Fractions and Percentages

Year (8)

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#MathsEveryoneCan
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	Week 1	Week 2	Week 3	Week 4	Week 5	Week 6	Week 7	Week 8	Week 9	Week 10	Week 11	Week 12
	Proportional Reasoning			Representations								
Autumn		and ale	i . '	licative nge	and di	olying ividing tions		rking in tesian p		Repres da		Tables & Probability
	Algebraic Techniques				Developing Number							
Spring	Brackets, equations and inequalities		Sequences	Indices	! !	ractions and Standard sercentages index form		Number sense				
	Developing Geometry			Reasoning with Data								
Summer	paralle	es in el lines olygons	Are trapez ciro	ia and	symr	ne netry flection	y The data handling cycle			res of tion		



Spring 2: Developing Number

Weeks 1 and 2: Fractions and Percentages

This block focuses on the relationships between fractions and percentages, including decimal equivalents, and using these to work out percentage increase and decrease. Students also explore expressing one number as a fraction and percentage of another. Both calculator and non-calculator methods are developed throughout to support students to choose efficient methods. Financial maths is developed through the contexts of e.g. profit, loss and interest The higher strand also looks at finding the original value given a percentage or after a percentage change.

National Curriculum content covered includes:

- develop their use of formal mathematical knowledge to interpret and solve problems, including in financial mathematics
- work interchangeably with terminating decimals and their corresponding fractions
- define percentage as 'number of parts per hundred', interpret percentages and percentage changes as a fraction or a decimal, interpret these multiplicatively, express one quantity as a percentage of another, compare two quantities using percentages, and work with percentages greater than 100%
- interpret fractions and percentages as operators

Weeks 3 and 4: Standard Index Form

Higher strand students have already briefly looked at standard form in year 7 and now this knowledge is introduced to all students, building from their earlier work on indices last term. The use of context is important to help students make sense of the need for the notation and its uses. The higher strand includes a basic introduction to negative and fractional indices.

National Curriculum content covered includes:

- use integer powers and associated real roots (square, cube and higher), recognise powers of 2, 3, 4, 5 and distinguish between exact representations of roots and their decimal approximations
- interpret and compare numbers in standard form $A\times 10^n$, $1\le A<10$, where n is a positive or negative integer or zero

Weeks 5 and 6: Number Sense

This block provides a timely opportunity to revisit a lot of basic skills in a wide variety of contexts. Estimation is a key focus and the use of mental strategies will therefore be embedded throughout. We will also use conversion of metric units to revisit multiplying and dividing by 10, 100 and 1000 in context. The higher strand will extend this to look at the conversion of area and volume units, as well as having an extra step on the use of error notation. We also look explicitly at solving problems using the time and calendar as this area is sometimes neglected leaving gaps in student knowledge.

National Curriculum content covered includes:

- use standard units of mass, length, time, money and other measures, including with decimal quantities
- round numbers and measures to an appropriate degree of accuracy [for example, to a number of decimal places or significant figures]
- use approximation through rounding to estimate answers and calculate possible resulting errors expressed using inequality notation $a < x \le b$
- use a calculator and other technologies to calculate results accurately and then interpret them appropriately



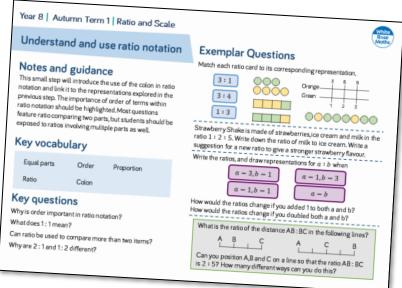
Why Small Steps?

We know that breaking the curriculum down into small manageable steps should help students to understand concepts better. Too often, we have noticed that teachers will try and cover too many concepts at once and this can lead to cognitive overload. We believe it is better to follow a "small steps" approach.

