Congruence, Similarity & Enlargement

Year 10

#MathsEveryoneCan

White Rose Maths

	Week 1	Week 2	Week 3	Week 4	Week 5	Week 6	Week 7	Week 8	Week 9	Week 10	Week 11	Week 12
Autumn	**Similarity**						**Developing Algebra**					
	Congruence, similarity and enlargement			Trigonometry			Equations and inequalities		Representing solutions		Simultaneous equations	
Spring	**Geometry**						**Proportions and Proportional Change**					
	Angles & bearings		Working with circles		Vectors		Ratios & fractions		Percentages and Interest		Probability	
Summer	**Delving into data**						**Using number**					
	Collecting, representing and interpreting data						Non-calculator methods		Types of number and sequences		Indices and Roots	

Autumn 1: Similarity

Weeks 1 & 2: Congruence, Similarity and Enlargement	Weeks 3 to 6: Trigonometry
Building on their experience of enlargement and similarity in previous years, this unit extends students' experiences and looks more formally at dealing with topics such as similar triangles. It would be useful to use ICT to demonstrate what changes and what stays the same when manipulating similar shapes. Parallel line angle rules are revisited to support establishment of similarity. Congruency is introduced through considering what information is needed to produce a unique triangle. Higher level content extends enlargement to explore negative scale factors, and also looks at establishing that a pair of triangles are congruent through formal proof. National curriculum content covered **(Higher content in bold):** • extend and formalise their knowledge of ratio and proportion in working with measures and geometry • compare lengths, areas and volumes using ratio notation and/or scale factors; make links to similarity • interpret and use fractional **{and negative}** scale factors for enlargements • apply the concepts of congruence and similarity, including the relationships between lengths, **{areas and volumes}** in similar figures • use mathematical language and properties precisely • make and test conjectures about the generalisations that underlie patterns and relationships; look for proofs or counter-examples • develop their mathematical knowledge, in part through solving problems and evaluating the outcomes, including multi-step problems	Trigonometry is introduced as a special case of similarity within right-angled triangles. Emphasis is placed throughout the steps on linking the trig functions to ratios, rather than just functions. This key topic is introduced early in Year 10 to allow regular revisiting e.g. when looking at bearings. For the Higher tier, calculation with trigonometry is covered now and graphical representation is covered in Year 11 National curriculum content covered: • extend and formalise their knowledge of ratio and proportion, including trigonometric ratios • apply Pythagoras' Theorem and trigonometric ratios to find angles and lengths in right-angled triangles {and, where possible, general triangles} in two **{and three}** dimensional figures • know the exact values of $\sin\theta$, $\cos\theta$, $\tan\theta$ for required angles • **{know and apply the sine rule and cosine rule to find unknown lengths and angles}** • **{know and apply to calculate the area, sides or angles of any triangle}** • develop their mathematical knowledge, in part through solving problems and evaluating the outcomes, including multi-step problems • make and use connections between different parts of mathematics to solve problems • model situations mathematically and express the results using a range of formal mathematical representations, reflecting on how their solutions may have been affected by any modelling assumptions • select appropriate concepts, methods and techniques to apply to unfamiliar and non-routine problems; interpret their solution in the context of the given problem.

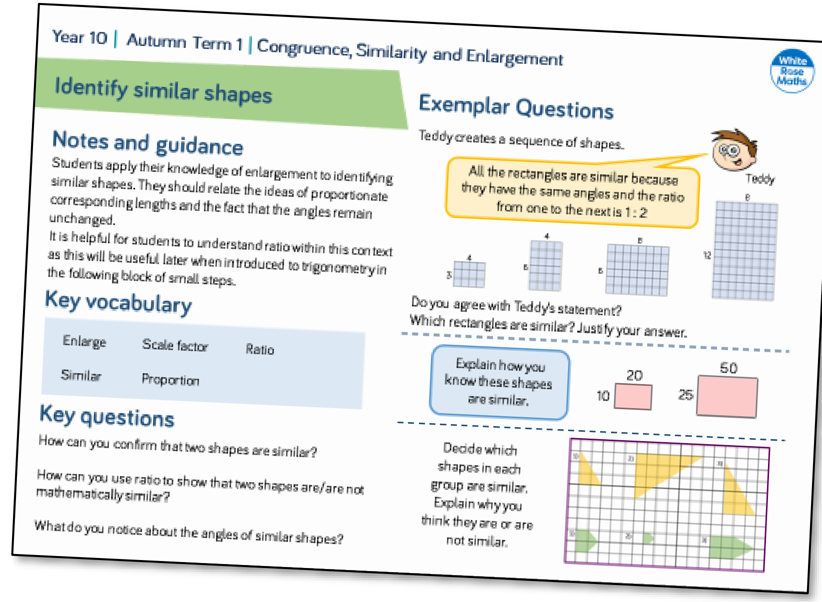
Why Small Steps?

We know that breaking the curriculum down into small manageable steps should help students to understand concepts better. Too often, we have noticed that teachers will try and cover too many concepts at once and this can lead to cognitive overload. We believe it is better to follow a "small steps" approach.

As a result, for each block of content in the scheme of learning we will provide a "small step" breakdown. ***It is not the intention that each small step should last a lesson – some will be a short step within a lesson, some will take longer than a lesson.*** We would encourage teachers to spend the appropriate amount of time on each step for their group, and to teach some of the steps alongside each other if necessary.

