Algebraic Notation

Year 7

#MathsEveryoneCan

2019-20

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Autumn 1: Algebraic thinking

Week 1: Exploring Sequences

Rather than rushing to find rules for n^{th} term, this week is spent exploring sequences in detail, using both diagrams and lists of numbers. Technology is used to produce graphs so students can appreciate and use the words "linear" and "non-linear" linking to the patterns they have spotted. Calculators are used throughout so number skills are not a barrier to finding the changes between terms or subsequent terms. Sequences are treated more formally later this unit.
National curriculum content covered:

- move freely between different numerical, algebraic, graphical and diagrammatic representations
- make and test conjectures about patterns and relationships
- use a calculator and other technologies to calculate results accurately and then interpret them appropriately
- generate terms of a sequence from a term-to-term rule
- recognise arithmetic sequences
- recognise geometric sequences and appreciate other sequences that arise

Weeks 2 to 4: Understanding and using algebraic notation

The focus of these three weeks is developing a deep understanding of the basic algebraic forms, with more complex expressions being dealt with later. Function machines are used alongside bar models and letter notation, with time invested in single function machines and the links to inverse operations before moving on to series of two machines and substitution into short abstract expressions.
National curriculum content covered:

- move freely between different numerical, algebraic, graphical and diagrammatic representations
- use algebra to generalise the structure of arithmetic, including to formulate mathematical relationships
- recognise and use relationships between operations including inverse operations
- model situations or procedures by translating them into algebraic expressions
- substitute values in expressions, rearrange and simplify expressions
- use and interpret algebraic notation, including:
  - ab in place of $a \times b$
  - $3y$ in place of $y + y + y$ and $3 \times y$
  - a^2 in place of $a \times a$
  - ab in place of $a \times b$
  - $\frac{a}{b}$ in place of $a \div b$
- generate terms of a sequence from a term-to-term rule
- produce graphs of linear functions of one variable

Weeks 5 and 6: Equality and equivalence

In this section students are introduced to forming and solving one-step linear equations, building on their study of inverse operations. The equations met will mainly require the use of a calculator, both to develop their skills and to ensure understanding of how to solve equations rather than spotting solutions. This work will be developed when two-step equations are met in the next place value unit and throughout the course. The unit finishes within consideration of equivalence and the difference between this and equality, illustrated through collecting like terms.
National curriculum content covered:

- use algebra to generalise the structure of arithmetic, including to formulate mathematical relationships
- simplify and manipulate algebraic expressions to maintain equivalence by collecting like terms
- use approximation through rounding to estimate answers
- use algebraic methods to solve linear equations in one variable

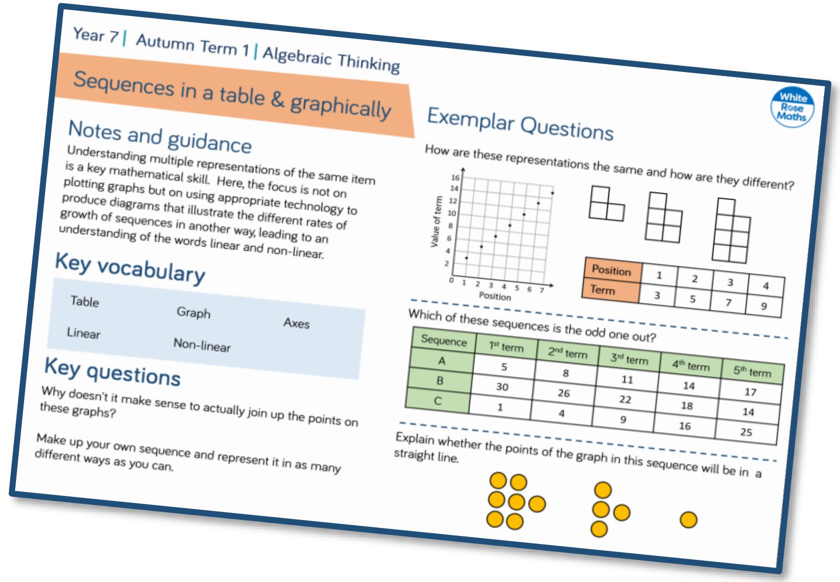
Why Small Steps?

