Small Steps Guidance – Ratios and fractions

Year 10

#MathsEveryoneCan

White Rose Maths

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Spring 2: Proportions and Proportional Change

Weeks 1 and 2: Ratios and Fractions

This block builds on KS3 work on ratio and fractions, highlighting similarities and differences and links to other areas of mathematics including both algebra and geometry. The focus is on reasoning and understanding notation to support the solution of increasingly complex problems that include information presented in a variety of forms. The bar model is a key tool used to support representing and solving these problems.

National curriculum content covered:

- Consolidating subject content from key stage 3:
  - Use ratio notation, including reduction to simplest form.
  - Divide a given quantity into two parts in a given $part : part$ or $part : whole$ ratio; express the division of a quantity into two parts as a ratio.
  - Relate the language of ratios and the associated calculations to the arithmetic of fractions and to linear functions.
  - Use compound units such as speed, unit pricing and density to solve problems.
- Compare lengths, areas and volumes using ratio notation and/or scale factors; make links to similarity.
- Apply the concepts of congruence and similarity, including the relationships between lengths, **{areas and volumes}** in similar figures.

Weeks 4 and 5: Percentages and Interest

Although percentages are not specifically mentioned in the KS4 national curriculum, they feature heavily in GCSE papers and this block builds on the understanding gained in KS3. Calculator methods are encouraged throughout and are essential for repeated percentage change/growth and decay problems. Use of financial contexts is central to this block, helping students to maintain familiarity with the vocabulary they are unlikely to use outside school.

National curriculum content covered:

- Consolidating subject content from key stage 3:
  - Interpret these multiplicatively, express one quantity as a percentage of another, compare two quantities using percentages, and work with percentages greater than 100%.
  - Solve problems involving percentage change, including: percentage increase, decrease and original value problems and simple interest in financial mathematics.
- Set up, solve and interpret the answers in growth and decay problems, including compound interest **{and work with general iterative processes}.**

Weeks 5 and 6: Probability

This block also builds on KS3 and provides a good context in which to revisit fraction arithmetic and conversion between fractions, decimals and percentages. Tables and Venn diagrams are revisited and understanding and use of tree diagrams is developed at both tiers, with conditional probability being a key focus for Higher tier students.

National curriculum content covered:

- Apply the property that the probabilities of an exhaustive set of mutually exclusive events sum to one.
- Use a probability model to predict the outcomes of future experiments; understand that empirical unbiased samples tend towards theoretical probability distributions, with increasing sample size.
- Calculate the probability of independent and dependent combined events, including using tree diagrams and other representations, and know the underlying assumptions.
- **{Calculate and interpret conditional probabilities through representation using expected frequencies with two-way tables, tree diagrams and Venn diagrams}.**

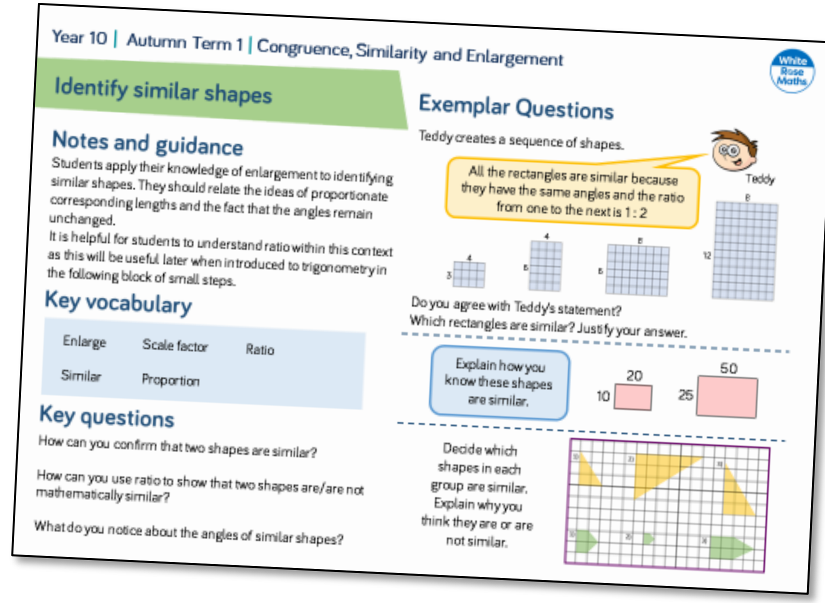
Why Small Steps?

We know that breaking the curriculum down into small manageable steps should help students to understand concepts better. Too often, we have noticed that teachers will try and cover too many concepts at once and this can lead to cognitive overload. We believe it is better to follow a "small steps" approach.