What We Provide

- Some ***brief guidance*** notes to help identify key teaching and learning points
- A list of ***key vocabulary*** that we would expect teachers to draw to students' attention when teaching the small step,
- A series of ***key questions*** to incorporate in lessons to aid mathematical thinking.
- A set of questions to help ***exemplify*** the small step concept that needs to be focussed on.


Year 10 | Autumn Term 1 | Congruence, Similarity and Enlargement

Identify similar shapes

Notes and guidance

Students apply their knowledge of enlargement to identifying similar shapes. They should relate the ideas of proportionate corresponding lengths and the fact that the angles remain unchanged.
It is helpful for students to understand ratio within this context as this will be useful later when introduced to trigonometry in the following block of small steps.

Key vocabulary

Enlarge, Scale factor, Ratio, Similar, Proportion

Key questions

How can you confirm that two shapes are similar?

How can you use ratio to show that two shapes are/are not mathematically similar?

What do you notice about the angles of similar shapes?

Exemplar Questions

Teddy creates a sequence of shapes.

All the rectangles are similar because they have the same angles and the ratio from one to the next is 1 : 2

Teddy

Do you agree with Teddy's statement? Which rectangles are similar? Justify your answer.

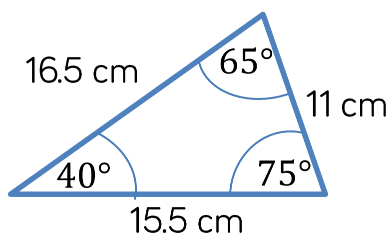
Explain how you know these shapes are similar.

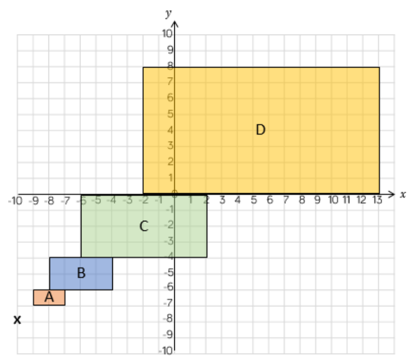
Decide which shapes in each group are similar. Explain why you think they are or are not similar.

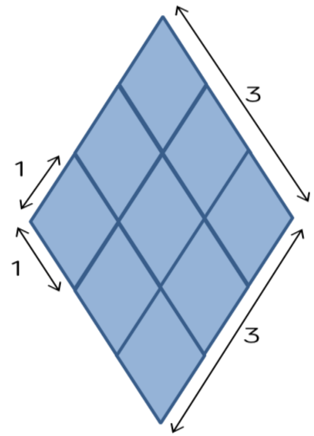
- These include reasoning and problem-solving questions that are fully integrated into the scheme of learning. Depending on the attainment of your students, you many wish to use some or all of these exemplars, which are in approximate order of difficulty. Particularly challenging questions are indicated with the symbol .
- For each block, we also provide ideas for key representations that will be useful for all students.

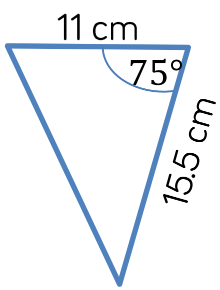
Some of the small steps are in **bold** and labelled with H to indicate this is Higher tier GCSE content. We would encourage teachers to use these with as many students as possible – if you feel your class can access any particular small step, then please include it in your planning. Steps that review content covered at Key Stage 3 are labelled R .

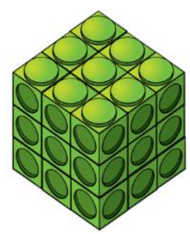
Key Representations

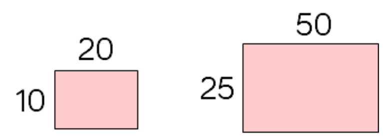


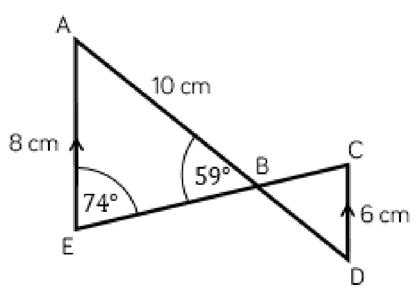


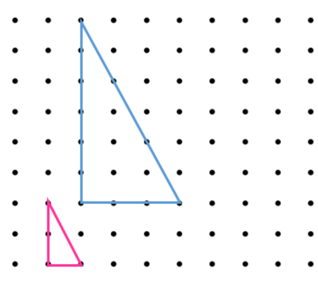


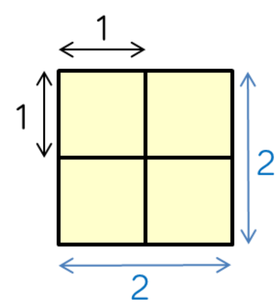












Pictorial representation is essential to support conceptual understanding of enlargement, similarity and congruence.

Students should be encouraged throughout to draw sketches or diagrams to help them visualise information. They should also be reminded to represent new information in diagrammatical form to help see further lines of enquiry that could move them towards solving the problem.

Manipulatives such as a geoboard and pattern blocks could be used to explore enlargements, similarity and area scale factor. Multi-link cubes could be useful in exploring volume scale factor.