As a result, for each block of content in the scheme of learning we will provide a "small step" breakdown. It is not the intention that each small step should last a lesson – some will be a short step within a lesson, some will take longer than a lesson. We would encourage teachers to spend the appropriate amount of time on each step for their group, and to teach some of the steps alongside each other if necessary.

What We Provide

- Some *brief guidance* notes to help identify key teaching and learning points.
- A list of *key vocabulary* that we would expect teachers to draw to students' attention when teaching the small step.
- A series of **key questions** to incorporate in lessons to aid mathematical thinking.
- A set of questions to help *exemplify* the small step concept that needs to be focussed on.

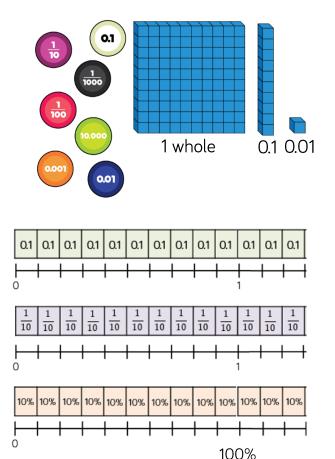


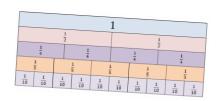
- These include reasoning and problem-solving questions that are fully integrated into the scheme of learning. Depending on the attainment of your students, you many wish to use some or all of these exemplars, which are in approximate order of difficulty. Particularly challenging questions are indicated with the symbol
- For each block, we also provide ideas for key representations that will be useful for all students.

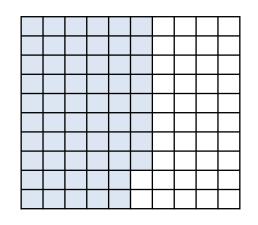
In many of the blocks of material, some of the small steps are in **bold**. These are content aimed at higher attaining students, but we would encourage teachers to use these with as many students as possible – if you feel your class can access any particular small step, then please include it in your planning.

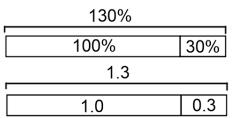


Key Representations









Concrete, pictorial and abstract representations are an important part of developing students' conceptual understanding.

Number lines are a useful way of assessing whether children understand the size of a fraction, decimal or percentage. Extending the number line above 1 is an option for some students.

Bar models and paper strips can be folded to represent different fractions, decimals or percentages and are particularly useful when making comparisons. Bar models are particularly useful to show when an amount has increased above or decreased below 100%

Number lines can be used to find original amounts for specific given percentage change problems.



Fractions and Percentages

Small Steps

Convert fluently between key fractions, decimals and percentages

W

Calculate key fractions, decimals and percentages of an amount without a calculator

- R
- Calculate fractions, decimals and percentages of an amount using calculator methods
- R

- Convert between decimals and percentages greater than 100%
- Percentage decrease with a multiplier
- Calculate percentage increase and decrease using a multiplier
- Express one number as a fraction or a percentage of another without a calculator
- Express one number as a fraction or a percentage of another using calculator methods
 - Denotes Higher Tier GCSE content
 - R Denotes 'review step' content should have been covered in Year 7



Fractions and Percentages

Small Steps

- Work with percentage change
- Choose appropriate methods to solve percentage problems
- Find the original amount given the percentage less than 100%
- Find the original amount given the percentage greater than 100%
- Choose appropriate methods to solve complex percentage problems

- Denotes Higher Tier GCSE content
- R Denotes 'review step' content should have been covered in Year 7



Fluently convert F, D & P



Notes and guidance

This small step revises year 7 work on mental conversion of key fractions, decimals and percentages. Use of diagrams such as the 100 square, and number lines to compare these will help to secure understanding; bead strings are also useful. Students should be confident in articulating their methods and using them to compare different forms e.g. which is larger $\frac{3}{5}$ or 65%

Key vocabulary

Fraction	Decimal	Percentage
Equivalent	Denominator	Numerator

Key questions

Why do we use all three representations of fractions, decimals and percentages? Explain why one third is not the same as 0.3 or 30% Can you draw a diagram to show the meaning of 0.7? Which is greater in value 0.5 or 50%?