As a result, for each block of content in the scheme of learning we will provide a "small step" breakdown. ***It is not the intention that each small step should last a lesson – some will be a short step within a lesson, some will take longer than a lesson.*** We would encourage teachers to spend the appropriate amount of time on each step for their group, and to teach some of the steps alongside each other if necessary.

What We Provide

- Some ***brief guidance*** notes to help identify key teaching and learning points
- A list of ***key vocabulary*** that we would expect teachers to draw to students' attention when teaching the small step,
- A series of ***key questions*** to incorporate in lessons to aid mathematical thinking.
- A set of questions to help ***exemplify*** the small step concept that needs to be focussed on.


Year 10 | Autumn Term 1 | Congruence, Similarity and Enlargement

Identify similar shapes

Notes and guidance

Students apply their knowledge of enlargement to identifying similar shapes. They should relate the ideas of proportionate corresponding lengths and the fact that the angles remain unchanged.
It is helpful for students to understand ratio within this context as this will be useful later when introduced to trigonometry in the following block of small steps.

Key vocabulary

Enlarge Scale factor Ratio
Similar Proportion

Key questions

How can you confirm that two shapes are similar?

How can you use ratio to show that two shapes are/are not mathematically similar?

What do you notice about the angles of similar shapes?

Exemplar Questions

Teddy creates a sequence of shapes.

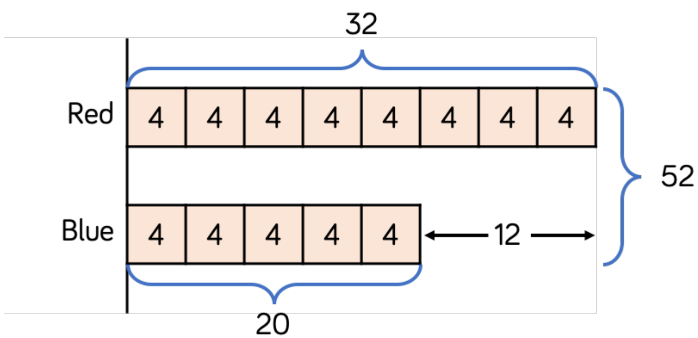
Do you agree with Teddy's statement?
Which rectangles are similar? Justify your answer.

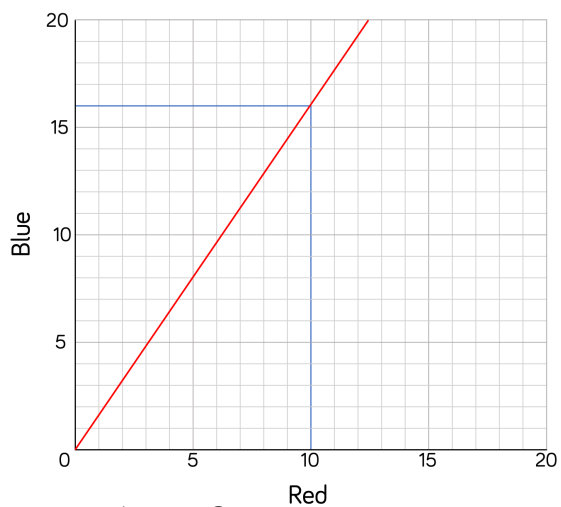
Decide which shapes in each group are similar. Explain why you think they are or are not similar.

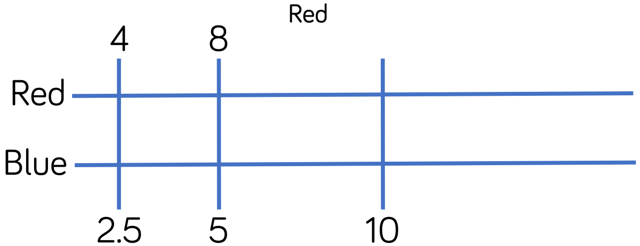
- These include reasoning and problem-solving questions that are fully integrated into the scheme of learning. Depending on the attainment of your students, you many wish to use some or all of these exemplars, which are in approximate order of difficulty. Particularly challenging questions are indicated with the symbol 💡.
- For each block, we also provide ideas for key representations that will be useful for all students.

Some of the small steps are in **bold** and labelled with H to indicate this is Higher tier GCSE content. We would encourage teachers to use these with as many students as possible – if you feel your class can access any particular small step, then please include it in your planning. Steps that review content covered at Key Stage 3 are labelled R .

Key Representations



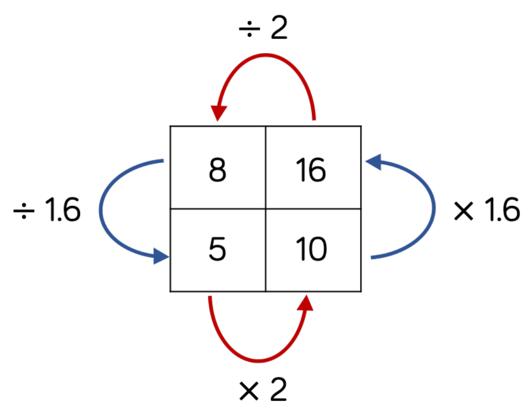




"For every 5 blue, there are 8 red."