Congruence, Similarity and Enlargement

Small Steps

- Enlarge a shape by a positive integer scale factor (R)
- Enlarge a shape by a fractional scale factor (R)
- **Enlarge a shape by a negative scale factor** (H)
- Identify similar shapes
- Work out missing sides and angles in a pair given similar shapes (R)
- Use parallel line rules to work out missing angles
- Establish a pair of triangles are similar

(H) denotes Higher Tier GCSE content

(R) Denotes 'review step' – content should have been covered at KS3

White Rose Maths

Congruence, Similarity and Enlargement

Small Steps

- Explore areas of similar shapes (1) H
- Explore areas of similar shapes (2) H
- Explore volumes of similar shapes H
- Solve mixed problems involving similar shapes H
- Understand the difference between congruence and similarity
- Understand and use conditions for congruent triangles
- Prove a pair of triangles are congruent H

H denotes Higher Tier GCSE content

R Denotes 'review step' – content should have been covered at KS3

Positive scale factors R

Notes and guidance

Students start year 10 with a review of enlargement. This understanding will, in later steps, be built on as similar shapes are introduced. Therefore it would be useful to highlight the fact that angles do not change when enlarging shapes and that the ratio of lengths is the same for corresponding lengths. Dynamic geometry could be used to see what is happening when shapes are enlarged.

Key vocabulary

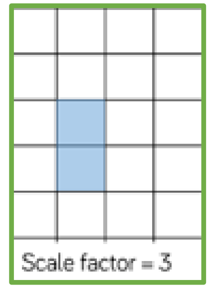
Enlarge	Scale factor	Ratio
Origin	Object	Image

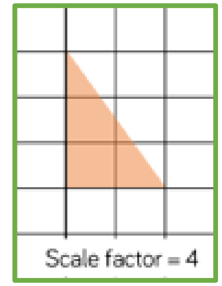
Key questions

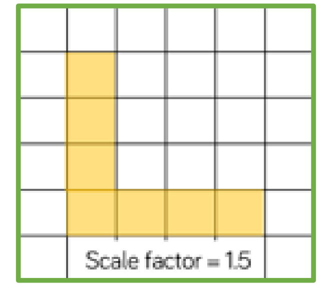
What are the size of the angles in each shape?
Do they stay the same or change when the shape is enlarged? Is this true for all shapes?
What is the ratio of sides?
Does this change depending on which lengths you are comparing?

Exemplar Questions

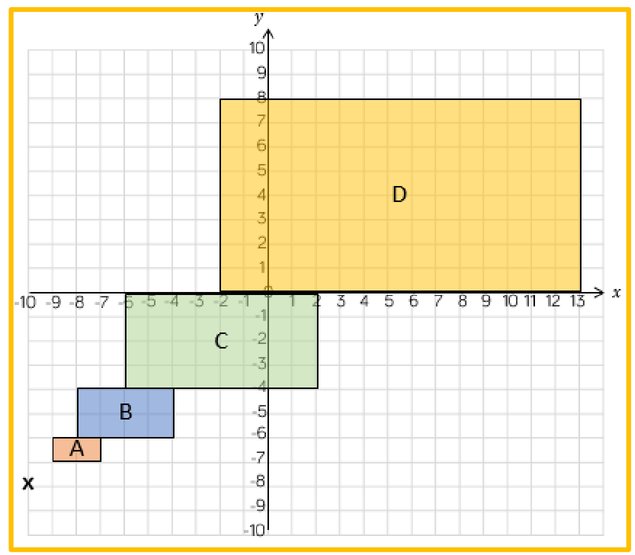
Enlarge each shape by the given scale factor.







Rectangle B, C, and D are each enlargements of A from the point marked at $(-10, -8)$.
What is the scale factor for each enlargement?



Write ratios for the following relationships.

- Width A : Width B
- Width B : Width C
- Width C : Width D
- Width A : Width C
- Width A : Width D

What is the same and what is different?
How do these connect to scale factor?

Fractional scale factors

R

Notes and guidance

Students review their understanding of enlargement in relation to fractional scale factors.

Dynamic geometry could be used to explore how the image changes in relation to the fractional scale factor (including proper and improper fractional scale factors) and in relation to the centre of enlargement.

Key vocabulary

Enlarge	Fractional scale factor	Image
Origin	Centre of enlargement	Object

Key questions

Does enlargement always make a shape bigger?

Which scale factors make the shape larger/smaller/stay the same?

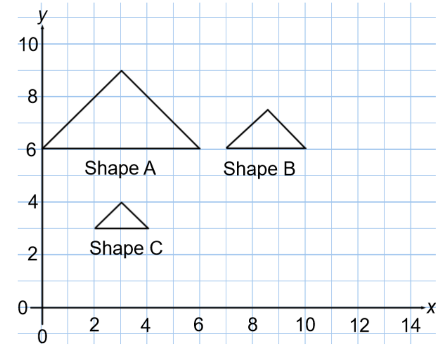
Do fractional scale factors always make the shape smaller?

Exemplar Questions

Shape A has been enlarged onto shape B.

Ron

The scale factor of enlargement is 2



Ron is incorrect.
What mistake has he made?

Write down the correct scale factor.
What is the scale factor of enlargement from Shape A to Shape C?

To find the centre of enlargement, I can join corresponding vertices on Shape A and Shape B with a straight line. The centre of enlargement is the intersection point of these lines.

Annie

Discuss Annie's statement. Is she correct? Why?

On a set of axes, draw rectangle A with coordinates $(4, -2)$, $(8, -2)$, $(4, -4)$ and $(8, -4)$. Enlarge this shape by scale factor $\frac{3}{4}$, centre the origin. Label this rectangle B.