Exemplar Questions

72% of the Earth's surface is covered by water. Tick all answers below which represent the percentage of earth which is not covered by water.

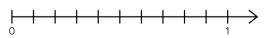


Complete the statements using <, > or =

1) 0.37
$$\frac{3}{8}$$
 3) 0.4 4%

2)
$$0.35$$
 $\frac{3}{5}$ 4) 0.6 60%

Label on the number line $\frac{4}{5}$, 0.7 and 75%



In a bag, $\frac{2}{5}$ of the counters are red. 0.15 of the counters are green. The rest of the counters are blue. What percentage of the counters are blue?

Rosie says that $\frac{1}{4}$ is equivalent to 25% so $\frac{1}{8}$ is 12.5%

Use this information to write $\frac{3}{8}$ as a percentage and a decimal.

Huan thinks $\frac{1}{3} = 30\%$. Prove that Huan is wrong.



Calculate F, D & P mentally



Notes and guidance

Students will have visited finding fractions and percentages of amounts during year 7. This step will provide a further opportunity to consolidate their understanding and revisit key ideas and supporting diagrams such as the bar model. Decimal multiplication can sometimes cause confusion, but using their knowledge of conversions and starting with $0.1 \times \cdots = \cdots \div 10$ and building from this is helpful.

Key vocabulary

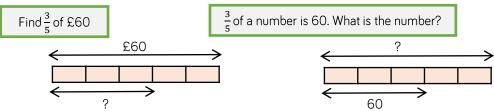
Fraction	Decimal	Percentage
Equivalent	Denominator	Numerator

Key questions

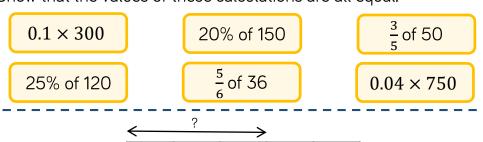
Explain how to find $\frac{3}{7}$ of an amount. Is it possible to find $\frac{6}{5}$ of a number? If so, how? Explain why is it that when we divide an amount by 10 it gives 10%, but if you divide by 20 it does not give 20%? Is it true that 45% of 60 is equal to 60% of 45? Does this work for other pairs of numbers?

Exemplar Questions

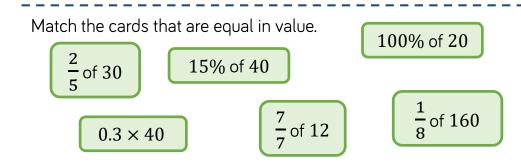
What is the same and what is different about the calculations for the questions below?



Show that the values of these calculations are all equal.



What might the question for the diagram be? How many questions can you find? Include fractions, decimals and percentages.





F, D & P with calculator



Notes and guidance

Teachers should model the use of calculators so students gain awareness of efficient methods and using estimation before calculating. Comparison of the fraction and percentages keys will be useful. When solving problems, students will have access to a calculator but may still need access to supporting tools, such as the bar model, to compliment their understanding.

Key vocabulary

Fraction key	Decimal	Percentage

Estimate Rounding Conversion

Key questions

How do you use the percentage key on your calculator? How does this compare to using decimal equivalents?

How do you use the fraction key on your calculator?

What keys could you press to find 23% of 45?

Exemplar Questions

Teddy and Mo are asked to calculate 35% of 150 cm. Which of their methods do you prefer and why?

