Sequences

Year 8

#MathsEveryoneCan

White Rose Maths

	Week 1	Week 2	Week 3	Week 4	Week 5	Week 6	Week 7	Week 8	Week 9	Week 10	Week 11	Week 12
Autumn	**Proportional Reasoning**						**Representations**					
	Ratio and scale		Multiplicative change		Multiplying and dividing fractions		Working in the Cartesian plane			Representing data		Tables & Probability
Spring	**Algebraic Techniques**						**Developing Number**					
	Brackets, equations and inequalities				Sequences	Indices	Fractions and percentages		Standard index form		Number sense	
Summer	**Developing Geometry**						**Reasoning with Data**					
	Angles in parallel lines and polygons		Area of trapezia and circles		Line symmetry and reflection		The data handling cycle				Measures of location	

Spring 1: Algebraic Techniques

Weeks 1 to 4: Brackets, Equations & Inequalities

Building on their understanding of equivalence from Year 7, students will explore expanding over a single bracket and factorising by taking out common factors. The higher strand will also explore expanding two binomials. All students will revisit and extend their knowledge of solving equations, now to include those with brackets and for the higher strand, with unknowns on both sides. Bar models will be recommended as a tool to help students make sense of the maths. Students will also learn to solve formal inequalities for the first time, learning the meaning of a solution set and exploring the similarities and differences compared to solving equations. Emphasis is placed on both forming and solving equations rather than just looking at procedural methods of finding solutions.

National curriculum content covered:

- identify variables and express relationships between variables algebraically
- begin to model situations mathematically and express the results using a range of formal mathematical representations
- substitute numerical values into formulae and expressions, including scientific formulae
- understand and use the concepts and vocabulary of expressions, equations, inequalities, terms and factors
- simplify and manipulate algebraic expressions to maintain equivalence by:
    - collecting like terms
    - multiplying a single term over a bracket
    - taking out common factors
    - expanding products of two or more binomials
- understand and use standard mathematical formulae
- use algebraic methods to solve linear equations in one variable

Week 5: Sequences

This short block reinforces students' learning from the start of Year 7, extending this to look at sequences with more complex algebraic rules now that students are more familiar with a wider range of notation. The higher strand includes finding a rule for the n^{th} term for a linear sequence, using objects and images to understand the meaning of the rule.

National curriculum content covered:

- generate terms of a sequence from either a term-to-term or a position-to-term rule
- recognise arithmetic sequences and find the n^{th} term
- recognise geometric sequences and appreciate other sequences that arise

Week 6: Indices

Before exploring the ideas behind the addition and subtraction laws of indices (which will be revisited when standard form is studied next term), the groundwork is laid by making sure students are comfortable with expressions involving powers, simplifying e.g. $3x^2y \times 5xy^3$. The higher strand also looks at finding powers of powers.

National curriculum content covered:

- use and interpret algebraic notation, including a^3 in place of $a \times a \times a$; a^2b in place of $a \times a \times b$
- use language and properties precisely to analyse algebraic expressions
- begin to model situations mathematically and express the results using a range of formal mathematical representations
- substitute values in expressions, rearrange and simplify expressions, and solve equations

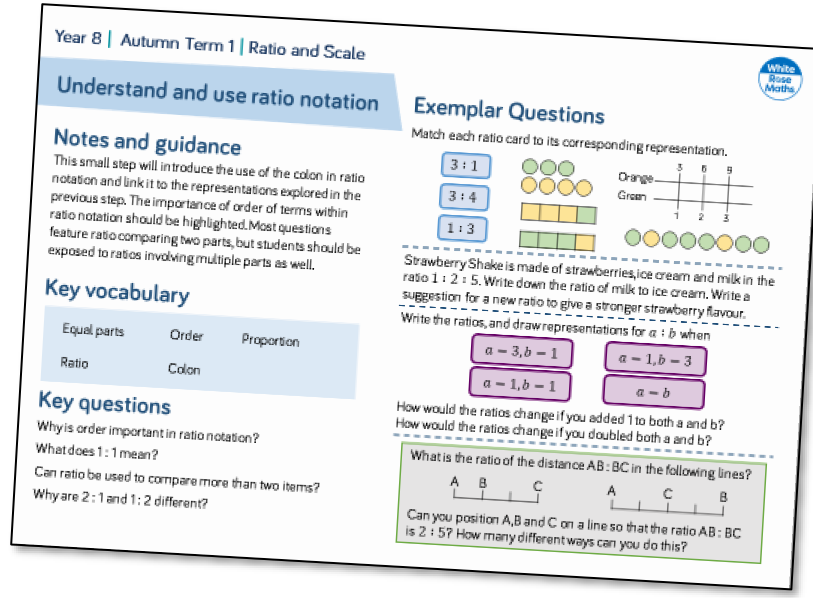
Why Small Steps?

