + 3$

Use x tiles to complete each diagram to make a rectangle.

Write down the expression, and the factorisation that each represents.



What's the same and what's different about each one?

Mo says: "To factorise $x^2 + 8x + 12$ I need to think about which factors of 12 will give the x term a coefficient of 8".

Use the diagram to explain his thinking.

List the factors of −12

Factorise the expressions.

- $x^2 + x - 12$
- $x^2 - x - 12$
- $x^2 - 4x - 12$
- $x^2 + 4x - 12$
- $x^2 - 11x - 12$
- $x^2 + 11x - 12$

Complex quadratic expressions H

Notes and guidance

In this Higher tier step, students realise that both the factors of the coefficient of x^2 and the factors of the constant term need to be considered when factorising. Algebra tiles support this thinking. Students could then consider a more abstract approach to complex factorising by using trial and improvement to establish the correct combination of pairs of factors. Encourage students to expand the brackets to check the factorisation.

Key vocabulary

Term	Quadratic	Trial and improvement
Coefficient	Factor	Factorise

Key questions

Why is it efficient to start with the tiles representing x^2 and ones when forming the rectangle?

Why is the coefficient of x dependent on the factors of x^2 and the constant term?

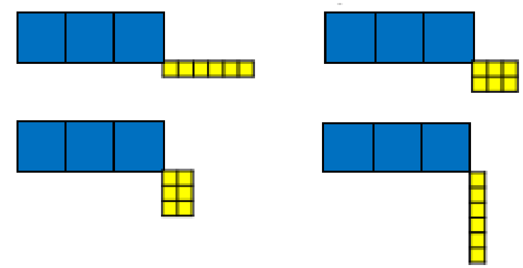
How can trial and improvement be used efficiently to factorise? Are there any pairs that obviously won't work?

Exemplar Questions

Eva is factorising $3x^2 + 11x + 6$

Using algebra tiles, she tries different factors of 6

Which arrangement works?



Eva says, "To factorise $3x^2 + 11x + 6$ I need to think about factors of 6 **and** factors of 3". Explain why Eva is right.

Factorise $3x^2 + 11x + 6$

Use algebra tiles to factorise.

- $2x^2 + 6x + 4$
- $4x^2 + 10x + 4$

Whitney is factorising $3x^2 + 11x - 20$

List the factors of −20

Try different pairs of factors in the brackets and expand.

$$(3x \ \square\ )(x \ \square\ ) \equiv 3x^2 \ \square\ x - 20$$

Which pair of factors in the expression give $11x$?

Factorise.

- $3x^2 - 28x - 20$
- $3x^2 - 32x - 20$
- $3x^2 - 17x - 20$
- $3x^2 + 59x - 20$

Solve equations equal to 0

Notes and guidance

The purpose of this small step is to prepare students for solving quadratics by factorisation. Firstly students practise solving linear equations equal to zero. They then need to understand that is the product of two numbers or terms is zero then at least one of the two numbers/terms must be zero. This supports understanding of why there are 2 solutions to a quadratic equation.

Key vocabulary

Expression	Term	Quadratic
Solve	Solutions	Product

Key questions

If two numbers/terms multiply to give 0, what do we know about one of the numbers/terms?

Why are there two solutions for x in equations such as $x(x+1)=0$?

How can we find each solution? How can we check the solutions are correct?

Exemplar Questions

Teddy is solving the equation $2 - x = 0$

Teddy says, "The answer must be 2 because 2 – 2 = 0 so $x = 2$"

Solve these equations.

- $x + 3 = 0$
- $x - 3 = 0$
- $3 - x = 0$
- $3 + x = 0$

Dexter and Amir are solving $1 - 3x = 0$

Dexter's method

$1 - 1 = 0$

So $3x = 1$

$x = \frac{1}{3}$

Amir's method

$1 - 3x = 0$

$1 = 3x$

$\frac{1}{3} = x$

+ 3x to both sides

÷ both sides by 3

Which method do you prefer? Use your chosen method to solve

- $3x - 1 = 0$
- $3x + 1 = 0$
- $2 - 3x = 0$

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a	b	ab
2		0
	2	0
$(x + 2)$		0
	$(x + 2)$	0

Complete the table.

What do you notice?

What do you know about a and b if $ab = 0$?

Solve these equations.