<u>Mo</u> 35 ÷ 100 = 0.35 0.35 x 150 = 52.5 cm

Teddy

$$150 \div 10 = 15 \text{ cm}$$
 $10\% = 15 \text{ cm}$
 $5\% = 7.5 \text{ cm}$
 $30\% = 45 \text{ cm}$
 $30\% + 5\% = 45 \text{ cm} + 7.5 \text{ cm}$
 $35\% = 52.5 \text{ cm}$

Rosie is working out 37% of £2800

She estimates the answer as $0.4 \times £3000 = £1200$

Is this a good way of estimating? Why or why not?

Estimate and then find the answers to the calculations on the cards.

23% of 800

2.3% of 800

68% of 600

18.5% of 61

The calculations below are used to find $\frac{3}{8}$ of £16000

How many other different ways can you find to calculate $\frac{3}{8}$ of £16000?

 $16000 \times 3 \div 8$

 0.375×16000

 $37.5\% \times 16000$

Jack says that 27% of 500 is the same as 54% of 1000 Show that Jack is wrong using a calculator and using a diagram.



Convert D & P both < and > 100%

Notes and guidance

Students should already be fluent in converting between decimals and percentages up to 100% and now explore the equivalence of percentages above 100%. This will support later use of multipliers for percentage increase. Physical resources and pictures, particularly the hundred square are very useful. It is good to link e.g. 130% = 100% + 30% to the decimal addition 1 + 0.3

Key vocabulary

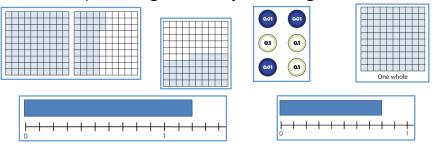
Fraction	Decimal	Percentage
Equivalent	Hundredth	Tenth

Key questions

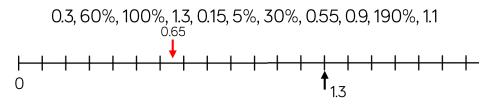
Why is 0.3 the same as 30% and not 3%? Is it possible to have a percentage greater than 100? How might 140% look like as a decimal multiplier? Why does multiplying a decimal by 100 give you an equivalent percentage? How can you order mixed decimals and percentages?

Exemplar Questions

Write down the percentage shown by each diagram.



Use the given information to add these decimals and percentages to the number line. Label each one.



Match the equivalent decimals and percentages. Write the equivalent percentage and decimal for any cards that are not paired up.



Fill in the blanks.

$$_{---}\% = 0.9$$
 $100\% = 1.00$
 $110\% = _{---}$



Percentage decrease: multipliers

Notes and guidance

For percentage decrease, students will need to understand that they are subtracting the given percentage from 100%. This concept should be represented using bar models and number lines to help reinforce how to find the correct multiplier. This should also avoid the misconception of e.g. multiplying by 0.2 to find a 20% decrease.

Key vocabulary

Decimal Percentage Reduce

Equivalent Decrease Multiplier

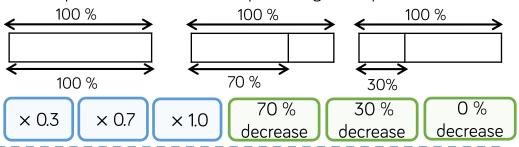
Key questions

Why is decreasing by 46% the same as finding 54%? If I am multiplying by 0.2 why is this an 80% decrease? What mistakes might happen if we are decreasing by 1.5%?

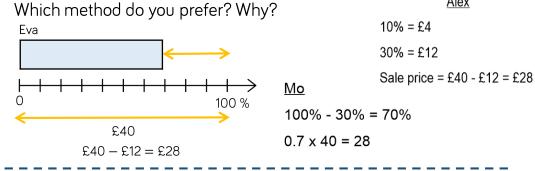
What happens if I decrease an amount by 0%? What does the word 'discount' mean?

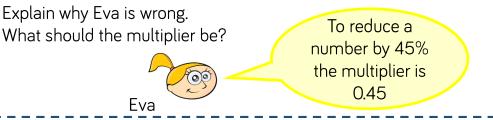
Exemplar Questions

Match up the bar model with the percentage multiplier and statement.