Dora

The ratio of sides A : B is 4 : 3

No, it isn't. The ratio is 3 : 4

Jack

Who's right, Dora or Jack? Justify your answer.

Negative scale factors

H

Notes and guidance

Dynamic software can be used to explore what happens as the scale factor moves between positive, fractional and negative scale factors. Students can explore different centres of enlargement and the effect this has on the new image. Asking students to explain their approach is helpful to aid others in strategies for visualising this transformation.

Key vocabulary

Enlarge	Negative scale factor
Reflection	Centre of enlargement

Key questions

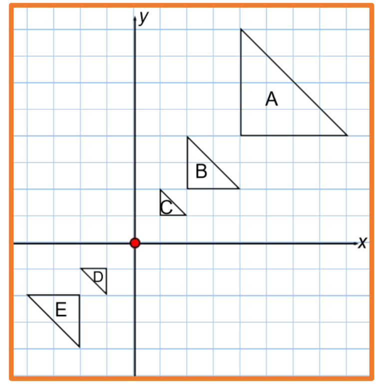
What happens to the shape using a scale factor of -1?

How would the shape change if the shape was enlarged by a negative fractional scale factor e.g. $-\frac{1}{2}$

Can you predict the position of each shape before drawing it? How could you find the centre of enlargement from a diagram?

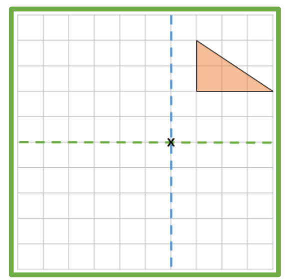
Exemplar Questions

Shape A has been enlarged onto Shapes B, C, D and E.
How do you think the scale factor will change when transforming Shape A to Shapes D and E? Complete the table.



Shape A enlarged onto:	Scale Factor
Shape B	
Shape C	
Shape D	
Shape E	

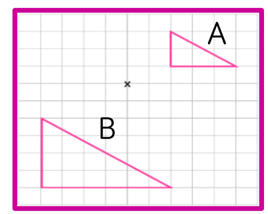
What's the same and what's different about these transformations?



Reflect the shape in the vertical line. Then reflect the shape in the horizontal line.

Enlarge by a scale factor of -1

Abdul has enlarged shape A by a scale factor of –2 to make the image B. Explain and correct Abdul's mistake.



Identify similar shapes

Notes and guidance

Students apply their knowledge of enlargement to identifying similar shapes. They should relate the ideas of proportionate corresponding lengths and the fact that the angles remain unchanged.
It is helpful for students to understand ratio within this context as this will be useful later when introduced to trigonometry in the following block of small steps.

Key vocabulary

Enlarge	Scale factor	Ratio
Similar	Proportion	

Key questions

How can you confirm that two shapes are similar?

How can you use ratio to show that two shapes are/are not mathematically similar?

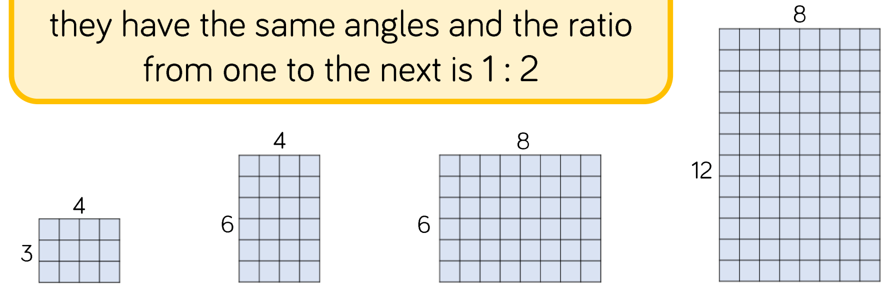
What do you notice about the angles of similar shapes?

Exemplar Questions

Teddy creates a sequence of shapes.

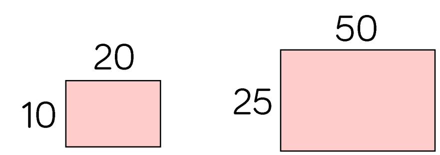
Teddy

All the rectangles are similar because they have the same angles and the ratio from one to the next is 1 : 2

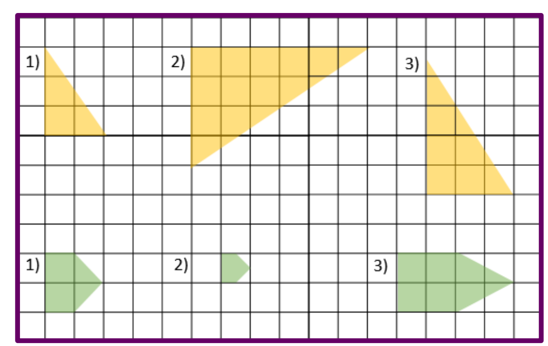


Do you agree with Teddy's statement?
Which rectangles are similar? Justify your answer.

Explain how you know these shapes are similar.



Decide which shapes in each group are similar. Explain why you think they are or are not similar.



Information in similar shapes

Notes and guidance

Students use their knowledge of similar shapes to calculate missing lengths and angles. They should be encouraged to look at scale factors both within and between shapes.

They should see similar shapes in a range of orientations and therefore have practice to ensure they correctly identify corresponding points. Careful labelling will assist this.