We know that breaking the curriculum down into small manageable steps should help students to understand concepts better. Too often, we have noticed that teachers will try and cover too many concepts at once and this can lead to cognitive overload. We believe it is better to follow a "small steps" approach.

As a result, for each block of content in the scheme of learning we will provide a "small step" breakdown. ***It is not the intention that each small step should last a lesson – some will be a short step within a lesson, some will take longer than a lesson.*** We would encourage teachers to spend the appropriate amount of time on each step for their group, and to teach some of the steps alongside each other if necessary.

What We Provide

- Some ***brief guidance*** notes to help identify key teaching and learning points
- A list of ***key vocabulary*** that we would expect teachers to draw to students' attention when teaching the small step,
- A series of ***key questions*** to incorporate in lessons to aid mathematical thinking.
- A set of questions to help ***exemplify*** the small step concept that needs to be focussed on.



Year 7 | Autumn Term 1 | Algebraic Thinking

Sequences in a table & graphically

Notes and guidance

Understanding multiple representations of the same item is a key mathematical skill. Here, the focus is not on plotting graphs but on using appropriate technology to produce diagrams that illustrate the different rates of growth of sequences in another way, leading to an understanding of the words linear and non-linear.

Key vocabulary

Table, Graph, Axes, Linear, Non-linear

Key questions

Why doesn't it make sense to actually join up the points on these graphs?

Make up your own sequence and represent it in as many different ways as you can.

Exemplar Questions

How are these representations the same and how are they different?

Position	1	2	3	4
Term	3	5	7	9

Which of these sequences is the odd one out?

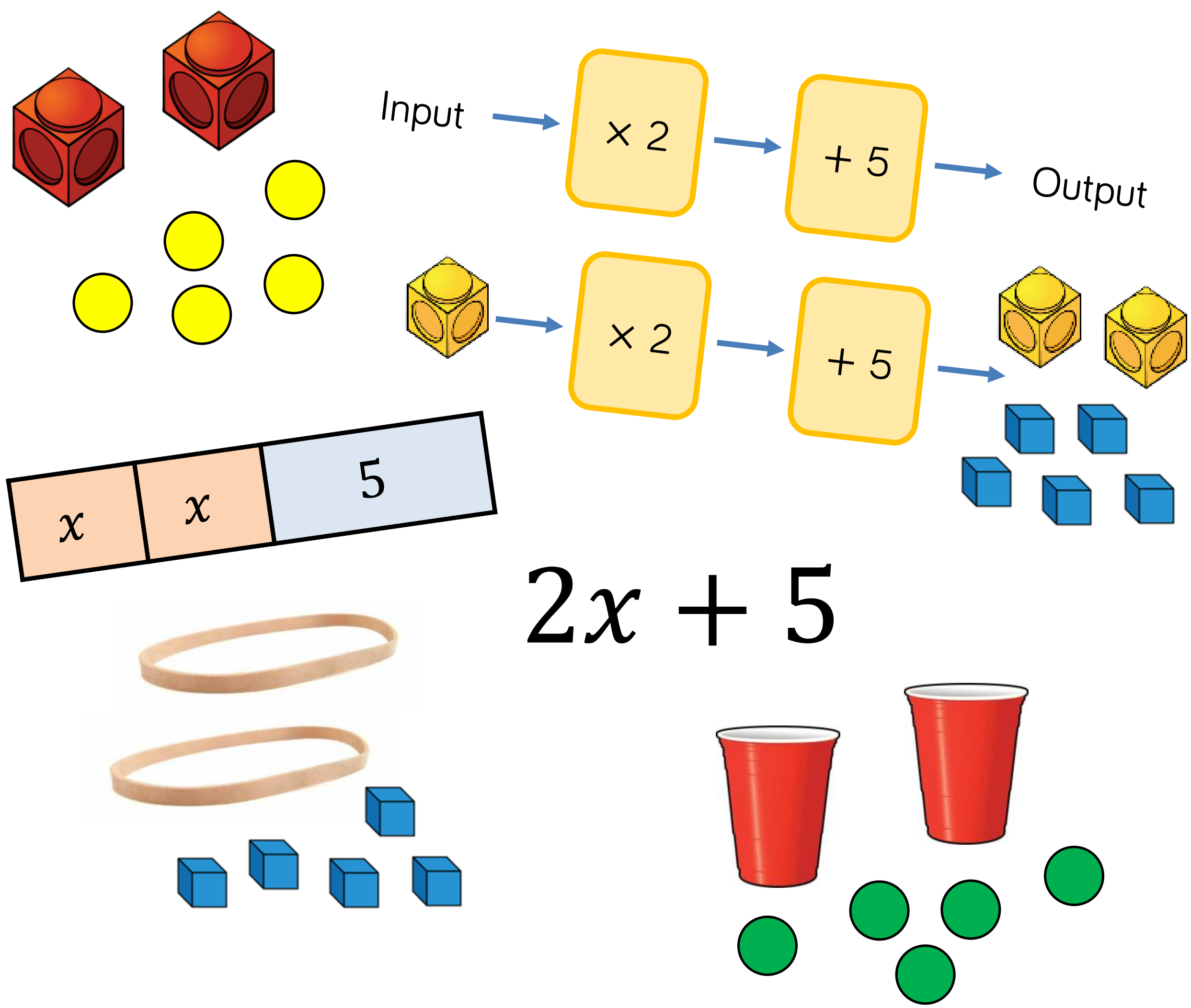
Sequence	1st term	2nd term	3rd term	4th term	5th term
A	5	8	11	14	17
B	30	26	22	18	14
C	1	4	9	16	25