In a sale, 30% is taken off all prices. Eva, Mo and Alex are calculating the sale price of a shirt that costs £40 before the sale.







What is the multiplier for a 10% reduction followed by another 10% reduction?



Increase & decrease: multipliers

Notes and guidance

Students build on the last two steps using multipliers above one to increase an amount by a given percentage.

It is worth discussing the similarities and differences between percentage increase and decrease and mixing questions so that students are thinking carefully rather than just using a procedure. Starting with a bar representing 100% can help access worded problems.

Key vocabulary

Multiplier	Decimal	Percentage
Equivalent	Increase	Growth

Key questions

When increasing an amount by a given percentage, how do we calculate the multiplier?

What is the percentage increase if you double a number? Will a number always get bigger if we increase it by a given percentage?

Can you represent this question with a bar model?

Exemplar Questions

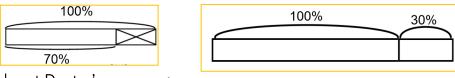
Match the multiplier with the correct percentage statement.



Dexter earns £30 a week for his paper round.

His employer gives him a 30% pay rise.

Which of the bar models shows this?



Work out Dexter's new wage.

Aisha earns £35000 a year. Her boss offers her a pay rise of 6% a year, but a rival employer offers to pay her £180 more per month.

Which offer should she accept to get the most money?

Alex increases 30 g by 20%

She then decreases her answer by 20%

Dora says she will have less than her original amount of 30 g

Alex disagrees. Who is correct? Justify your answer.



Express as a %: Non-calculator

Notes and guidance

As a first step on the way to expressing one number as a percentage of another, students will firstly explore writing one number as a fraction of another. In this step, the focus will be to support students to express fractions as percentages where the fraction denominators are factors or multiples of 100 This is another good opportunity to make links to probability and simple conversions.

Key vocabulary

Express Fraction Percentage

Equivalent Factor Multiple

Key questions

Why can we convert quarters, fifths and tenths easily to a percentage but not thirds?

Why can't we compare a mark out of 20 and a mark out of 25 directly? What are the factors of 100? Is it possible to convert fortieths to hundredths? Why or why not?

Exemplar Questions

Tommy saves £13 of his £20 pocket money each week. He gives £3 to his sister.

What fraction of his pocket money does he have left to spend?
What percentage of his pocket money does he have left to spend?

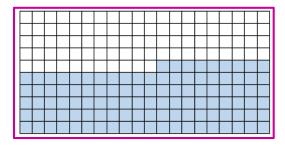
Eva has a bag of 200 counters. 34 counters are red, 46 are blue and the rest are green.

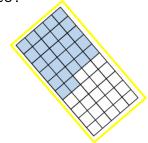
What proportion of the counters are green?

Give your answer as a fraction and a percentage. She takes out the blue counters.

By what fraction has the number of counters in the bag been decreased by? Express this fraction as a percentage.

Which shape has the larger percentage shaded?





A bag contains green and red counters in the ratio 12:13 What percentage of the counters are green?



Express as a %: Calculator

Notes and guidance

Building on from the previous step, students are asked to consider a number as a percentage of another both from fractions that can be converted mentally and those that are best converted using a calculator. To keep the focus on conversion rather than rounding, it might be best to give non-exact answers to the nearest whole number percentage; this skill may need revising in starters.

Key vocabulary

Fraction	Decimal	Percentage
Equivalent	Round	Integer

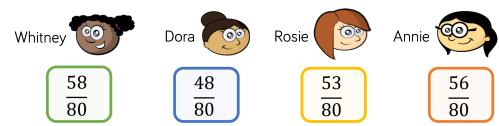
Key questions

Why might we need a calculator to calculate the percentage of a test mark out of 30, but not for a mark out of 50?

How do we use a calculator to convert a fraction to a decimal and then to a percentage? Is it possible to work out e.g. 70 as a percentage of 65?