Key vocabulary

Enlargement	Scale factor	Ratio
Correspond	Similar	

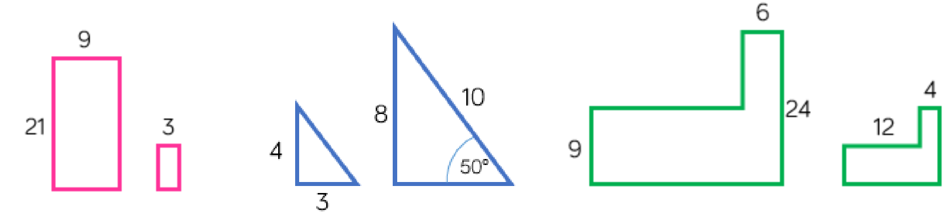
Key questions

Which angles/lengths correspond to each other? How do you know?

How does the order of the letters of the shape e.g. ABC and FGH help you decide which lengths/angles match up (correspond)?

Exemplar Questions

Find the missing information in the these pairs of similar shapes.



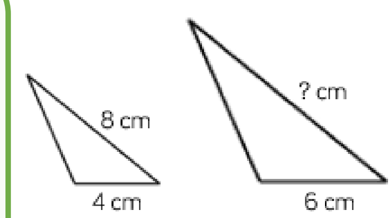
Scale factor = ____

Scale factor = ____

Scale factor = ____

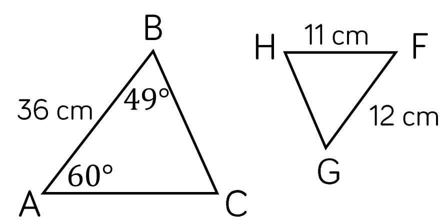
Here are two similar triangles with a missing length.
Compare the two methods. Why do they both work?

The longest side is double the length of the smallest side, so the missing length must be 6 × 2 = 12 cm



The ratio of sides is 4 : 6 so there is a scale factor of 1.5
The missing length must be 8 × 1.5 = 12 cm

Triangle ABC is similar to FGH.



Which length in triangle FGH corresponds to AB?
Which angle in triangle FGH corresponds to ∠BAC?
Calculate the length AC.

Parallel line rules

R

Notes and guidance

This review of year 8 content will support students to show pairs are triangles are similar in the following step. Students are encouraged to explain their reasoning for their steps and review angle and side notation. It will be useful to distinguish between 'corresponding angles' (that are equal because of parallel lines) and 'angles that correspond' (matching pairs of angles in two shapes).

Key vocabulary

Parallel	Corresponding
Alternate	Co-interior

Key questions

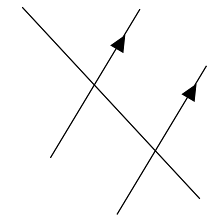
Where are the parallel lines in the diagram?

Which angles would be corresponding/alternate/co-interior?

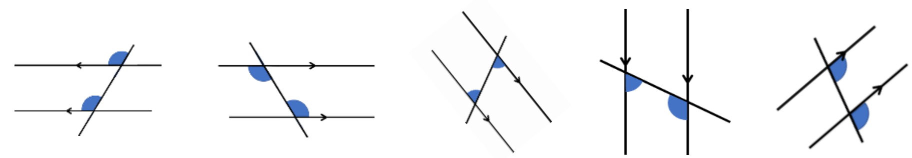
What other angle rules do you know?

Exemplar Questions

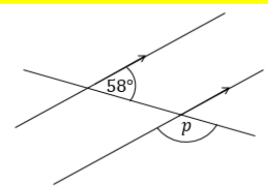
Use colour coding to identify all the equal angles.
Use colour coding to identify vertically opposite, corresponding, alternate and co-interior angles.
What are the relationships?

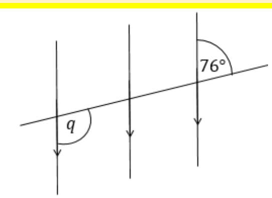


State whether each pair of angles are corresponding, alternate or co-interior angles. State what this tells you about the angles.



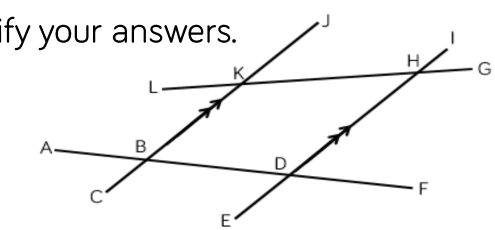
Work out the missing angles.
Hint: what other angles do you need to work out first?





Which statements are true. Justify your answers.

- $\angle CBD = \angle BDH$
- $\angle LKB = \angle KBD$
- $\angle KHD + \angle HDB = 180°$
- $\angle KBD = \angle JKH$



Establish triangles are similar

Notes and guidance

Students use their understanding of angles in parallel lines to show that a pair of triangles are similar. They may need support to work out which vertex in one triangle corresponds to which in the other and to distinguish this from 'corresponding angles' in parallel lines.
Students should also recognise that using side ratios is an equally valid method of establishing similarity.

Key vocabulary

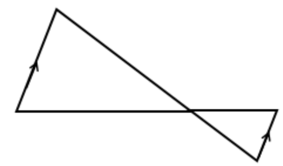
Corresponding angles	Similar
Alternate angles	Parallel

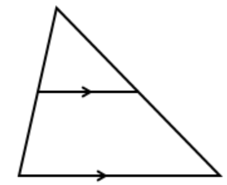
Key questions

Why do you only need two pairs of equal angles to show that two triangles are similar?
What's the same and what's different about the pairs of triangles?
Which diagrams include pairs of parallel lines? How do we show they are parallel?