We know that breaking the curriculum down into small manageable steps should help students to understand concepts better. Too often, we have noticed that teachers will try and cover too many concepts at once and this can lead to cognitive overload. We believe it is better to follow a "small steps" approach.

As a result, for each block of content in the scheme of learning we will provide a "small step" breakdown. ***It is not the intention that each small step should last a lesson – some will be a short step within a lesson, some will take longer than a lesson.*** We would encourage teachers to spend the appropriate amount of time on each step for their group, and to teach some of the steps alongside each other if necessary.

What We Provide

- Some ***brief guidance*** notes to help identify key teaching and learning points
- A list of ***key vocabulary*** that we would expect teachers to draw to students' attention when teaching the small step,
- A series of ***key questions*** to incorporate in lessons to aid mathematical thinking.
- A set of questions to help ***exemplify*** the small step concept that needs to be focussed on.


Year 8 | Autumn Term 1 | Ratio and Scale

Understand and use ratio notation

Notes and guidance

This small step will introduce the use of the colon in ratio notation and link it to the representations explored in the previous step. The importance of order of terms within ratio notation should be highlighted. Most questions feature ratio comparing two parts, but students should be exposed to ratios involving multiple parts as well.

Key vocabulary

Equal parts, Order, Proportion, Ratio, Colon

Key questions

Why is order important in ratio notation?
What does 1 : 1 mean?
Can ratio be used to compare more than two items?
Why are 2 : 1 and 1 : 2 different?

Exemplar Questions

Match each ratio card to its corresponding representation.

3 : 1, 3 : 4, 1 : 3

Strawberry Shake is made of strawberries, ice cream and milk in the ratio 1 : 2 : 5. Write down the ratio of milk to ice cream. Write a suggestion for a new ratio to give a stronger strawberry flavour.

Write the ratios, and draw representations for $a : b$ when

$a = 3, b = 1$; $a = 1, b = 3$; $a = 1, b = 1$; $a = b$

How would the ratios change if you added 1 to both a and b?
How would the ratios change if you doubled both a and b?

What is the ratio of the distance AB : BC in the following lines?

Can you position A, B and C on a line so that the ratio AB : BC is 2 : 5? How many different ways can you do this?

- These include reasoning and problem-solving questions that are fully integrated into the scheme of learning. Depending on the attainment of your students, you many wish to use some or all of these exemplars, which are in approximate order of difficulty. Particularly challenging questions are indicated with the symbol .
- For each block, we also provide ideas for key representations that will be useful for all students.

In many of the blocks of material, some of the small steps are in **bold**. These are content aimed at higher attaining students, but we would encourage teachers to use these with as many students as possible – if you feel your class can access any particular small step, then please include it in your planning.

Sequences

Small Steps

- Generate sequences given a rule in words
- Generate sequences given a simple algebraic rule
- Generate sequences given a complex algebraic rule
- **Find the rule for the n^{th} term of a linear sequence** H

H denotes higher strand and not necessarily content for Higher Tier GCSE

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Sequences from rules in words

Notes and guidance

Building on from year 7, students revisit the idea of forming a sequence given a rule in words. They should now be able to deal with more complex multi-step rules, and operations such as cubing and rooting. This step is a good chance to revisit the vocabulary of sequences, and students should also be able to use correct language to fully describe a given simple sequence. Exploring Fibonacci sequences is worthwhile.

Key vocabulary

Sequence	Position	Term
Linear	Non-linear	Fibonacci
Difference	Constant	Term-to-term

Key questions

What's the name for a sequence where there is a constant difference between successive terms?
What would the graph of such a sequence look like?
What information do you need to give to fully describe a sequence? Why is e.g. 'it goes up in 3s' not enough?

Exemplar Questions

Compare these sequences by working out the first five terms.

Sequence A
The first term is 10
Each term is four greater than the previous term.

Sequence B
The first term is 10
Each term is four smaller than the previous term.

Sequence C
The first term is 10
Each term is four multiplied by the previous term.

Which of the sequences are linear and which are not?

Describe each of these sequences.

Sequence D 10, 15, 20, 25, 30…

Sequence E 10, 10, 10, 10, 10…

Sequence F 10, 4, −2, −8, −14..

Work out the first five terms of each sequence. What do you notice? For which sequence can you easily work out the 100th term?

Double take 5
The first term is 7
Each term is five less than double the previous term.

Square add 1
The first term is 10
Each term is one more than the square of the previous term.

Take from 15
The first term is 4
Each term is the result of subtracting the previous term from 15

Investigate the Fibonacci sequences.

1, 1, 2, 3, 5, 8…

3, 7, 10, 17, 27…

Sequences from algebraic rules

Notes and guidance

As well as providing practice in substitution, this step provides plenty of opportunity for students to develop their reasoning. They can observe the behaviour of the linear sequences in preparation for the later higher step of finding the rule, and solve equations to determine whether a number is a term in a sequence or not by considering if the solutions are integers. Similarly, they could also practice forming and solving inequalities.

