Standard Index Form

Year 8

#MathsEveryoneCan

White Rose Maths

	Week 1	Week 2	Week 3	Week 4	Week 5	Week 6	Week 7	Week 8	Week 9	Week 10	Week 11	Week 12
Autumn	**Proportional Reasoning**						**Representations**					
	Ratio and scale		Multiplicative change		Multiplying and dividing fractions		Working in the Cartesian plane			Representing data		Tables & Probability
Spring	**Algebraic techniques**						**Developing Number**					
	Brackets, equations and inequalities				Sequences	Indices	Fractions and percentages		Standard index form		Number sense	
Summer	**Developing Geometry**						**Reasoning with Data**					
	Angles in parallel lines and polygons		Area of trapezia and circles		Line symmetry and reflection		The data handling cycle				Measures of location	

Spring 2: Developing Number

Weeks 1 and 2: Fractions and Percentages

This block focuses on the relationships between fractions and percentages, including decimal equivalents, and using these to work out percentage increase and decrease. Students also explore expressing one number as a fraction and parentage of another. Both calculator and non-calculator methods are developed throughout to support students to choose efficient methods. Financial maths is developed through the contexts of e.g. profit, loss and interest. The Higher strand also looks at finding the original value given a percentage or after a percentage change.

National Curriculum content covered includes:

- develop their use of formal mathematical knowledge to interpret and solve problems, including in financial mathematics
- work interchangeably with terminating decimals and their corresponding fractions
- define percentage as 'number of parts per hundred', interpret percentages and percentage changes as a fraction or a decimal, interpret these multiplicatively, express one quantity as a percentage of another, compare two quantities using percentages, and work with percentages greater than 100%
- interpret fractions and percentages as operators

Weeks 3 and 4: Standard index form

Higher strand students have already briefly looks at standard form in Year 7 and now this knowledge is introduced to all students, building from their earlier work on indices last term. The use of context is important to help students make sense of the need for the notation and its uses. The Higher strand includes a basic introduction to negative and fractional indices.

National Curriculum content covered includes:

- use integer powers and associated real roots (square, cube and higher), recognise powers of 2, 3, 4, 5 and distinguish between exact representations of roots and their decimal approximations
- interpret and compare numbers in standard form $A \times 10^n$, $1 \leq A < 10$, where n is a positive or negative integer or zero

Weeks 5 and 6: Number sense

This block provides a timely opportunity to revisit a lot of basic skills in a wide variety of contexts. Estimation is a key focus and the use of mental strategies will therefore be embedded throughout. We will also use conversion of metric units to revisit multiplying and dividing by 10, 100 and 1000 in context. The Higher strand will extend this to look at the conversion of area and volume units, as well as having an extra step on the use of error notation. We also look explicitly at solving problems using the time and calendar as this area is sometimes neglected leaving gaps in student knowledge.

National Curriculum content covered includes:

- use standard units of mass, length, time, money and other measures, including with decimal quantities
- round numbers and measures to an appropriate degree of accuracy [for example, to a number of decimal places or significant figures]
- use approximation through rounding to estimate answers and calculate possible resulting errors expressed using inequality notation $a < x \leq b$
- use a calculator and other technologies to calculate results accurately and then interpret them appropriately.

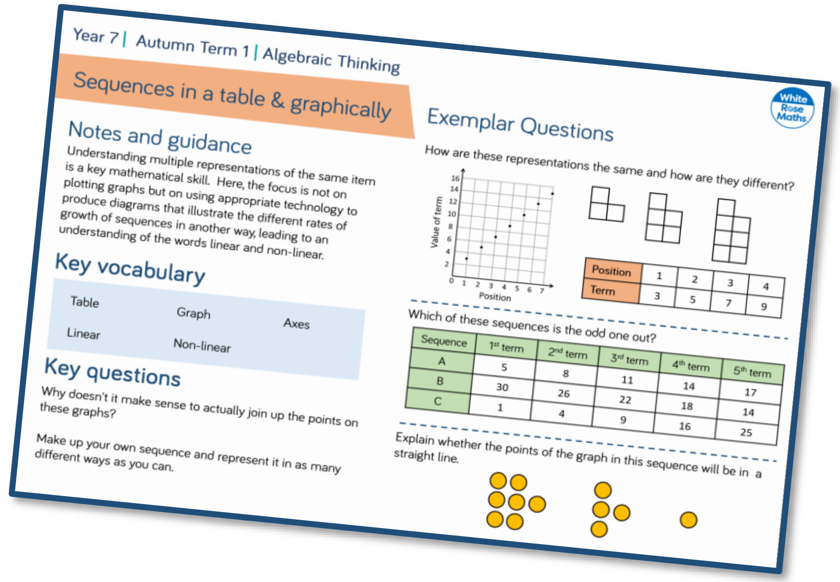
Why Small Steps?

We know that breaking the curriculum down into small manageable steps should help students to understand concepts better. Too often, we have noticed that teachers will try and cover too many concepts at once and this can lead to cognitive overload. We believe it is better to follow a "small steps" approach.

As a result, for each block of content in the scheme of learning we will provide a "small step" breakdown. ***It is not the intention that each small step should last a lesson – some will be a short step within a lesson, some will take longer than a lesson.*** We would encourage teachers to spend the appropriate amount of time on each step for their group, and to teach some of the steps alongside each other if necessary.