Exemplar Questions

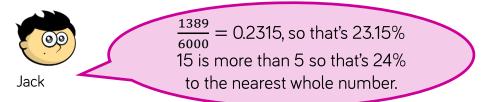
Here are the marks from a test.



Convert the marks to percentages.

Why are some of the percentages integers and others not?

In a local election 1389 out of 6000 residents vote. Jack is working out the percentage of the village that voted.



Explain why Jack has rounded this incorrectly.

The attendance of three classes one Friday was:

- Class X had 3 people missing out of 29
- Class Y had 4 people missing out of 31
- Class Z had 2 people missing out of 30

Work out the percentage attendance of each class, giving your answers to the nearest whole percent.

Which class had the highest percentage attendance?



Work with percentage change

Notes and guidance

Students continue to express one number as a percentage of another, this time in the context of change. Good contexts to consider include percentage profit and loss and interest to remind students of these words. It is also useful to look at situations that can be worked out using both calculator and non-calculator methods allowing the students to choose the most appropriate method.

Key vocabulary

Profit	Loss	Interest	Change
Original	Invest	Numerator	Denominator

Key questions

What's the difference between profit and loss? How can you represent this percentage change question on a bar model?

Why is it important to identify the original amount before doing the calculation for percentage change questions?

Exemplar Questions

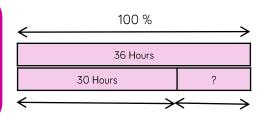
Amir sells his mobile phone for £240 He paid £480 for the mobile phone when it was new. Has he made a profit or loss?

What percentage profit or loss has he made?

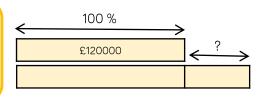
When new, Dani's phone battery would last 36 hours between charges. Now it only lasts 30 hours between charges.

Calculate the percentage decrease

in battery life.



Jack buys a house for £120000
He sells the house one year later
for £135000
What is the percentage profit that
Jack has made?



Brett invests £80000 in a new business.

At the end of the year he is paid back £81140

What is his percentage return on his investment?

Alex invests £60000 in a saver account that pays 2.5% interest per year. Show that Alex's investment earns more than Brett's, and work out by how much.



Choose appropriate methods

Notes and guidance

In this step students will use all the skills gained from the previous steps to apply to various percentage problems. It is worth investing time in analysing and discussing what questions are being asked and how to choose methods, to avoid students rushing into an inappropriate procedure. In particular, students need to decide whether a question asks for finding a percentage or express as a percentage.

Key vocabulary

Original	Percentage	Increase
Decrease	Profit/Loss	Express

Key questions

Describe the different calculation processes involved in these questions.

How can you represent this on a bar model?
What is the same and what is different in these questions?
What type of percentage question is this problem?
How can you tell?

Exemplar Questions

What's the same and what's different?

Work out 30% of £70

Uncrease £70 by 30%

What percentage of £70 is £30?

Eva scores 60% on a test.

Which of the cards could have been her score?

54 out of 90

31 out of 50

20 out of 30

14 out of 20

9 out of 15

33 out of 55

The sign shows the cash price of a TV set and a monthly payment plan.

How much more does it cost altogether to buy the TV set using the monthly payment plan? Express this as a percentage of the cash price. Pay £1200 today

OR

Pay a 15% deposit + 12 payments of £99

Ron bakes some cakes.

He pays £15 altogether for the ingredients.

He sells 10 of the cakes, but makes a loss of 20% overall.

How much did Ron charge for each cake?

Ron reviews his pricing strategy and bakes some more cakes.

He again pays £15 altogether for the ingredients.

This time he sells 23 of the cakes for 90p each.

What percentage profit did Ron make this time?









Find original less than 100%



Notes and guidance

It is useful to concentrate on questions where a calculator is not required so students can interpret rather than just follow a procedure. Common errors include finding the given percentage of the given number rather than working backwards towards an original. Bar models are a useful model as they show both the reduction and the remainder providing a strong visual clue as to how to find the original.