$R : B$
$8 : 5$

$8b = 5r$



Red	8	16	4	20
Blue	5	10	2.5	12.5

Pictorial support is essential to support conceptual understanding of ratio and fractions.

The bar model is useful to visually represent ratio problems. They help students to see the equal parts and conceptually understand how to share between equal parts and more complex questions involving comparison.

Double number lines and ratio tables can be helpful tools to show proportionality. They are a structured way for students to represent their mathematical thinking when working through problems and are a consistent tool that can be used when working with proportionate reasoning.

It is important to still keep reinforcing the language of ratio and using this to help aid conceptual understanding.

Ratios and Fractions

Small Steps

- Compare quantities using a ratio R
- Link ratios and fractions R
- Share in a ratio (given total or one part) R
- Use ratios and fractions to make comparisons
- Link ratios and graphs R
- Solve problems with currency conversion
- Link ratios and scales R
- Use and interpret ratios of the form $1 : n$ and $n : 1$
- Solve 'best buy' problems
- Combine a set of ratios

H Denotes Higher Tier GCSE content

R Denotes 'review step' – content should have been covered at KS3

Ratios and Fractions (2)

Small Steps

- Link ratio and algebra
- **Ratio in area problems** H
- **Ratio in volume problems** H
- Mixed ratio problems

H Denotes Higher Tier GCSE content

R Denotes 'review step' – content should have been covered at KS3

Compare using a ratio

R

Notes and guidance

In this small step, students review expressing information in a ratio. They also encounter questions where the units are not the same and discuss why it is important to use equivalent units in these situations. Contextualising these kinds of questions aids student understanding of why units should be the same when comparing. A recap of unit conversions could be useful here.

Key vocabulary

Ratio	Simplest Form	Convert
Unit	Equivalent	

Key questions

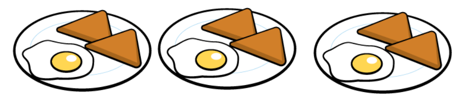
Why do the units need to be the same in order to write a ratio?

Can a ratio compare more than two quantities?

Why is (e.g.) $2 : 1$ different from $1 : 2$?

Exemplar Questions

What is the same and what is different about each of these representations?



$1 : 2$

$3 : 6$

There are twice as many pieces of toast as eggs.

Eva takes 30 minutes and Teddy takes one hour to do the same homework.
Eva says the ratio of time taken to do homework is $30 : 1$
Explain why this is incorrect.

A group of children choose their favourite colour.
30% choose red, 35% choose blue, 25% choose green, and the rest choose yellow.
Express the ratio of colour choice red : blue : green : yellow in its simplest form.

There are three piles of books.
Pile 1 has twice as many books as pile 2
Pile 3 has half as many books as pile 2
Find the ratio of books in Pile 1 : Pile 2 : Pile 3

Write the ratios in simplest form.

4 kg : 500 g

25 ml : 2 litres

35 p : £4.20

3 hours : 45 mins

600 mm : 20 cm : 3 m

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Link ratios and fractions

R

Notes and guidance

When looking at a ratio, it is important for students to look at both the relationships between the parts and the relationships to the whole e.g. in the ratio $a : b = 1 : 3$, a is a $\frac{1}{3}$ of b, b is 3 times the size of a, a is $\frac{1}{4}$ of the whole etc. Pictorial representations help to unpick any misconceptions as fractional relationships are clearly highlighted.

Key vocabulary

Ratio	Simplest Form	Convert
Unit	Equivalent	

Key questions

Why do the units need to be the same in order to write a ratio?

Can a ratio compare more than two quantities?

Why is (e.g.) $2 : 1$ different from $1 : 2$?

Exemplar Questions

A farmer has 20 goats, 30 sheep and 50 cows.
Decide whether each statement is true or false, and explain why.

G	G	S	S	S	C	C	C	C	C

30% of the animals are sheep.

50% of the animals are cows.

$\frac{1}{5}$ of the animals are goats.

The ratio of goats to sheep to cows is 20 : 30 : 50

Other animals	Cows

What other fractions, percentages and ratios can you write down?

