Non-linear graphs

Year 11

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White Rose Maths

White Rose Maths

	Week 1	Week 2	Week 3	Week 4	Week 5	Week 6	Week 7	Week 8	Week 9	Week 10	Week 11	Week 12
Autumn	**Graphs**						**Algebra**					
	Gradients & lines		Non-linear graphs		Using graphs		Expanding & factorising		Changing the subject		Functions	
Spring	**Reasoning**						**Revision and Communication**					
	Multiplicative		Geometric		Algebraic		Transforming & constructing		Listing & describing		Show that...	
Summer	**Revision**						**Examinations**					

Autumn 1: Graphs

Weeks 1 and 2: Gradients and lines

This block builds on earlier study of straight line graphs in years 9 and 10. Students plot straight lines from a given equation, and find and interpret the equation of a straight line from a variety of situations and given information. There is the opportunity to revisit graphical solutions of simultaneous equations. Higher tier students also study the equations of perpendicular lines.
National Curriculum content covered includes:

- move freely between different numerical, algebraic, graphical and diagrammatic representations
- plot and interpret graphs
- interpret the gradient of a straight line graph as a rate of change
- use the form $y = mx + c$ to identify parallel **{and perpendicular}** lines; find the equation of the line through two given points, or through one point with a given gradient
- find approximate solutions to two simultaneous equations in two variables (linear/linear **{or linear/quadratic}**) using a graph

Weeks 3 and 4: Non-linear graphs

Students develop their knowledge of non-linear graphs in this block, looking at quadratic, cubic and reciprocal graphs so they recognise the different shapes. They find the roots of quadratics graphically, and will revisit this when they look at algebraic methods in the Functions block during Autumn 2, where they will also look at turning points. Higher tier students also look at simple exponential graphs and the equation of a circle. Note that the equation of the tangent to a circle is covered later when the circle theorem of tangent/radius is met. Higher students also extend their understanding of gradient to include instantaneous rates of change considering the gradient of a curve at a point.
National Curriculum content covered includes:

- move freely between different numerical, algebraic, graphical and diagrammatic representations
- recognise, sketch and interpret graphs of linear functions, quadratic functions, simple cubic functions, the reciprocal function 1 **{the exponential function $y = k^x$ for positive values of k}**
- plot and interpret graphs (including reciprocal graphs **{and exponential graphs}**)
- find approximate solutions using a graph
- identify and interpret roots, intercepts of quadratic functions graphically
- **{recognise and use the equation of a circle with centre at the origin;}**

Weeks 5 and 6: Using graphs

This block revises conversion graphs and reflection in straight lines. Students also study other real-life graphs, including speed/distance/time, constructing and interpreting these. Higher tier students also investigate the area under a curve.
National Curriculum content covered includes:

- plot and interpret graphs of non-standard functions in real contexts, to find approximate solutions to problems such as simple kinematic problems involving distance, speed and acceleration
- **{interpret the gradient at a point on a curve as the instantaneous rate of change; apply the concepts of instantaneous and average rate of change (gradients of tangents and chords) in numerical, algebraic and graphical contexts}**
- **{calculate or estimate gradients of graphs and areas under graphs (including quadratic and other non-linear graphs), and interpret results in cases such as distance-time graphs, velocity-time graphs and graphs in financial contexts}**

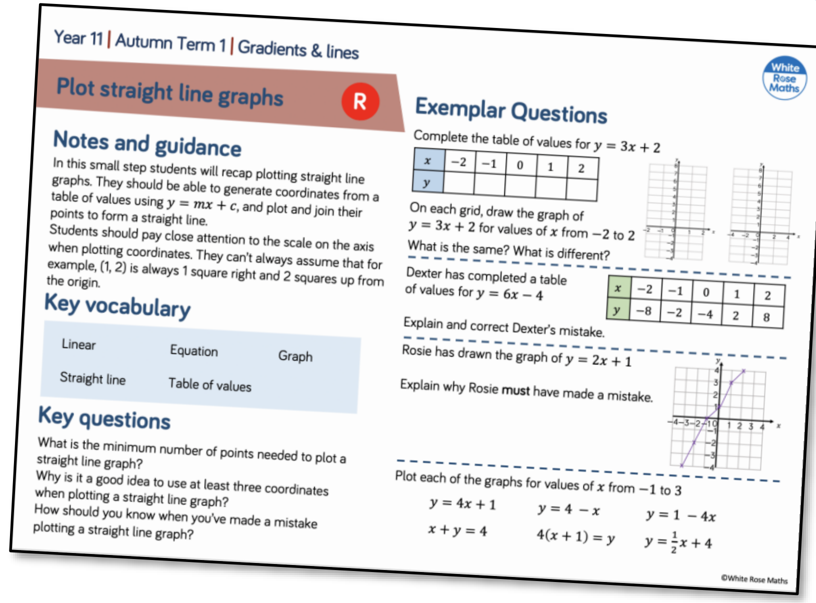
Why Small Steps?