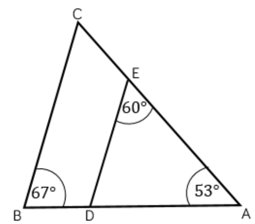
Exemplar Questions

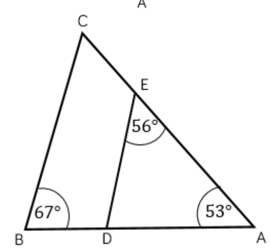
In each diagram, show that the triangles are similar.

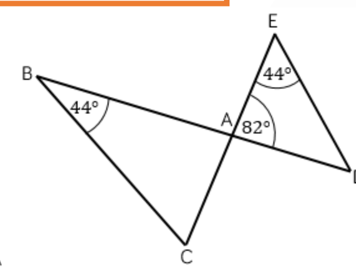


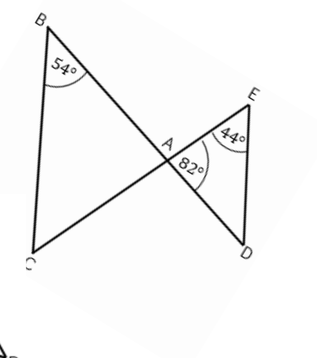


In each diagram, is ABC similar to ADE?



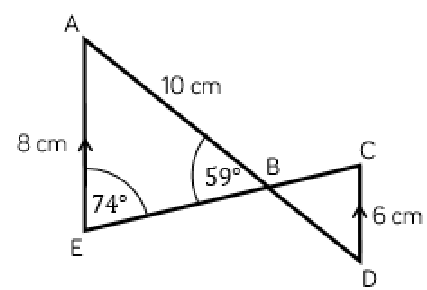






AE is parallel to CD.
Work out the following:

- ∠BDC
- Length BD



Areas of similar shapes (1)

H

Notes and guidance

Students explore how area changes as the scale factor between two shapes changes. A common misconception is that if, for example, the lengths double, the area will also double. Visual representations and use of manipulatives, such as pattern blocks or multilink cubes, are very helpful here to support student understanding of the links between the length scale factor and area scale factor. This will be revisited in the spring term of year 10

Key vocabulary

Enlarge	Length scale factor	Object
Ratio	Area scale factor	Image

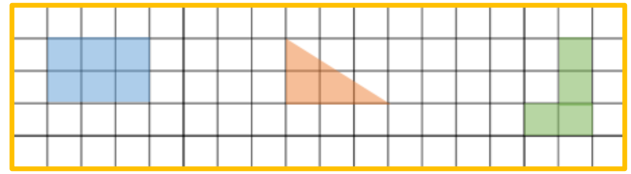
Key questions

If we know the length scale factor between two similar shapes, how can you find the area scale factor of the shapes? What about the other way round?

Can you draw a diagram to show your understanding?

Exemplar Questions

Draw the following shapes. Then enlarge each by scale factor 2



For each enlargement, how many times does each object fit into its corresponding image?
Repeat the activity using a scale factor of 3
What does this tell you about the area of an enlarged shape compared to the area of the original shape?

Complete the table.

Original shape	Enlarged shape	Length Scale factor	Area scale factor
Rectangle 5 cm by 3 cm Area = ____ cm²	Rectangle 10 cm by 6 cm Area = ____ cm²		
Triangle 4 cm, 2 cm Area = ____ cm²	Triangle 6 cm, 3 cm Area = ____ cm²		
Parallelogram 6 cm, 4 cm Area = ____ cm²	Parallelogram 3 cm, 2 cm Area = ____ cm²		

Areas of similar shapes (2) H

Notes and guidance

Teachers may decide to extend this by using area scale factors to find missing areas and to work backwards to finding missing lengths.

This is revisited in the spring term of year 10

Key vocabulary

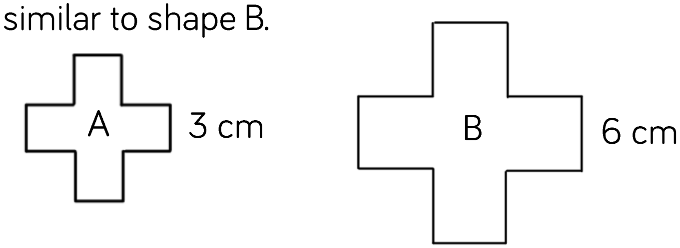
Enlarge	Length scale factor	Object
Ratio	Area scale factor	Image

Key questions

If we know the length scale factor between two similar shapes, how can you find the area scale factor of the shapes?
What about the other way round?

Exemplar Questions

Shape A is similar to shape B.



Write down the length scale factor of enlargement from Shape A to Shape B.
State the area scale factor of enlargement from Shape A to Shape B.
The area of Shape A is 45 cm^2. Find the area of Shape B?

Shape C is similar to Shape A above. The area of Shape C is 281.25 cm^2. Rosie is calculating the side length on Shape C which corresponds to the labelled 3 cm side length on Shape A. Complete her steps.

Area scale factor: 281.25 ÷ 45 = __________.
To find the length scale factor, I need to __________ the area scale factor. The length scale factor is __________.
To find the side length on shape C which corresponds to the labelled 3 cm on shape A, I need to multiply 3 by my length scale factor: 3 × ____ = _____ cm

Volumes of similar shapes H

Notes and guidance

This small step leads on directly from student reasoning around area of similar shapes in the previous step.