Match the statements to the corresponding ratios.
You may use bar models to help you.

b is $\frac{1}{5}$ of a

$a : b = 5 : 1$

$a : b = 1 : 4$

$a : b = 4 : 1$

a is $\frac{1}{5}$ of b

a is $\frac{1}{5}$ of the whole

b is $\frac{1}{5}$ of the whole

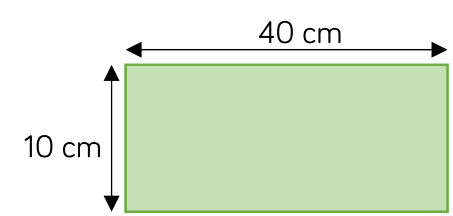
$a : b = 1 : 5$

Explain how each card relates to the rectangle.

$\times 4$

$1 : 4$

$\frac{1}{4}$



Share in a ratio

R

Notes and guidance

Students should be familiar with this step from KS3. Encouraging the use of bar models and emphasising the importance of labelling them helps students to understand the structure of ratio problems, highlighting when the total or when one of the parts is known. This also provides a good opportunity to revisit other topics such as geometry.

Key vocabulary

Ratio	Share	More/less than
Part	Whole	

Key questions

Can you represent this with a bar model? What information can you label on the diagram?

Do you always need to add the numbers of parts first when solving a ratio problem? Why or why not?

Can you tell if the answer is going to be more or less than the value(s) in the question? How?

Exemplar Questions

The ratio of pink to blue beads on a bracelet is 7 : 1
Could there be exactly 28 beads on the bracelet? Explain your answer.

What's the same and what's different about these questions?
Draw a bar model and solve each one.

Rosie, Tommy and Alex share £90 in the ratio $6 : 5 : 4$
How much more money does Rosie get than Alex?

Rosie, Tommy and Alex share some money in the ratio $6 : 5 : 4$
Rosie gets £90
How much money does Tommy get?

Rosie, Tommy and Alex share some money in the ratio $6 : 5 : 4$
Rosie gets £90 more than Alex.
How much money does Tommy get?

- The angles in a triangle are in the ratio 14 : 18 : 13
  Find the size of the largest angle.
- The exterior angles of a triangle are in the ratio 3 : 4 : 5
  Calculate the size of the interior angles of the triangle.

Dora and Amir share some money in the ratio 10 : 9

They shared £x, where x is an integer that satisfies the inequality

$$100 < x < 120$$

How much money did they share? Explain how you know.

Make comparisons

Notes and guidance

Students might need to review comparing fractions before ratios. They should be reminded that there are different ways to compare fractions (e.g. using common numerators or common denominators or decimals). Students should be encouraged to draw bar models, and to write parts of a ratio as a fraction of the whole, to support their comparisons.

Key vocabulary

Proportion	Ratio	Fraction
Convert	Compare	Equivalent

Key questions

If the numerators/denominators of two fractions are the same, how can you identify the greater fraction?

Is it more efficient/easier to use fractions to compare?

How can you decide which is the biggest and which is the smallest proportion?

Exemplar Questions

In each pair, which is the larger fraction? Justify your answer.

$\frac{5}{11}$ or $\frac{7}{11}$ $\frac{3}{5}$ or $\frac{3}{7}$ $\frac{2}{5}$ or $\frac{3}{10}$ $\frac{8}{9}$ or $\frac{7}{8}$

Compare the ratios $3 : 2$ and $3 : 4$

Blue and yellow paint are used to make tins of green paint.

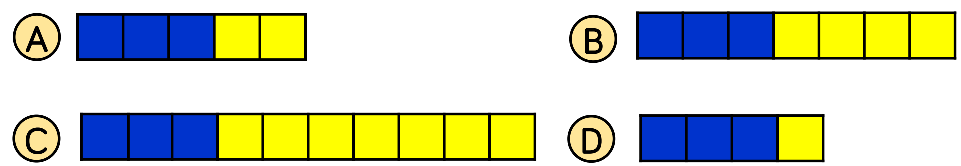
A $b : y = 3 : 2$

B $b : y = 3 : 4$

C $\frac{3}{10}$ of the green paint is blue

D $\frac{3}{4}$ of the green paint is blue

The greater the proportion of blue the darker the paint will be. Annie draws bar models to help her decide which tin will be darkest green.



Write down the fraction of blue paint in each tin.
Now put the tins in order from darkest to lightest green.

Eva, Mo and Ron make some drinks using blackcurrant and lemonade.

Eva has 2 parts blackcurrant and 6 parts lemonade.
Mo has 3 parts blackcurrant and 5 parts lemonade.
Ron has 3 parts blackcurrant and 7 parts lemonade.

Whose drink has the strongest blackcurrant flavour?
Justify your answer.

Link ratios and graphs

R

Notes and guidance

This step reviews the idea of direct proportion met at KS3, and how this links to graphical representation. Students can revisit the notion of gradient and see how this links to the ratio of the pairs of values $\frac{y}{x}$. Examples of values that are not in direct proportion are important here, observing that these do not produce a constant ratio.