We know that breaking the curriculum down into small manageable steps should help students to understand concepts better. Too often, we have noticed that teachers will try and cover too many concepts at once and this can lead to cognitive overload. We believe it is better to follow a "small steps" approach.

As a result, for each block of content in the scheme of learning we will provide a "small step" breakdown. ***It is not the intention that each small step should last a lesson – some will be a short step within a lesson, some will take longer than a lesson.*** We would encourage teachers to spend the appropriate amount of time on each step for their group, and to teach some of the steps alongside each other if necessary.

What We Provide

- Some ***brief guidance*** notes to help identify key teaching and learning points
- A list of ***key vocabulary*** that we would expect teachers to draw to students' attention when teaching the small step,
- A series of ***key questions*** to incorporate in lessons to aid mathematical thinking.
- A set of questions to help ***exemplify*** the small step concept that needs to be focussed on.



Year 11 | Autumn Term 1 | Gradients & lines

Plot straight line graphs R

Notes and guidance

In this small step students will recap plotting straight line graphs. They should be able to generate coordinates from a table of values using $y = mx + c$, and plot and join their points to form a straight line.
Students should pay close attention to the scale on the axis when plotting coordinates. They can't always assume that for example, (1, 2) is always 1 square right and 2 squares up from the origin.

Key vocabulary

Linear	Equation	Graph
Straight line	Table of values	

Key questions

What is the minimum number of points needed to plot a straight line graph?
Why is it a good idea to use at least three coordinates when plotting a straight line graph?
How should you know when you've made a mistake plotting a straight line graph?

Exemplar Questions

Complete the table of values for $y = 3x + 2$

x	-2	-1	0	1	2
y					

On each grid, draw the graph of $y = 3x + 2$ for values of x from -2 to 2
What is the same? What is different?

Dexter has completed a table of values for $y = 6x - 4$

x	-2	-1	0	1	2
y	-8	-2	-4	2	8

Explain and correct Dexter's mistake.

Rosie has drawn the graph of $y = 2x + 1$
Explain why Rosie **must** have made a mistake.

Plot each of the graphs for values of x from -1 to 3

$y = 4x + 1$ $y = 4 - x$ $y = 1 - 4x$
$x + y = 4$ $4(x + 1) = y$ $y = \frac{1}{2}x + 4$

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- These include reasoning and problem-solving questions that are fully integrated into the scheme of learning. Depending on the attainment of your students, you may wish to use some or all of these exemplars, which are in approximate order of difficulty. Particularly challenging questions are indicated with the symbol .
- For each block, we also provide ideas for key representations that will be useful for all students.

Some of the small steps are in **bold** and labelled with H to indicate this is Higher tier GCSE content. We would encourage teachers to use these with as many students as possible – if you feel your class can access any particular small step, then please include it in your planning. Steps that review content covered at Key Stage 3 are labelled R.

Non-linear graphs

Small Steps

- Plot and read from quadratic graphs
- Plot and read from cubic graphs
- Plot and read from reciprocal graphs
- Recognise graph shapes
- Identify and interpret roots and intercepts of quadratics
- **Understand and use exponential graphs** H
- **Find and use the equation of a circle centre (0, 0)** H
- **Find the equation of the tangent to any curve** H

H denotes Higher Tier GCSE content

R denotes ‘review step’ – content should have been covered at KS3

Quadratic graphs

Notes and guidance

Check that students can substitute a negative into an expression containing x^2 and/or $-kx$. Students may also need to revise using a calculator for this. When plotting the graph, make explicit that the points are joined with a smooth curve. In addition, students need to be aware of the shape of curve. In addition, students need to be aware of the shape of the curve so that they avoid just joining up two points either side of a turning point. Before reading from a quadratic graph, check they know equations of vertical and horizontal lines.