Again, visual support will ensure students can see the link between the length, area and volume scale factors of similar shapes. Multilink cubes could be used to explore this concept.

Key vocabulary

Enlarge	Similar
Length scale factor	Volume scale factor

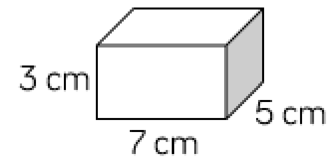
Key questions

Are the cuboids similar? How do you know?

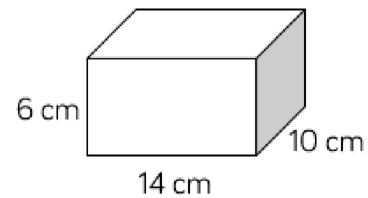
If you know the length scale factor between two similar shapes, how can you find the volume scale factor? Draw a diagram to explain why.

Exemplar Questions

Explain how the written calculations match the diagrams.



$3 \times 7 \times 5 = 105 \text{ cm}^3$



$6 \times 14 \times 10$

$= 3 \times 2 \times 7 \times 2 \times 5 \times 2$

$= 105 \times 8$

$= 840 \text{ cm}^3$

What is the length scale factor for these cuboids?
What is the volume scale factor for these cuboids? Why?

Complete:
To find the volume scale factor, we can ______ the length scale factor.

Complete the following table.

Volume of original cuboid	Length scale factor	Volume scale factor	Volume of enlarged cuboid
12 cm^3	$\times 3$	$\times 3^3$	
25 m^3	$\times 1.5$		
310 cm^3		$\times 343$	
	$\times 5$		$8\,375 \text{ cm}^3$

Similar shape problems

H

Notes and guidance

This small step brings together the previous steps to consolidate and extend student understanding of the topics while interleaving other topics.

Students should be encouraged to discuss their approaches and reasoning when solving the problems.

Key vocabulary

Length scale factor	Parallel	Similar
Alternate angles	Corresponding angles	

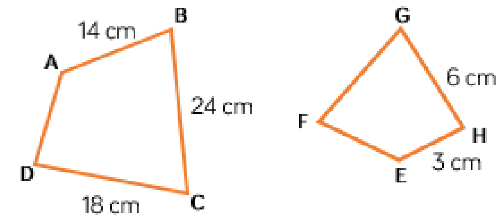
Key questions

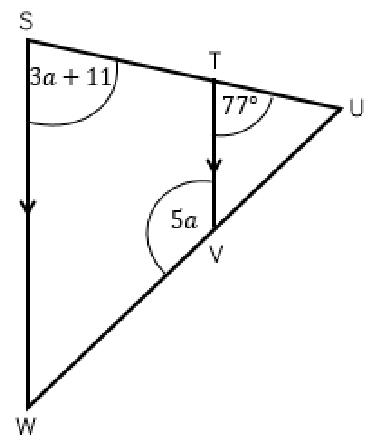
How does the order of the letters of the shape e.g. ABCD and EFGH help you decide which lengths/angles are corresponding?

If you know two shapes are similar, what do you know about those shapes?

Exemplar Questions

Quadrilateral ABCD is similar to EFGH.
Work out the length FG.
Work out the length AD.





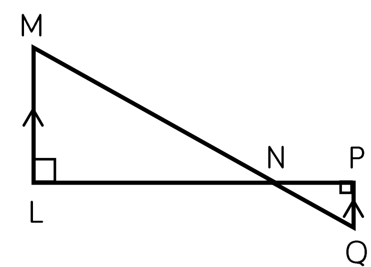
SW is parallel to TV.

- Explain why TUV and SUW are similar triangles.
- Find ∠SWV

Length UV is 50 cm and length UW is 125 cm.

- If length TV is 42 cm, what is the length SW?
- If length SU is 100 cm, what is the perimeter of triangle TUV?

Triangle LMN is similar to PQN.
The ratio of LN : NP is 3 : 1
If NP is 4 cm and PQ is 3.5 cm, what is the area of triangle LMN?
Can you find another way to calculate the answer?



Congruence and similarity

Notes and guidance

Within this small step, students bring together the ideas of similarity and congruence and through categorising, are able to distinguish between them.

By reasoning and distinguishing in this way, students will have a better idea of where the concepts overlap and what characteristics are unique to each.

Key vocabulary

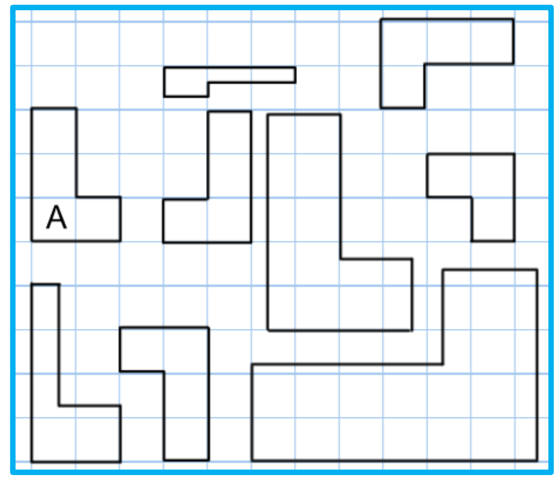
Congruent	Similar	Scale factor
In proportion	Ratio	Corresponding

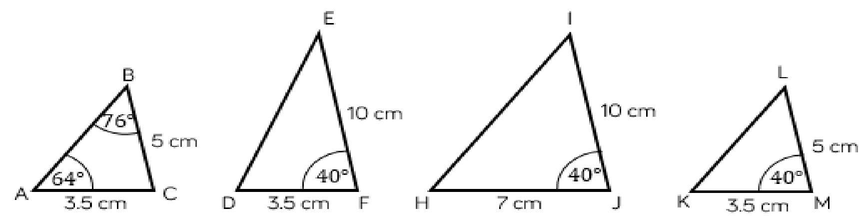
Key questions

If you know two shapes are congruent, what else do you know about the shapes?
If you know two shapes are similar, what else do you know about the shapes?
What is the ratio of corresponding lengths in a congruent shape?