Key vocabulary

Direct proportion	Ratio	Gradient
Equation	Origin	$y = mx\ (+c)$

Key questions

If two variables are directly proportional, what will the graph look like?

Can a direct proportion graph have a negative gradient?

How can you quickly find the gradient of a straight line that passes through the origin?

Exemplar Questions

1 metre of electrical cable costs £3

The table of values shows the cost, y, in pounds for some values of x metres of cable.

x	1	2	3	4
y	3	6	9	12

Plot the points given by the table of values and join them with a straight line.

What is the equation of the line?

How does this relate to the ratio $x : y$ for each pair of values?

Investigate the ratios and graphs given by these tables of values.

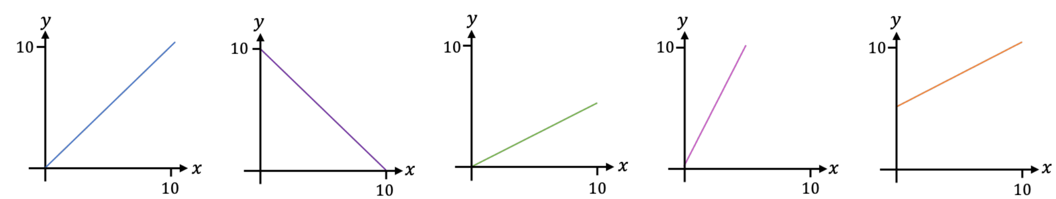
x	5	10	15	20
y	7.5	15	22.5	30

x	2	4	6	8
y	1	2	3	4

x	1	2	3	4
y	3	5	7	9

Describe how one graph and set of ratios is different to the other two.

In which of these line graphs is x <u>not</u> directly proportional to y? Explain how you know.



Currency conversion

Notes and guidance

This small step gives students the opportunity to revisit reading information from graphs and also gives them the opportunity to reinforce their understanding and use of multiplicative reasoning. Double number lines are particularly helpful in aiding students to build up to higher quantities using multiplicative reasoning and to think about how they can use what they know to find other values, linking this to their knowledge of ratio.

Proportion	Convert
Double number line	Exchange rate

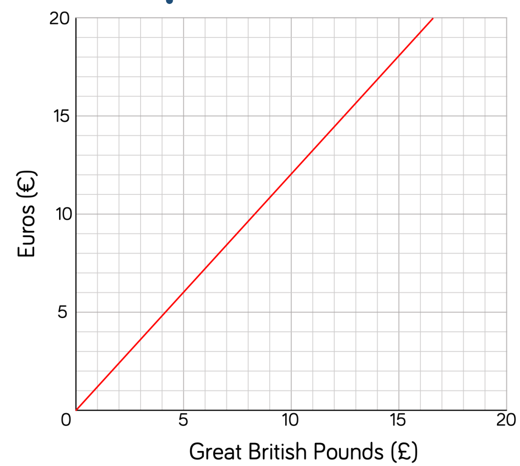
Key questions

How can you find values that cannot be read from the graph?

How can you use what you already know to build up to other values?

Are currency conversion graphs an example of direct proportion? Why/Why not?

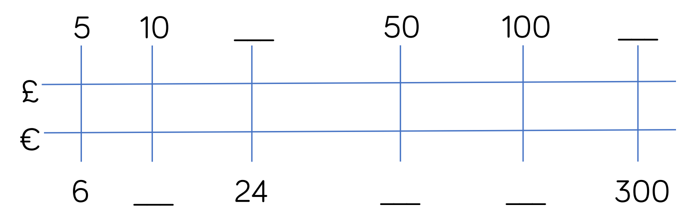
Exemplar Questions



The graph shows the conversion from pounds to euros. Use the graph to complete the sentences:

- For every £10, you get €___
- For every €15, you get £___

Complete the double number line. What connections can you see?



The exchange rate for pounds to Mexican Peso is £1 = 25 Pesos

- How many Pesos can you buy for £200?
- How many Pounds can you buy for 200 Pesos?
- Which is greater in values, £75 or 1850 Pesos?

The exchange rate for pounds to Canadian dollars is £1 = $1.70

Dora wants to buy a new tablet.

In which country is the tablet best value for money?

UK price: £299

Canada price: $525

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Link ratios and scales

R

Notes and guidance

Students may need reminding about unit conversions as a precursor to this step. It is good practice to use full size maps rather than just extracts normally seen in examination and textbook questions. Using applications like Google maps to extend students' experience of different scales may also be useful. This is also a good opportunity to revisit/reinforce drawing and reading bearings.

Key vocabulary

Ratio	Scale	Map
Represent	Bearing	

Key questions

How many cm are there in a m/km?

How do you know whether to divide or multiply when doing calculations involving scales?