Key vocabulary

Quadratic	Parabola	Curve	Substitute
Equation	Vertical	Horizontal	Estimate

Key questions

Why is $(-3)^2$ the same as 3^2?

Is $2x^2$ the same as $2 \times x^2$ or $(2 \times x)^2$?

How could I tell if one of my coordinates was incorrect, or if I had plotted it incorrectly?

Why do we join the points with a smooth curve?

Describe the shape of a parabola.

Exemplar Questions

x^2 $2x^2$ $x^2 - x$

Eva substitutes $x = 3$ into each expression.
Jack substitutes $x = -3$ into each expression.
Jack thinks that he will get the same answers as Eva each time.
Do you agree with Jack? Justify your answer.

Complete the table for $y = x^2 - 2x + 2$

x	−3	−2	−1	0	1	2	3	4
y	17				1			10

Amir plots each coordinate and joins his points with a ruler.
Why is this incorrect?

Draw the graph of $y = x^2 - 2x + 2$ for values of x from -3 to 4

Draw the graph of $y = x^2 + x - 2$ for values of x from -3 to 3

x	−3	−2	−1	0	1	2	3
y		0				4	

On your graph, show that when $x = -0.5$, an estimate for y is -2.3

Why is there more than one answer when estimating x if $y = 1.5$? Draw the line $y = 1.5$ onto your graph and estimate the value of x.

How can you check whether your estimates are accurate?

Cubic graphs

Notes and guidance

Using interactive dynamic software is a powerful way of supporting students to notice features of cubic graphs. Remind students that cubing a negative gives a negative result. A common mistake is for students to multiply by 3 instead of cubing. Ensure that they use a smooth curve to join points. Students sometimes join points either side of a turning point with a flat line; to avoid this error, remind students of the shape of a cubic graph.

Key vocabulary

Cube	Cubic	Estimate
Curve	Substitute	

Key questions

What mistakes can be made when substituting?

How would these 'stick out' when you draw the graph?

Why is it important to use a smooth curve to join the points?

Exemplar Questions

$y = x^3$

x	−3	−2	−1	0	1	2	3
y		−8					27

Complete the table of values.
Draw the graph of $y = x^3$ for values of x from −3 to 3

$y = x^3 + x$

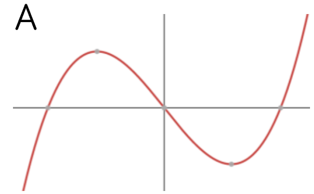
x	−3	−2	−1	0	1	2	3
y		−10					30

Ron thinks that when $x = 1, y = 4$, but Alex thinks that when $x = 1, y = 2$. Who is correct? How do you know?

- Complete the table of values.
- Draw the graph of $y = x^3 + x$ for values of x from −3 to 3
- How could you tell if one of your coordinates is incorrect?
- Describe the features of the graph.
- Use your graph to estimate the value of x when $y = 5$
- What do you notice about x when $y = -5$?

Teddy thinks that A and C are cubic graphs, but B isn't as it doesn't go through (0,0).
Why is Teddy incorrect?

A



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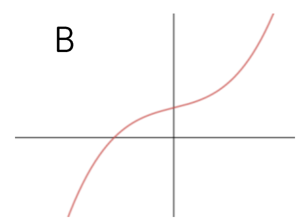

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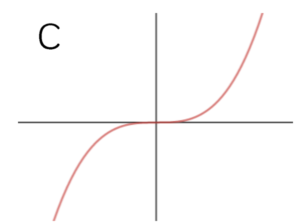


Reciprocal graphs

Notes and guidance

Again, using interactive dynamic software is a powerful way of supporting students to notice features of reciprocal graphs $y = \frac{k}{x}$ and become familiar with the concept of asymptotes. Allow students time to investigate the reciprocal function using their calculators. It is useful to introduce concepts such as infinity and negative infinity to describe the behaviour of the curves at extreme values.

Key vocabulary

Asymptote | Infinity | Reciprocal | Tends towards

Key questions

Why doesn't the graph of $y = \frac{1}{x}$ meet the axes?

What happens at $x = 0$?

What do we mean by infinity?

What are the key features of this graph?

Exemplar Questions

$y = \frac{1}{x}$

x	−4	−3	−2	−1	1	2	3	4
y								

Complete the table of values.