Exemplar Questions

In pairs, discuss which shapes are similar to shape A, which are congruent to shape A, and which are neither similar or congruent. Label the similar shapes, S, and the congruent shapes C.





Complete the sentences.
Triangle ABC and triangle _______ are congruent.
Triangle ABC and triangle _______ are similar.
Triangle ______ is neither similar nor congruent to triangle ABC.

Conditions for congruent triangles

Notes and guidance

The conditions for congruence are formalised within this step. Students will have come across the language of SSS, ASA etc. in previous years, but will not have used them to show congruence of triangles.

Students should understand the minimum information needed to establish congruence between triangles.

Key vocabulary

Side-side-side	Angle-side-angle
Side-angle-side	Right angle-hypotenuse-side

Key questions

What is the minimum information needed for triangles to be congruent?

Does it matter which two angles and sides are given for the angle-side-angle condition to be true?

Exemplar Questions

In pairs or groups, after constructing each triangle, discuss if the triangles are congruent or not.

Construct a triangle which has one side 8 cm long and another 5 cm

Construct a triangle which has the following sides: 8 cm, 5 cm and 6 cm

Construct a triangle with one side of length 5 cm, another of length 8 cm and an angle of $50°$ between them.

Construct a triangle which has the following angles: $40°$, $80°$ and $60°$

Construct a triangle which has one side of length 6.5 cm and angles of $40°$ and $55°$ at either end.

Construct a right-angled triangle with hypotenuse of length 10 cm and base of length 6 cm

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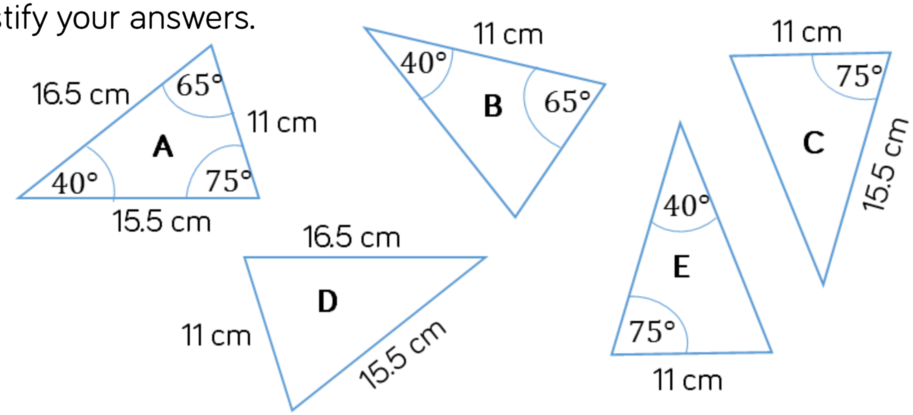
Which of the following triangles are congruent to triangle A? Justify your answers.



Prove triangles are congruent H

Notes and guidance

Students prove that triangles are congruent using the conditions of congruence. Teachers should model the process in the first instance and then scaffold by providing writing frames before expecting students to produce formal proofs independently. It may be useful to remind students of the properties of special quadrilaterals in preparation for this step.

Key vocabulary

SSS	ASA	SAS	RHS
Prove	Conditions of congruence		

Key questions

Would drawing a sketch help you?

What angle facts do we know about a parallelogram?

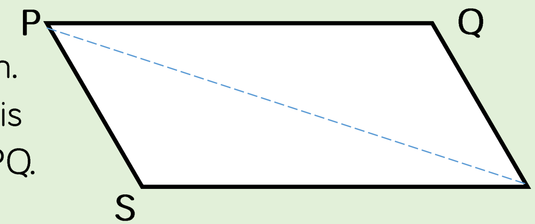
Can you prove it any other way using the conditions of congruence?

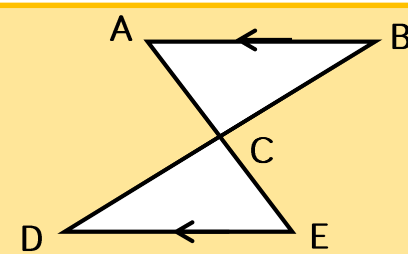
Exemplar Questions

Which two of the following triangles are congruent? Prove it.

- In triangle ABC, AB $= 5$ cm, $\angle ABC = 40°$ and AC $= 3$ cm
- In triangle DEF, DE $= 5$ cm, $\angle DEF = 40°$ and EF $= 3$ cm
- In triangle GHI, GH $= 5$ cm, $\angle GHI = 40°$ and GI $= 3$ cm

PQRS is a parallelogram.
Prove that triangle PRS is congruent to triangle RPQ.





AB and DE are parallel lines of equal length.
Prove that ABC is congruent to EDC.

LMNO is a rectangle.
X is the mid-point of LM and Z is the mid-point of ON.
Prove that triangle LXY and NZY are congruent.