Why do maps have different scales?

Exemplar Questions

Match up each scale with the corresponding ratio.

1 cm represents 1 km	1 : 100
1 cm represents 1 m	1 : 10 000
1 cm represents 100 m	1 : 100 000

Maps A and B cover the same area.
Which map has more detail? How do you know?

Map A
Scale = 1 : 1000

Map B
Scale = 1 : 100 000

On a street map of a town, 2 cm represents 140 metres.

- Express the scale of the map a ratio is its simplest form
- Find the actual distance between two points that are 30 cm apart on the map.
- The actual distance between the town hall and the park is 595 metres. How far apart will they be on the map?

Dora is standing 600 metres away from Tom.
Her bearing from Tom is 125°
Jack is standing 400 metres away from Dora.
His bearing from Dora is 195°
Draw a diagram with a scale of 1 : 10 000 to show the positions of the three children.

Ratios of the form $1 : n$ and $n : 1$

Notes and guidance

Students sometimes find this tricky as answers do not always conform to the usual simplifying of ratios where both parts are integers. Students may need some guidance on deciding which has the highest proportion or whether a criteria is met and using stem sentences, such as 'for every 1 red, there are _____ green' can be a helpful way for students to interpret the information a bit more easily.

Key vocabulary

Ratio

For every … , there are …

Integer

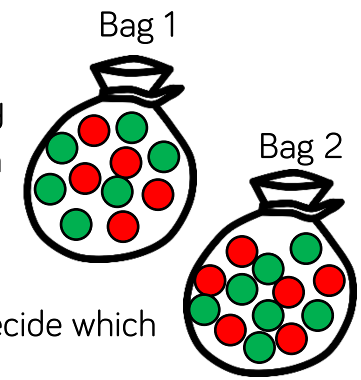
Non-integer

Key questions

How does getting the ratio into the form $1 : n$ help you to compare ratios?

What is different about reducing a ratio to the from $1 : n$ as opposed to the form $n : 1$? How do you know what to divide by?

Exemplar Questions



Alex has two bags of counters with some red and green counters. He wants to know which has the highest proportion of red counters.

- Find the ratio of red : green in each bag
- Write each ratio in the form $1 : n$ and decide which bag has the highest proportion of red

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A school is planning three school trips.
There must be at least 1 teacher for every 12 students.
Which of the trips can go ahead?

	Trip to university	Trip to museum	Trip to adventure park
Number of students	38	126	274
Number of teachers	4	10	23

Write the ratios in the form $1 : n$ and the form $n : 1$
Where necessary, give n to 3 significant figures.

40 g : 1 kg

£5 : 80 p

12 hours : 1 week

Length of the side of a square : perimeter of the square

Solve 'best buy' problems

Notes and guidance

In this small step, students compare prices to find best value. Students will have different methods for comparing and it is useful to share these as a group. Thinking in terms of efficiency and discussing these with students can be a powerful way to show alternative methods others may not have considered. Use of double number lines or ratio tables can be useful for structuring mathematical thinking.

Key vocabulary

Compare	Proportion	Best value
Unit cost		

Key questions

Is it the largest or smallest number that tells you which is the best value for money?

What is the difference between 'cost per item' and 'number of items per £/p'?

Why might factors or multiples be useful in this problem?

Exemplar Questions

Circle which one would be the best buy for each item:

1 pen for 45p or 3 pens for £1.20

4 litres of juice for £1.80 or 3 litres of juice for £1.50

2 kg of carrots for £1.28 or 7 kg of carrots for £4.20

---

9 chocolates cost £2.25

Box A

20 chocolates cost £4.00

Box B

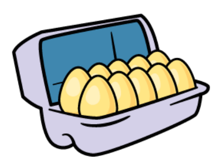
24 chocolates cost £4.30

Box C

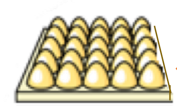
What does each of the calculations tell you?

$2.25 \div 9 = \square$	$20 \div 4 = \square$	$24 \div 4.30 = \square$
$4.30 \div 24 = \square$	$4 \div 20 = \square$	$9 \div 2.25 = \square$

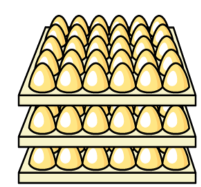
Use your answers to put the boxes in order of best value for money to worst value for money.
Check your answer by calculating the cost of 360 chocolates for each size of box.



10 eggs £1.60



25 eggs £3.70



90 eggs £13.50

Find three ways to work out which box of eggs is best value.

Combine a set of ratios

Notes and guidance

In order to combine ratios, students need to be secure in finding the lowest common multiple and in working with equivalent ratios. Pictorial methods are very helpful here and students could draw the objects (as in the sweets example), or use bar models to represent the number of parts. “Scaling up” the ratios until a common multiple is found is another very useful strategy.