Key vocabulary

Algebraic	Integer	Non-integer
Substitute	Linear	Non-linear

Key questions

How can you tell by looking at the rule for the nth term of a sequence whether it is linear or not?

Is it possible for n to take non-integer values? Why or why not?

How can we form an equation to see if the number is in the sequence?

Exemplar Questions

Find the value of these expressions when $n = 1, 2, 3$ and 100

$7n + 4$ | $20 - 3n$ | $\frac{n}{2} - 1$ | $n^2 + 1$

Work out the first five terms of the sequences given by these rules.

$3n - 1$ | $3n + 5$ | $4n + 5$

What connections do you see between your sequences and their algebraic rules?

Rosie: None of the terms in the sequence given by $5n + 2$ will end in 0

Is Rosie correct? How do you know?

A sequence is given by the rule $3n + 7$

- Work out the 45th term of the sequence.
- Form equations to determine which, if any, of these numbers are in the sequence. 113 | 213 | 313
- Form an inequality to find the position of the first term in the sequence that is greater than 1000
- Is the sequence linear? How do you know?

Complex algebraic rules

Notes and guidance

Students explored simple algebraic sequences in Year 7. They have since looked at more complex expressions involving squares, cubes and brackets in much more detail and so this step allows them to practice their substitution skills in the context of sequences; they may need reminders as to the behaviour of directed number. As well as the examples shown, students could also explore fractions e.g. $\frac{n}{n+3}$

Key vocabulary

Algebraic	Bracket	Expand
Substitute	Linear	Non-linear

Key questions

What is the difference between how we work out e.g. $3n^2$ and $(3n)^2$? How do you know?

Do we need to expand the brackets first in order to substitute e.g. $n = 5$ into an expression like $2(n + 3)$?

Exemplar Questions

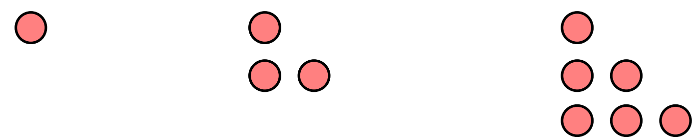
Work out the first five terms of each of the sequences given by the rules on the cards. What's the same and what's different?

n^2 | $(n - 1)^2$ | $(n + 1)^2$ | $(1 - n)^2$

Dora is working with the sequence given by the rule $n(n + 1)$

1st term, $n = 1$ so $n(n + 1) = 1 \times (1 + 1) = 1 \times 2 = 2$
2nd term, $n = 2$ so $n(n + 1) = 2 \times (2 + 1)) = 2 \times 3 = 6$

Complete the working to find the first six terms.
Dexter is using counters to make the triangle numbers.



etc.

Continue Dexter's pattern and work out the first six triangle numbers.

What is the connection between Dora's rule and the triangle numbers?

Compare the sequences given by the pairs of rules.

$3(n + 1)$ | $3n + 1$

$2n^2$ | $(2n)^2$

$5(2 - n)$ | $n(2 - n)$

$3n$ | n^3

Finding the algebraic rule H

Notes and guidance

This higher step should only be completed when students are comfortable with using rules for the n^{th} term and distinguishing between linear and non-linear sequences; this will be revisited in Year 9 and KS4. The aim is for students to understand the connection between the sequence and the associated multiplication table. Linking the sequences to pictures helps bring understanding to the rules.

Key vocabulary

Rule	Term-to-term	Position-to-term
Linear	Non-linear	Coefficient

Key questions

What does n represent here?
How can you tell the sequence is linear?
What is the constant difference in this sequence?
How does this relate to the coefficient of n?
How do the e.g. $3n$ and the $+1$ relate to the pattern?

Exemplar Questions

Work out the first five terms of the sequences give by these rules.

$4n$ $4n+3$ $4n-1$ $4n+7$

Compare the sequences to the 4 times table. What do you notice?

Match these sequences and rules, working out the missing number.

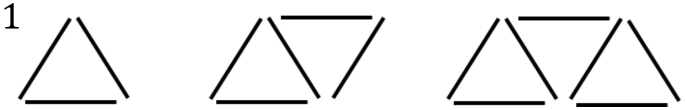
Sequence A	6, 10, 14, 18…	$4n-2$
Sequence B	1, 5, 9, 13…	$4n+2$
Sequence C	9, 13, 17, 13…	$4n+5$
Sequence D	2, 6, 10, 14…	$4n-$___

Which of these sequences follow a rule of the form $3n+$___ or $3n-$___? Why or why not?

A 4, 7, 10, 13…
B 3, 6, 10, 15…
D 1, 4, 7, 10, …
C 30, 27, 24, 21…
E 13, 16, 19, 22, …

Find the rules for the n^{th} term of the sequences that are of the form $3n+$___ or $3n-$___

The rule for the number of sticks needed to make the n^{th} triangle in this pattern is $2n+1$



Why does the number of sticks go up two each time you add a triangle? Why is there a "$+1$" in the rule?