- $x(x+1) = 0$
- $(x+1)(x-1) = 0$
- $(2x+1)(x-1) = 0$

Solve quadratics by factorisation

Notes and guidance

It's important to emphasise the difference between factorising and solving. Some students try to solve when they are asked to factorise. Students should make links between the solutions of a quadratic equation and the roots of a quadratic. They should also form quadratic expressions and equations using given information. They should solve quadratic equations in a context and choose the most sensible solution given the context e.g. avoiding negative lengths.

Key vocabulary

Expression	Equation	Factorise
Solve	Solutions	Roots

Key questions

What's the difference between factorising and solving?
What's the difference between the roots of a quadratic equation and the solutions of the same quadratic equation? Explain your answer.
How is an expression different to an equation?
What do we mean by "the difference of two squares"?

Exemplar Questions

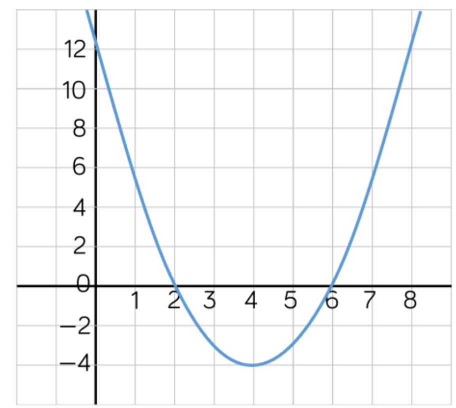
Dani solves $x^2 + 17x + 70 = 0$ and finds $x = 7$ or $x = -10$
Check her answers by substituting them into $x^2 + 17x + 70$
Which answer is incorrect? How do you know?
Solve $x^2 + 17x + 70 = 0$ correctly.

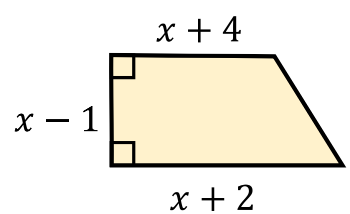
Show that one solution of $x^2 - 8x + 12 = 0$ is $x = 6$

Factorise $x^2 - 8x + 12$

Work out another possible solution for x.

The graph shows $y = x^2 - 8x + 12$
What are the roots of this equation?
What do you notice?





Show that the area of the trapezium is $x^2 + 2x - 3$
The area of the trapezium is 45 cm²
What equation can you form?
How can you find the value of x?
Are there one or two solutions?

Brett thinks you can't solve $x^2 - 36 = 0$ because it's impossible to factorise $x^2 - 36$ as it only has two terms.
Brett is wrong.
Find two ways to solve the equation $x^2 - 36 = 0$

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Solve complex quadratics

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Notes and guidance

Higher tier students need to solve quadratics where the coefficient of x^2 is greater than 1 by factorisation. Encourage students to make the link between the solutions of a quadratic and the roots illustrated on a graph. Explicitly discuss the possibility of simplifying some quadratic equations by dividing both sides by a common factor, but highlight that it is not possible to simplify an expression in the same way.

Key vocabulary

Quadratic equation	Factories	Solve
Simplify	Solutions	Roots

Key questions

Why do we often use fractions rather than decimals when writing solutions?
How can we check whether the solutions are correct?
How does this link to the graph of a quadratic equation?
Why can we sometimes simplify a quadratic equation, but not an expression?

Exemplar Questions

Eva is solving $2x^2 + 9x - 35 = 0$
Complete her workings.

$$(2x - 5)(x \ \square\ ) = 0$$

$2x - 5 = 0$ or $x \ \square = 0$

$2x = \square$ $\quad$ $x = \square$

$x = \frac{\square}{2}$

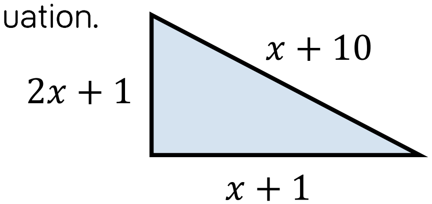
Solve the following quadratic equations.

- $3x^2 + 37x + 44 = 0$
- $3x^2 - x - 44 = 0$
- $3x^2 - 37x + 44 = 0$
- $3x^2 + x - 44 = 0$

Draw each graph using a dynamic geometry package.
What do you notice about the solutions and roots of each equation?

Ron is solving $4x^2 - 14x - 98 = 0$
He says that he can simplify the equation by dividing both sides by 2
Is he right? Factorise and solve this equation.