Key vocabulary

Original Percentage Reverse

Equivalent Multiple

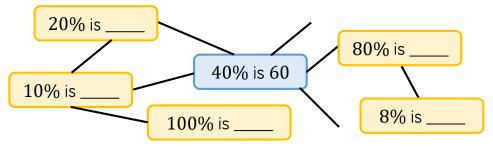
Key questions

Is the original value greater than or less than the given amount? What percentage is the original amount? How can we represent this using a bar model? From the percentage given, what other percentages can we easily work out?

How can we build on these to find 100%?

Exemplar Questions

40% of a number is 60. What other facts can you find?



After spending 60% of her money, Alex has £24 left.

After spending 50% of her money, Rosie has £27.50 left.

Who had most money to start with? Rosie How much more?



Amir is comparing the sale price of t-shirts from two different shops. In which shop was the t-shirt originally more expensive?





Whitney flips a coin and gets heads 45% of the time. She gets heads 54 times.

How many times did she flip the coin?



Find original more than 100%



Notes and guidance

This step is closely linked with the previous one and once again a heavy emphasis should be placed on adding the percentage increase to 100%. This will enable students to understand what percentage the value they are given represents. Common misconceptions include students finding the percentage increase of the given amount and subtracting it from the given amount.

Key vocabulary

Original Percentage Reverse

Equivalent Increase

Key questions

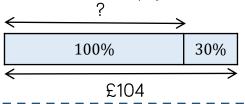
Is the amount given more or less than the new amount?

How can we represent this on a bar model?

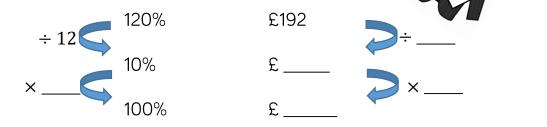
What is the same and what is different between these two bar models?

Exemplar Questions

After a 30% pay rise, Eva earns £104 a week. How much did she earn before the pay rise?



Valued Added Tax (VAT) is charged at 20% in the UK. Complete this calculation to find the cost of the jacket without VAT.



After a 20% change in price, a games console now costs £288 What was the original price if the change was a 20% increase? What was the original price if the change was a 20% decrease?



Annie has some sweets.

Teddy gives her some sweets and she now has 50% more. Rosie gives her some sweets and she now has an extra 40% Annie now has 63 sweets. How many did she have originally?







Complex percentages



Notes and guidance

This is another opportunity for students following the higher strand to practise interpretation of questions so that they can choose the correct method. They should look at a variety of situations including the 'reverse' percentage questions just studied mixed with percentage increase, decrease, finding a percentage and expressing as a percentage.

Key vocabulary

Original	Percentage	Reverse
Express	Decrease	Increase
Profit/Loss		

Key questions

How can you represent this problem using a bar model? How can you tell if a question involves finding an amount before a percentage change? How does this affect your approach to the question?

Exemplar Questions

Mo buys a rare comic for £120 and sells it again for £170 Compare these methods to work out his percentage profit.

Method 1

$$170 - 120 = 50$$

 $\frac{50}{120} = 0.41666 \approx 42\%$

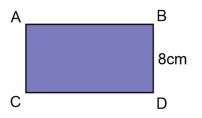
Method 2
$$\frac{170}{120} = 1.41666 \approx 142\%$$

$$142\% - 100\% = 42\%$$

After a 18% pay rise, Dora's salary is £38350 Which of these calculations will give her original salary?

Write a question that could be solved with each of the other calculations.

Ms Rose bought a house in 2012 for £120000 She sold the house five years later making a profit of 60% How much did she sell the house for?



ABCD is a rectangle.
The lengths of the sides AB and
BD are in the ratio 5:4
What percentage of the perimeter
of the rectangle is side AC?