Investigate what happens when x is close to 0 by completing this table of values:

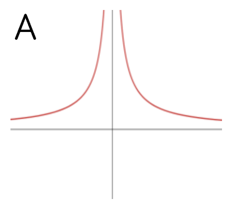
x	−0.4	−0.3	−0.2	−0.1	0.1	0.2	0.3	0.4
y								

Can $x = 0$ when $y = \frac{1}{x}$? Explain your answer.

Draw the graph of $y = \frac{1}{x}$ for x values from −4 to 4

Which of the following sketches would match the graph of $y = \frac{2}{x}$?

A



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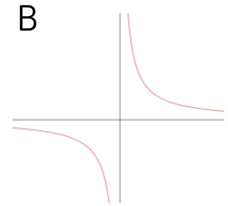

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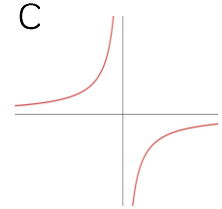


Describe the features of a reciprocal graph.

Complete the sentences about $y = \frac{1}{x}$

When x tends towards negative infinity, y tends towards _____
When x tends towards positive infinity, y tends towards _____
When x tends towards zero, y tends towards _____

Recognise graph shapes

Notes and guidance

In this small step it is important to make explicit the similarities and differences of straight line, quadratic, cubic and reciprocal graphs. Students need to consider the detail of a graph when comparing two of the same type, (e.g. $y = x^3 - 5x$ and $y = x^3$ have very different shapes). In these cases, students may need to substitute a couple of well chosen x values to check which graph matches the equation.

Key vocabulary

Gradient, y-intercept, Quadratic, Cubic, Reciprocal, Infinity, Asymptote

Key questions

What features of a graph help us to identify its equation?
Which types of graphs do you find easier to identify? Why?
If you're not sure which equation matches a graph, what could you do to find out more information?
What's different about a quadratic and a cubic graph?

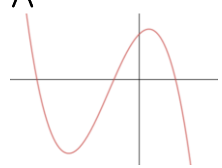
Exemplar Questions

Use a dynamic geometry package to plot these graphs.
Make a sketch of each one.

$y = x$	$y = -x$
$y = x^2$	$y = -x^2$
$y = x^3$	$y = -x^3$

- What's the same and what's different about each set of graphs?
- What's the same and what's different about each pair of graphs?
- Investigate $y = \pm kx, y = \pm kx^2$ and $y = \pm kx^3$ for different values of k.

Match each graph with its equation.

A

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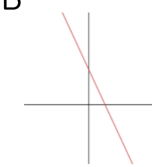
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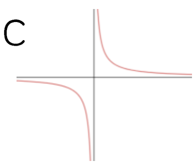
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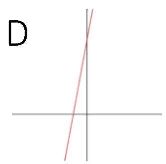
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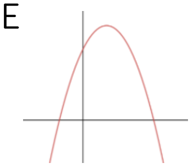
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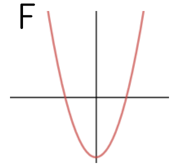


Equation	Letter	Type of Graph
$y = 10x + 10$		
$y = \frac{1}{x}$		
$y = x^2 - 10$		
$y = x^3$		
$y = -x^2 + 2x + 3$		
$y = -2x + 4$		
$y = -x^3 - 2x^2 + x + 1$		

One equation in the table doesn't have a match.
Sketch a graph to match this equation.

Roots and intercepts of quadratics

Notes and guidance

Students start by identifying a root from a graph. They understand that the root of an equation is given when $y = 0$, and should write these as $x = a$. They understand that quadratics can have 0, 1 or 2 roots. Students also locate the y-intercept from a graph and make the connection between this and substitution of $x = 0$ into the equation of the curve. It is important students write the y-intercept as a coordinate.

Key vocabulary

Quadratic	y-intercept	Coordinate
Roots	Solution	Meets

Key questions

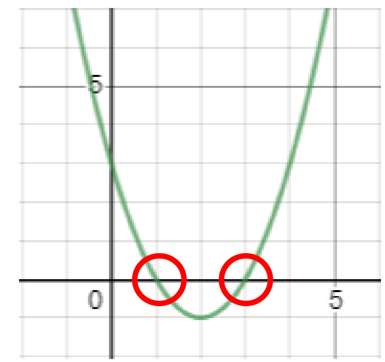
Why do we write the y-intercept as a coordinate?
How can we locate the y-intercept from a graph?
How can we locate the roots from a graph?
Why do we write the roots as $x = a$?
How many roots is it possible for a quadratic equation to have? Can a quadratic equation have more than 2 roots? 0 roots?