Key vocabulary

Ratio	Equivalent	Combine
For every … there are …		LCM

Key questions

Why are equivalent ratios useful in this question?

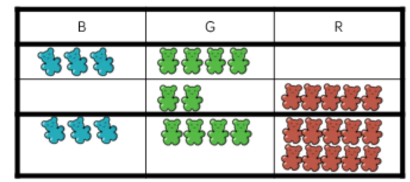
Are the parts already equivalent or do you need to use an equivalent ratio to make them equal?

How could you draw a bar model to help you?

Exemplar Questions

The ratio of the number of blue to green sweets is 3 : 4
The ratio of the number of green to red sweets is 2 : 5
Dora finds the ratio of blue sweets to green sweets to red sweets using multiples.

B : G = 3 : 4
G : R = 2 : 5 = 4 : 10
Now there are 4 red sweets in both ratios so,
B : G : R = 3 : 4 : 10



Use Dora’s method to solve this problem.

The ratio of the number of cats to dogs in a pet shop is 2 : 5
The ratio of the number of dogs to rabbits in the shop is 3 : 10
Find the ratio of Cats : Dogs : Rabbits.

The ratio of strawberry muffins to chocolate muffins is 2 : 3
The ratio of strawberry muffins to blueberry muffins is 6 : 5
What is the ratio of strawberry to chocolate to blueberry muffins?
Jack says that there are 96 muffins altogether. Is this possible?

The ratio of the number of pens to pencils in my pencil case is 5 : 2
There are three times as many pens as rubbers.

- Write the ratio of the number of pens to pencils to rubbers.
- There are 30 pencils in the case. I pick one object at random from the case . What is the probability it is a pencil?

Link ratios to algebra

Notes and guidance

This step explores both the use of algebraic notation within ratios and the linking of ratio questions to problems that need to be tackled through e.g. forming and solving equations. If the ratios $a : b$ and $c : d$ are equal then the key concept that $\frac{a}{b} = \frac{c}{d}$ is often useful to solve complex looking problems.

Key vocabulary

Variable	Unknown	Equation
Equivalent	Express	

Key questions

Express a in terms of b if (e.g.) $a : b = 2 : 3$
How is this different from expressing b in terms of a?

Can you draw a bar model to represent the ratio? Is it more useful to draw a single bar or a comparison bar?

Exemplar Questions

The ratio $a : b$ is equal to $4 : 3$
Explain which of the statements are true and which are false.

$a < b$	$\frac{a}{b} = \frac{4}{3}$	$b < a$	$a = \frac{4}{3}b$	$a = \frac{3}{4}b$

Create your own true/false puzzle if $a : b$ is equal to $2 : 1$

$x : y$ is equal to $5 : 3$
Work out x and y if
- $x + y = 240$
- $x - y = 240$

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b is 50% larger than a
Write $a : b$ in simplest form.
Write $a : b$ in the form $1 : n$

$a : b = 1 : 3$ and $b : c = 4 : 5$
Find the ratio $a : b : c$

Tom is twice as old as Kim.
Nijah is 10 years older than Tom.
The total age of all 3 people is 60 years.
Find the ratio of
Tom's age : Kim's age : Nijah's age

Amir and Mo share sweets in the ratio $5 : 3$
Amir gives Mo 5 of his sweets and the ratio is now $9 : 7$
Complete the solution to find how many sweets they shared.

At first, Amir has $5x$ sweets and Mo has $3x$ sweets
Then Amir has $5x - 5$ sweets and Mo has ☐ sweets
So $\frac{5x-5}{☐} = \frac{9}{7}$
$7(5x - 5) = 9(☐)$
etc.

Dora thinks you can solve the problem using multiples. Investigate Dora's claim.

White Rose Maths

Ratio in area problems

H

Notes and guidance

Students have explored the effect of enlargement on the areas of similar shapes in the Autumn term, looking at squaring scale factors. This is an opportunity to revisit this learning using ratio notation alongside that of scale factors. It can also be an opportunity to revisit area problems and those that involve Pythagoras' theorem and trigonometry.

Key vocabulary

Enlarge	Length/Area scale factor
Length/Area Ratio	Similar

Key questions

How can we use the ratio of the areas of two similar shapes to find the scale factor of their areas?

If we know the ratio of the areas of two shapes, how can we find the ratio of the lengths of their sides?

Exemplar Questions

Sketch a rectangle with dimensions 6 cm by 8 cm.
Enlarge the rectangle so the ratio of the side lengths of the new rectangle to the side lengths of the original rectangle are $3 : 2$

- Find the ratios of the areas of your rectangles
- Repeat by enlarging the original rectangle by the ratio $5 : 2$
- Generalise your findings

Triangle B is made by enlarging triangle A by a scale factor $\frac{1}{3}$
Which of the statements are true and which are false?