Explain whether the points of the graph in this sequence will be in a straight line.

- These include reasoning and problem-solving questions that are fully integrated into the scheme of learning. Depending on the attainment of your students, you many wish to use some or all of these exemplars, which are in approximate order of difficulty. Particularly challenging questions are indicated with the symbol .
- For each block, we also provide ideas for key representations that will be useful for all students.

In many of the blocks of material, some of the small steps are in **bold**. These are content aimed at higher attaining students, but we would encourage teachers to use these with as many students as possible – if you feel your class can access any particular small step, then please include it in your planning.

Key Representations



Concrete, pictorial and abstract representations are an important part of developing students' conceptual understanding.

Here are a few ideas for how you might represent algebra. Cups, cubes and elastic bands lend themselves well to representing an unknown, whereas ones (from Base 10) and counters work well to represent a known number.

Be careful to ensure that when representing an unknown students use equipment that does not have an assigned value – such as a Base 10 equipment and dice.

Understand and use notation

Small Steps

- Given a numerical input, find the output of a single function machine
- Use inverse operations to find the input given the output
- Use diagrams and letters to generalise number operations
- Use diagrams and letters with single function machines
- Find the function machine given a simple expression
- Substitute values into single operation expressions
- Find numerical inputs and outputs for a series of two function machines
- Use diagrams and letters with a series of two function machines
- Find the function machines given a two-step expression
- Substitute values into two-step expressions
- Generate sequences given an algebraic rule
- Represent one- and two-step functions graphically

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Single function machines (number)

Notes and guidance

The aim of this small step is for students to become fluent in the use of single function machines with numbers, working from left to right. Students also need to learn the associated vocabulary of "input" and "output". Wherever appropriate, calculator use should be encouraged.

Key vocabulary

Function	Input	Output
Estimate	Operation	Square

Key questions

How can we check if the answer from our calculator is reasonable?

What happens to the size of the outputs if we change the size of the inputs?

Exemplar Questions

Find the outputs when you input 0, 1, 2, 3, 4 and 5 into these machines. What's the same and what's different?

→ $+4$ →   → $\div 4$ →

Find the output for these function machines if 17 is input into each of them.

→ $+ 86$ →   → -37 →

→ $\div 5$ →   → Square →

Before doing the calculations, can you estimate which of these machines will have the biggest output for the given inputs?

1876 → $+ 12\ 000$ →

418 → $\times 96.12$ →

How many functions can you think of where the output is always the same as the input?

Find the input given the output

Notes and guidance

Using students' knowledge of inverse operations, we will now consider using a function machine from right to left to find the input for a given output. Again, calculator skills should be developed including how to use the square and square root functions.

Key vocabulary

Function	Input	Output
Estimate	Operation	Inverse

Key questions

What calculation can we do to check that our answer for the input is correct?