Exemplar Questions

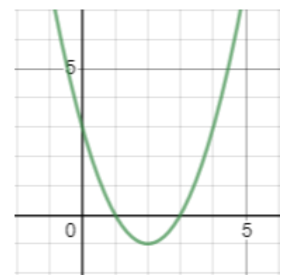
Ron circles in red where the graph intersects the line $y = 0$
One root of $y = x^2 - 4x + 3$ is $x = 1$
Write down another root.

Ron checks that $x = 1$ is a root by substituting $x = 1$ into $y = x^2 - 4x + 3$
$y = 1^2 - 4 \times 1 + 3$
$y = 0$, so $x = 1$ is a root as $y = 0$
Check that the second root also gives $y = 0$

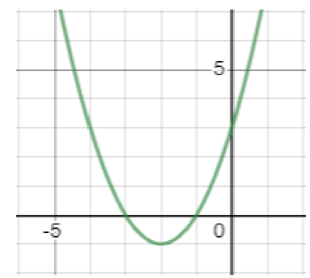
$y = x^2 - 4x + 3$



$y = x^2 - 4x + 3$



$y = x^2 + 4x + 3$



Each graph meets the y-axis at (0, 3)

Substitute $x = 0$ into each equation. What do you notice?
Explain how you can identify the y-intercept from the equation of a graph.
Annie thinks that the coordinate of the y-intercept of $y = x^2 + 3x + 4$ will be (4,0). What mistake has she made?

Exponential graphs

H

Notes and guidance

Students may need to revise negative powers and/or powers of a fraction before this step. Students will explore exponential graphs so they can spot similarities in features. Students may need strategies to draw a smooth curve such as 'keep your wrist on the table'. This step can be extended by using simultaneous equations to find a and b, given two coordinates, when the equation is in the form $y = ab^x$.

Key vocabulary

Exponential	Growth	Decay
Rapid	Tends	Infinity
Asymptote	y-intercept	

Key questions

Can you think of real-life situations that can be modelled using exponential graphs?

True/False: a graph of an equation in the form $y = a^x$ will always have a y-intercept of (0,1)

What does 'tend towards' mean? What's an asymptote?

How can I find a given the y-intercept, in an equation of the form $y = ab^x$?

Exemplar Questions

Complete the table of values for $y = 2^x$ and draw the graph for values from $x = -3$ to $x = 3$

x	−3	−2	−1	0	1	2	3
y		0.25				4	

- Find y when $x = 10, x = 20, x = 50, x = 100$
- Find y when $x = -10, x = -20, x = -50, x = -100$

Explain what happens to the graph in each case.

Alex says "y will never be 0"

Is she right? Explain your answer.

Use a dynamic geometry package to plot these graphs.

Make a sketch of each one.

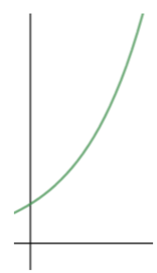
- $y = 2^x$
- $y = 3^x$
- $y = 4^x$
- $y = 5^x$

Write down the coordinates of the y-intercept for each graph.

What's the same and what's different about the graphs?

Write down the coordinates of the y-intercept of $y = 10^x$

How is the graph $y = 1^x$ different to these graphs?



The sketch shows a curve with equation $y = ab^x$ where a and b are constants and $b > 0$

The curve passes through the points (0, 2) and (1, 8)

What do you know? What can you find out?

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Equation of circle centre (0,0) H

Notes and guidance

Students start by finding the radius of circles with centre (0,0) and making the connection to Pythagoras' theorem. This reveals the generalised equation for a circle centre (0,0). Given an equation in the format $x^2 + y^2 = a$, students sometimes read a as the radius instead of $\sqrt{a}$. There are opportunities here to revisit simplification of surds, circumference and area of a circle.