Here is a right-angled triangle.



Show that $4x^2 - 14x - 98 = 0$

Work out the perimeter of the triangle.

Complete the square

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Notes and guidance

Students use algebra tiles to understand the structure of completing the square. They understand why halving the coefficient of x is necessary when completing the square. Students should then be encouraged to work abstractly, perhaps with scaffolded activities. They then consider how to solve quadratic equations, where a common mistake is to take only the positive square root, therefore missing a solution.

Key vocabulary

Complete the square	Quadratic	In the form
Coefficient	Factorise	Solve

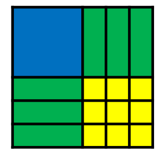
Key questions

Why is this method called completing the square?

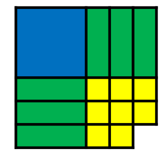
How does p in $(x+p)^2$ relate to the original expression/equation?

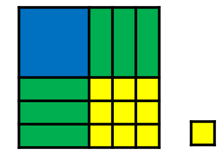
When solving, what's important to remember about square rooting both sides of the equation? How does this compare to the graph?

Exemplar Questions



Alex factorises $x^2 + 6x + 9$ using algebra tiles.
Write down the factorised expression.
How does the shape connect to the factorised expression?





Explain how the diagrams show that
$x^2 + 6x + 8 \equiv (x+3)^2 - 1$ and $x^2 + 6x + 10 \equiv (x+3)^2 + 1$

Use algebra tiles, to write the expressions in the form $(x+a)^2 + b$.

- $x^2 + 4x + 5$
- $x^2 + 6x + 7$
- $x^2 - 8x + 19$

What connections can you see between the expressions and the "completed square" form?

Complete the workings.

$$x^2 + 3x - 9 \equiv (x - \square)^2 - \frac{9}{4} - 9$$
$$x^2 + 3x - 9 \equiv (x - \square)^2 - \frac{9}{4} - \frac{\square}{4}$$
$$x^2 + 3x - 9 \equiv (x - \square)^2 - \frac{\square}{4}$$

Spot the error.

$$x^2 + 8x + 6 = 0$$
$$(x+4)^2 - 10 = 0$$
$$(x+4)^2 = 10$$
$$(x+4) = \sqrt{10} \therefore x = \sqrt{10} - 4$$

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Using the quadratic formula

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Notes and guidance

Teachers could introduce this step by exploring the derivation of the formula. Students use the quadratic formula to solve equations both with and without a calculator. Errors in substitution often occur when b is negative; this needs highlighting. Students should be encouraged to breakdown the calculation, even when using a calculator, as this minimises error. They may need to practise simplifying surds before using the quadratic formula without a calculator.

Key vocabulary

Formula	Substitute	Surd
Simplify	Significant figures	

Key questions

How do we know which number to substitute into the formula?

Why do we need to be careful, particularly if b is negative?

Why should we always calculate in more than one step?

If we're not using a calculator, how do we simplify our answer?

Exemplar Questions

Complete the method to solve $3x^2 + 8x - 5 = 0$ using the quadratic formula. Give your answer to 3 significant figures.

$$x = \frac{-8 \pm \sqrt{(64 - 4 \times \square \times \square)}}{2 \times \square}$$

$$x = \frac{-8 \pm \sqrt{\square}}{\square}$$

$x = 0.523$ or $x = \square$

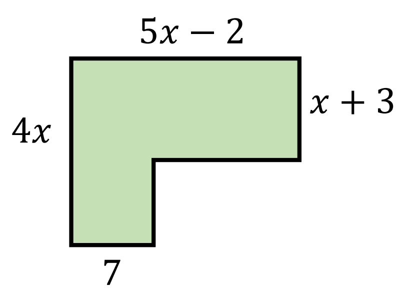
Huan is solving $2x^2 - 6x + 3 = 0$ using the quadratic formula. Find all of his errors.

$$x = \frac{-6 \pm \sqrt{(36 - 4 \times 2 \times 3)}}{2}$$

$$x = \frac{-6 \pm \sqrt{12}}{2}$$

$$x = -6 \pm \sqrt{6}$$

Correct his method and show that $x = \frac{3 \pm \sqrt{3}}{2}$



On the diagram, all measurements are in cm and the area of the hexagon is 100 cm^2.

Show that $5x^2 + 34x - 127 = 0$

Find x, giving your answer to 3 significant figures.