What happens to the size of the outputs if we change the size of the inputs?

Exemplar Questions

Find the input for these function machines if the output for each of them is 100

→ $+ 86$ →   → $\times 1.25$ →

→ $\div 5$ →   → -37 →   → Square →

Find two possible machines that give the output 10 for an input of 5

5 → ? → 10

What could the machines have been if the input had been 10 and the output had been 5?

Find the outputs for this function machine.

3.1 →
5 →
8.2 →

Subtract from 10 →

Put the outputs back in to the function machine. What do you notice?

Investigate inverse functions on your calculator.

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Use letters to generalise number

Notes and guidance

This small step is where students are explicitly taught algebraic notation and may well need a few lessons. At each stage it is important to keep reminding students that each representation stands for a number. Multiple representations including concrete materials and bar models should be used alongside each other to encourage flexible thinking, but emphasis needs to be placed on correct algebraic notation.

Key vocabulary & notation

Bar model	Variable	Coefficient
$3a$ for $a \times 3$	$\frac{a}{3}$ for $a \div 3$	a^2 for $a \times a$
ab for $a \times b$	Commutative	Expression

Key questions

What's different about using a letter to represent a number compared to using a bar?

Exemplar Questions

How are these sets of calculations the same and how are they different?

$10 + 10 + 10 + 10$
4×10
10×4

$6 + 6 + 6 + 6$
4×6
6×4

$a + a + a + a$
$4 \times a$
$a \times 4$

Write these expressions without mathematical operation signs.

$f + f + f + f + f + f$	$7 \times g$
$t \div 5$	$5 \div t$
$m \times m$	$d \times c$

Zeb says p^2, $p2$ and $2p$ are all exactly the same.

Explain why Zeb is **wrong**.

Use diagrams to help.

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Single function machines (algebra)

Notes and guidance

Here we are linking the last few steps to reinforce students understanding of algebraic notation and linking it to bar model representations. Use of concrete resources such as multi-link cubes to represent unknowns alongside should also be encouraged, but take care not to use objects like ten-sticks that have a pre-defined value to stand for variables as this can lead to confusion.

Key vocabulary & notation

Bar model	Variable	Coefficient
$3a$ for $a \times 3$	$\frac{a}{3}$ for $a \div 3$	a^2 for $a \times a$
ab for $a \times b$	Commutative	Expression

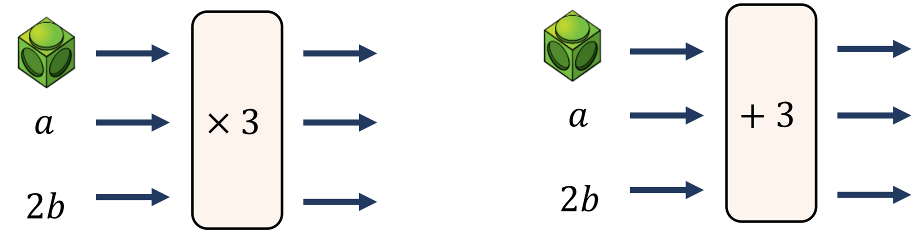
Key questions

What's different about using a letter to represent a number compared to using a bar?

Will outputs like $a + 3$ and $3a$ always, sometimes or never be the same?

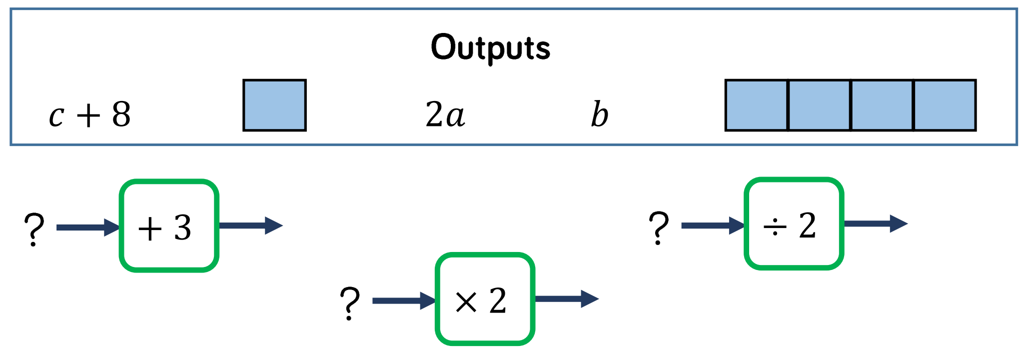
Exemplar Questions

Find the **output** for each of the function machines with these inputs.