Key vocabulary

Radius	Diameter	Pythagoras' theorem
Equation	Origin	Simplify

Key questions

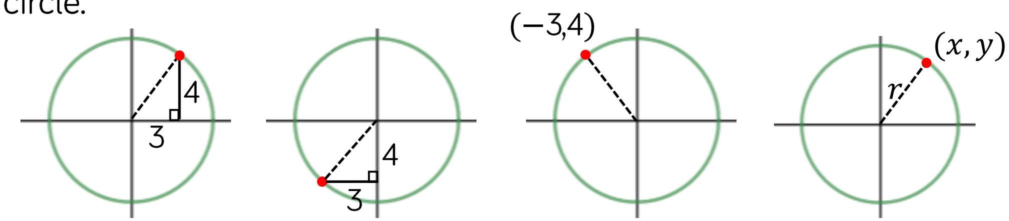
How is Pythagoras' Theorem connected to the equation of a circle?

How can I find the radius from the equation of a circle?

How can I write the equation of a circle given the diameter/circumference/area?

Exemplar Questions

Use the information given to write down the length of the radius of the circle.



Which is the correct equation of a circle, centre (0,0) with radius 5 cm?

A) $x^2 + y^2 = 5$ B) $x^2 + y^2 = 25$ C) $x^2 + y^2 = 10$

Match each equation of a circle with centre (0,0) to its radius.

$x^2 = 27 - y^2$	$r = 2\sqrt{3}$
$x^2 + y^2 + 100 = 0$	$r = 3\sqrt{3}$
$x^2 + y^2 - 100 = 0$	$r = 4\sqrt{3}$
$x^2 + y^2 - 48 = 0$	$r = 10$
$x^2 + y^2 = 12$	

Which equation doesn't have a match? Why?

The following circles all have centre (0,0).
Write down the equation of the circles.

- Radius = 0.5
- Diameter = $360^{\frac{1}{2}}$
- Radius = $\frac{1}{8}$
- Circumference 18π
- Area 40π
- Diameter = $\sqrt{20}$

Tangent to any curve

H

Notes and guidance

Using a dynamic software package and the 'zoom' function is an excellent way of highlighting why a tangent to a curve gives the gradient at a specific point on the curve. Ensure students understand the steps in drawing a tangent. They need to put their ruler on the point on the curve and adjust it so that near to the point, the ruler is equidistant from the curve on either side. Students then find the equation of the tangent using the gradient and the given point.

Key vocabulary

Tangent	Curve	Equidistant
Gradient	y-intercept	Equation

Key questions

What are the steps to draw a tangent to a curve at a given point?

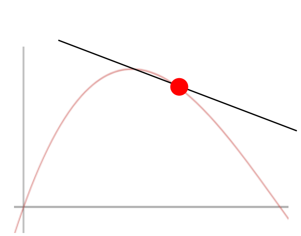
How do I find the gradient of the tangent? Why is this a good estimate of the gradient of a curve at a given point?

How do I know if the gradient is positive or negative?

Exemplar Questions

Annie is practising drawing tangents at given points on a curve.

A



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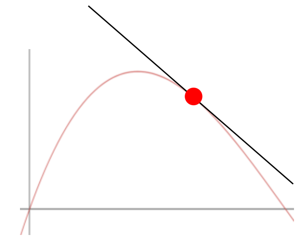
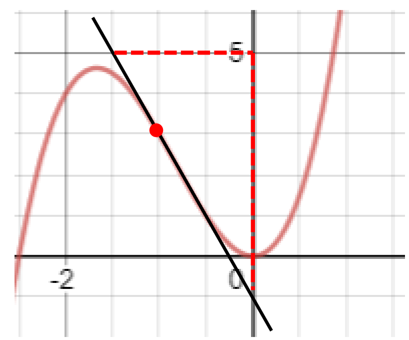


Which is her best attempt and why?



Dexter draws a tangent at $(-1,3)$.
Show that the gradient of the tangent is -4

Dexter finds the equation of the tangent.
Finish his workings.

$y = mx + c$

$y = -4x + c$

Substitute in $x = -1$ and $y = 3$ to find c.

Draw the graph $y = (x - 1)(x + 2)$ for values from $x = -1$ to $x = 4$

Find the equation of the tangent at the point on the curve with coordinate $(3, 10)$

Ron says that the gradient of the tangent to the curve when $x = -0.5$ is 0 and so the equation of the tangent at this point is $y = -2.25$

Is Ron correct? Explain your answer.