Perimeter A : Perimeter B $= 1 : 3$

Perimeter A : Perimeter B $= 3 : 1$

Area A : Area B $= 9 : 1$

Area A : Area B $= 1 : 6$

Area A : Area B $= 6 : 1$

Triangle C is an enlargement of triangle B by scale factor 4
Find the ratios:

Area B : Area C

Area C : Area A

Area A : Area C

The ratio of the surface area of solid X to the surface area of solid Y is $4 : 9$
The total length of the edges of solid Y is 180 cm.
Find the total length of the edges of solid Y

Ratio in volume problems

H

Notes and guidance

As with the previous step, students have explored the effect of enlargement on the volumes of similar shapes in the Autumn term, looking at cubing scale factors. This is an opportunity to revisit this learning using ratio notation alongside that of scale factors. It can also be an opportunity to revisit the use of volume formulae or the use of trigonometry in 3-D shapes.

Key vocabulary

Enlarge	Length/Volume scale factor
Length/Volume Ratio	Similar

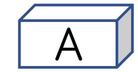
Key questions

If you know the ratio of one volume to another volume and that they are similar solids, how can you use this ratio to work out missing lengths or areas?

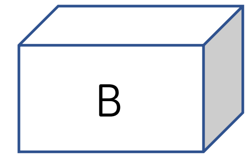
How can you find the ratio of the volumes of two shapes if you only know their surface areas?

Exemplar Questions

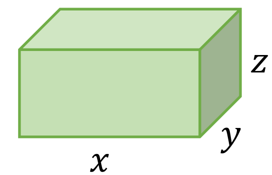
The dimensions of cuboid A are 1 cm, 2 cm and 3 cm.
Find the volume of cuboid A.



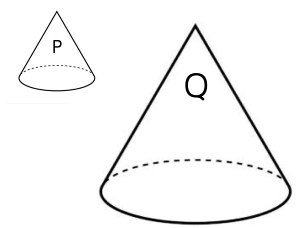
The ratio of the lengths of the sides of cuboid A to those of cuboid B is $1 : 8$
What are the dimensions of cuboid B?
What is the volume of cuboid B?
Write down the ratio Volume A : Volume B.



The ratio of $x : y : z = 5 : 3 : 2$
The volume of the cuboid is 240 cm^3
Find the side lengths x, y ad z.
State the volume of a cuboid of with sides $\frac{x}{2}, \frac{y}{2}$ and $\frac{z}{2}$



Cone P is similar to cone Q
The height of cone P is 4 cm.
The height of cone Q is 6 cm.
The volume of cone Q is 135 cm^3
Work out the volume of cone P.



The volumes of two similar cylinders are in the ratio $8 : 27$
If the surface area of the smaller cylinder is 40 cm^2, what is the surface area of the larger cylinder?

Mixed ratio problems

Notes and guidance

It is very useful for students to be able to reflect on a variety of topics covered rather than just see them discretely, so the purpose of this step is to provide opportunities to look again at various aspects of this unit to reinforce understanding. Teachers may use this to focus in on any areas of particular difficulty or to explore ratios in other topics that may need revision.

Key vocabulary

Enlarge	Scale factor	Ratio
Share	Similar	

Key questions

If two shapes are similar, what do we know about the ratios of the side lengths?

How could a bar model represent this problem?

Exemplar Questions

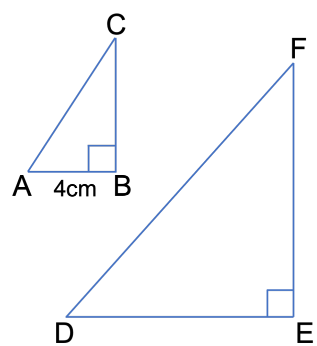
80 students study either French or Spanish.
There are 52 girls altogether.
12 of the boys study French.
Of the students studying Spanish, the ratio of boys to girls is $2 : 3$
Draw and complete a two-way table showing this information.
Find the ratio of the number of students studying French to the number of students studying Spanish.

The ratio of the angles in a triangle is $3 : 4 : 5$
Show that the triangle does not contain a right angle.

The angles in a quadrilateral are a, b, c and d.

$a = 90°$ | $a : b = 3 : 5$ | $c : d = 2 : 3$

Work out the sizes of angles a, b, c and d.



Triangle ABC and DEF are right-angled triangles.

AB : DE $= 1 : 3$
BC : EF $= 1 : 2$
AB : BC $= 2 : 3$

- Find the area of triangle DEF.
- Find the length AC.
- Explain why ABC and DEF are not similar triangles.