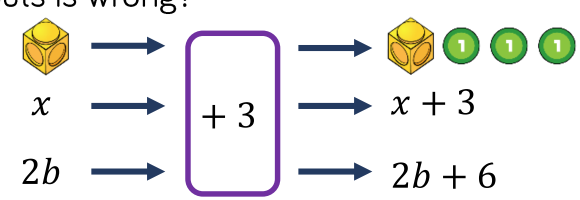


Investigate other function machines e.g. "÷ 2"

Find the **input** for each of the function machines with each of these outputs.



Which of these outputs is wrong?



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Find functions from expressions

Notes and guidance

In this small step students are developing their fluency and understanding by reversing the process of the previous step.

Given an expression involving a single operation applied to a variable, they identify the function that has taken place and so find the function machine.

Key vocabulary & notation

Bar model	Variable	Coefficient
$3a$ for $a \times 3$	$\frac{a}{3}$ for $a \div 3$	a^2 for $a \times a$
ab for $a \times b$	Commutative	Expression

Key questions

What does the expression $6a$ mean?

Why are the expressions $\frac{a}{2}$ and $\frac{2}{a}$ different?

Exemplar Questions

For each of these function machines, find the function that gives the outputs shown for the given inputs

$a \rightarrow$? $\rightarrow 5a$ $b \rightarrow$? $\rightarrow b - 3$

$10c \rightarrow$? $\rightarrow 2c$ $y \rightarrow$? $\rightarrow xy$

$x \rightarrow$? $\rightarrow x^2$ $d \rightarrow$? $\rightarrow d - g$

Do any of the machines have more than one possible answer?

Complete the missing information for this function machine.

Input		Output
4	?	8
10		20
?		16
$10c$		?
?		$6b$

Fred says the machine is "$\times$ 2", Bertha says it's "$+ a$".
Who do you agree with?

$a \rightarrow$? $\rightarrow 2a$

White Rose Maths

Substitute into single expressions

Notes and guidance

In this small step, students are practising their calculator skills and using the expressions they have learnt in the more abstract context of stand-alone expressions. These can be used in conjunction with function machine diagrams if needed. Comparing answers of different expressions will link to sequences studied earlier, and inform later work on equivalence.

Key vocabulary & notation

Expression	Evaluate	Substitute
$3a$ for $a \times 3$	$\frac{a}{3}$ for $a \div 3$	a^2 for $a \times a$
ab for $a \times b$		

Key questions

Are $t + 5$ and $5 + t$ always, sometimes or never equal?

Are $2p$ and p^2 always, sometimes or never equal?

What is different about the expressions $p - 4$ and $4 - p$?

Exemplar Questions

Substitute $a = 5$ into each of these expressions.

$7a$	$\frac{7}{a}$	$19.8 - a$	a^2
$2a$	$a - 3.6$	$a + 3.6$	

Which of these expressions will be equal when $x = 2$?

$2x$	$\frac{x}{2}$	$\frac{2}{x}$	$x + 2$
$2 + x$	$x - 2$	$2 - x$	x^2

Put the expressions in order from smallest to largest for different values of x (Try $x = 1,\ x = 0.4,\ x = 100,\ x = 0 \dots$)
Which expressions will always be equal, whatever the value of x?

Substitute $n = 1, n = 2, n = 3,\ n = 4$ and $n = 5$ into all of these expressions.

$n + 7$	$3n$	n^2
$20 - n$	$\frac{n}{2}$	$\frac{2}{n}$

What do you notice about each set of answers?

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2-step function machines (number)

Notes and guidance

Students now move on to using two function machines in a row, so that the output of the first machine is the input of the second machine. Students need to become fluent in this process with numbers, both forward and backward, before moving on to the next step where they use concrete objects, diagrams and letters.

Key vocabulary

Input	Output	Inverse

Key questions

Why do you do the inverse operations in reverse order when finding the input to a pair of function machines?

Exemplar Questions

Find the output of this series of two function machines.

3.7 → $\times 5$ → $+1$ →

I think of a number, double it and then add on 9
The result is 22.4

- Show this using a series of function machines..
- Use inverse operations to work out the number I started with.

Aisha says these pairs of function machines will have the same output as they are the same functions.

Input → $\times 3$ → $+7$ → Output

Input → $+7$ → $\times 3$ → Output

Give an example to show that Aisha is wrong.

White Rose Maths

2-step function machines (algebra)

Notes and guidance

Students now build on their experience of two machines in the previous step by using objects, bar models and letters. They will need to be taught that the order in which the functions are applied is important and will need to be introduced to brackets in algebraic expressions to distinguish between e.g. $2x + 5$ and $2(x + 5)$. Formal expanding of brackets is not expected at this stage.

Key vocabulary

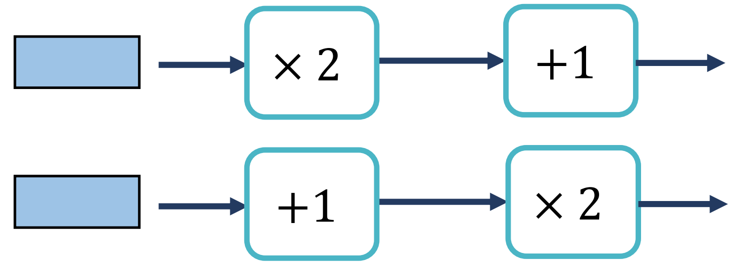
Input	Output	Order
Bracket	Variable	Expression

Key questions

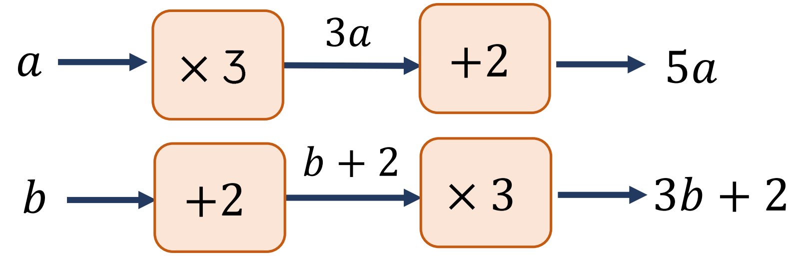
Does it sometimes, always or never make a difference if you change the order of a pair of function machines?

Exemplar Questions

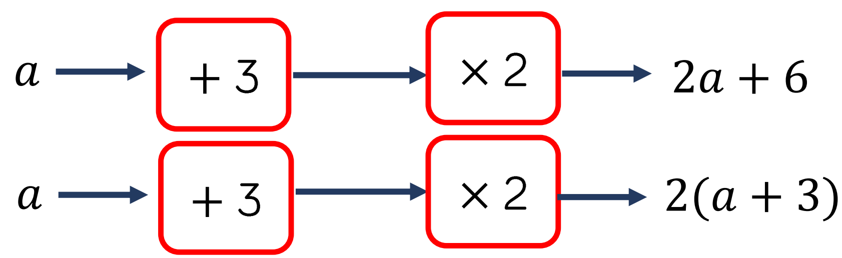
Compare the outputs of these pairs of function machines.



Correct the mistakes in the working below.



Use bars or concrete materials to show that both these answers are correct.



White Rose Maths

Find functions from expressions

Notes and guidance

In this small step, students show their understanding of two-step expressions by reversing the process of the previous step and finding the operations that formed the expressions. It should again be reinforced that the letter represents any number, possibly by teaching the next small step alongside this one.

Key vocabulary

Input	Output	Order
Bracket	Variable	Expression

Key questions

What's the difference between $\frac{a+4}{2}$ and $\frac{a}{2}+4$?

Is there more than one way of applying two consecutive functions to x and obtaining $2x+4$?

Exemplar Questions

Fill in the gaps in these function machines.

$x \rightarrow$? $\rightarrow$? $\rightarrow 5x-6$

$y \rightarrow$? $\rightarrow$? $\rightarrow \frac{y}{2}-4$

$w \rightarrow$? $\rightarrow$? $\rightarrow 3(w+1)$

$t \rightarrow$? $\rightarrow$? $\rightarrow \frac{t-2}{4}$

Complete the missing information for this function machine.

$6 \rightarrow$? $\rightarrow +2 \rightarrow 20$

$? \rightarrow$? $\rightarrow +2 \rightarrow 3x+2$

$? \rightarrow$? $\rightarrow +2 \rightarrow 9y+2$

Investigate.

$\rightarrow \times 5 \rightarrow \div 5 \rightarrow$

White Rose Maths

Substitute into two-step expressions

Notes and guidance

Students are again practising their calculator skills, now using the two-step expressions they have learnt. They can compare the similarities and differences between e.g. $3a + 2$ and $3(a + 2)$ for a wide variety of inputs. Substituting repeatedly into the same expression is a valuable experience with opportunities for discovery.

Key vocabulary

Expression	Evaluate	Substitute
Variable	Constant	

Key questions

How would you use your calculator to wok out the value of the square of a number?

When do you need to use brackets when substituting into expressions using a calculator?

Exemplar Questions

Substitute different values of x into these two expressions – include integers, decimals, negatives and fractions.

$2(x + 4)$ $2x + 8$

What do you notice?
Can you use function machines and diagrams to explain why?

Which of these is the largest when $a = 1$ and $b = 0.1$?

ab $\frac{a}{b}$ $\frac{b}{a}$ $a + b$ $a - b$

How would this change if $a = 0.1$ and $b = 0.01$?
Investigate for other values of a and b

Pick values of a and b to substitute into this expression.

$a^2 + 2b$

- How do the values of the expression change if you keep a the same and vary b?
- How do the values of the expression change if you keep b the same and vary a?

Generate sequences from a rule

Notes and guidance

In this small step, students revisit the ideas from week 1 combining their knowledge with that of the substitution they have just learnt. At this stage students do not need to learn a procedure for finding a rule for the n^{th} term of a linear sequence, but they may well make connections between the sequences found and the rules given. The language of sequences can also be reinforced in this step.

Key vocabulary

Sequence	Non-linear	Linear
Rule	Term-to-term	Position-to-term

Key questions

What feature of the difference between terms tells us if a sequence is linear?

Which type of rule is better for finding the 100^{th} term of a sequence?

Exemplar Questions

Substitute $n = 1,\ n = 2,\ n = 3,\ n = 4$ and $n = 5$ into the expression $3n + 5$

- What do you notice about your answers?

Repeat for $3n + 6$ and then $2n + 5$

- What stays the same? What changes?

Use your calculator to find the first ten terms of the sequences given by these rules.

n^2 $\qquad 2^n$ $\qquad n^2 - 4$ $\qquad (n - 4)^2$

What are the similarities and differences?

Which of these rules do you think will produce linear sequences?

$\frac{n}{2} + 4$ $\qquad 150 - 8n$ $\qquad 6n + 0.2$

$3 + n^2$ $\qquad \frac{n-3}{4}$

Check by substituting several consecutive values of n.

Represent functions graphically

Notes and guidance

In this small step, students use technology to plot the graphs of some of the functions they have been working with to reinforce the vocabulary of linear and non-linear. There is no need to formally investigate the equations of lines at this stage, but students should be encouraged to spot similarities and differences.

Key vocabulary

Graph	Axis	Axes	Scale
Equation	Linear	Non-linear	Curve

Key questions

How can you tell from an equation whether the graph is going to be linear?

How does this link to linear and non-linear sequences?

Exemplar Questions

Use a graphing program to compare the graph of the sequence given by the rule $2n + 1$ with the graph given by the equation $y = 2x + 1$

What are the similarities and differences?

Compare the graphs of $y = 2x$ and $y = x^2$

What are the similarities and differences?

Without using a graph plotter, decide which of these equations will produce a straight line graph

- $y = 3x + 2$
- $y = 2 + 3x$
- $y = x^2 + 3$
- $y = 6 - \frac{x}{2}$
- $y = \frac{2}{x} + 6$
- $y = 5 - x$

Check your answers with a graph plotter.
Which shapes were most surprising?