What We Provide

- Some ***brief guidance*** notes to help identify key teaching and learning points
- A list of ***key vocabulary*** that we would expect teachers to draw to students' attention when teaching the small step,
- A series of ***key questions*** to incorporate in lessons to aid mathematical thinking.
- A set of questions to help ***exemplify*** the small step concept that needs to be focussed on.



- These include reasoning and problem-solving questions that are fully integrated into the scheme of learning. Depending on the attainment of your students, you many wish to use some or all of these exemplars, which are in approximate order of difficulty. Particularly challenging questions are indicated with the symbol .
- For each block, we also provide ideas for key representations that will be useful for all students.

In many of the blocks of material, some of the small steps are in **bold**. These are content aimed at higher attaining students, but we would encourage teachers to use these with as many students as possible – if you feel your class can access any particular small step, then please include it in your planning.

Standard Form

Small Steps

- Investigate positive powers of 10
- Work with numbers greater than 1 in standard form
- Investigate negative powers of 10
- Work with numbers between 0 and 1 in standard form
- Compare and order numbers in standard form
- Mentally calculate with numbers in standard form
- Add and subtract numbers in standard form
- Multiply and divide numbers in standard form
- Use a calculator to work with numbers in standard form

H denotes Higher Tier GCSE content

R Denotes “review step” – content should have been covered in Year 7

Standard Form

Small Steps

- Understand and use negative indices (H)
- Understand and use fractional indices (H)

(H) denotes Higher Tier GCSE content

(R) Denotes "review step" – content should have been covered in Year 7

Positive powers of 10

Notes and guidance

Using experimentation students will explore powers of ten. This recaps and builds on students' understanding from the indices unit and the work they did in year 7. It is useful to explore why $10 \times 10^3 \neq 10^3$ as this is a common misconception. Discussion around the benefits of writing numbers in powers of ten can be demonstrated using large numbers such as 1 billion $= 1000\,000\,000 = 10^9$

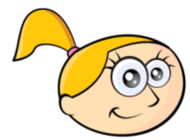
Key vocabulary

Base	Index/Indices	Power
Exponent		

Key questions

How many times bigger than 1000 is 10^8?
Why are (e.g.) $(10^2)^3$ and $10^2 \times 3$ different?
Is there a simpler way to write (e.g.) 10000×100000?
What calculations could give an answer of (e.g.) 10^{12}?

Exemplar Questions



$10 \times 10 \times 10 = 10^3$ and
$10 \times 10 \times 10 \times 10 = 10^4$
Therefore this must mean that
$10 \times 10 \times 10 \times 10 \times 10 = 10^5$

- What connections do you see in Eva's examples?
- Use the examples to help work out what is meant by 10^7
- How can we use this understanding to work out $10^7 \times 10$?
- What is the meaning of 10^{20}?

Fill in the blanks.

$10^3 \times 10^3 = 10^{\square} \times 10^5$

$10^{6+4} = 10^3 \times 10^{\square}$

Solve the equations.

$10^6 \times b = 10^{10}$

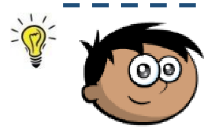
$100a = 10^9$

$10^4 \times c = 10^{12}$

$100 = \frac{10^6}{d}$

$\frac{10^6}{e} = 10^3$

$\frac{10^6}{f} = 10^6$



5^3 is the same as $\frac{10^3}{2}$

Convince me that Amir is incorrect.

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Standard form with numbers > 1

Notes and guidance

Students will now write large numbers in standard form. Students should be exposed to correct examples in the form $A \times 10^n$ where A is a number between 1 and 10 and n is an integer. It is important to look at how standard form works rather than just counting zeros. Teachers can deepen this understanding by looking at non-examples such as 0.8×10^4 and $4 \times 10^{0.8}$

Key vocabulary

Base	Index/Indices	Power
Exponent	Standard (index) form	

Key questions

What is one gigabyte (1 GB) written in standard form?
What is the same and what is different about how 75 000 and 70 000 are written in standard form?
Why is it more efficient to write 4×10^{50} in standard form rather than as an ordinary number?

Exemplar Questions

Fill in the blanks.

300000
$= 3 \times 10000$
$= 3 \times 10^4$

6000000
$= 6 \times \square$
$= 6 \times 10^{\square}$

70000
$= \square$
$= \square$

5000 is the same as 5×10^3 and 6000 is 6×10^3

5500 must be 5.5×10^3 as it is half way between 5000 and 6000

- How would 5200 be written in standard form?
- How would 52 000 be written in standard form?
- Write 5.2×10^6 as an ordinary number

Fill in the blanks.

$40000 = 4 \times 10^4$
$\square = 4 \times 10^3$
$400 = \square$

$\square = 4.2 \times 10^2$
$425 = \square$
$\square = 4.25 \times 10^3$

Neptune is 2.8 billion miles from the sun.

- Write this number in standard from.
- Write 2.8 billion miles in kilometres in standard form.

Negative powers of 10

Notes and guidance

During this step students will look at decreasing powers of ten then investigate what happens if you get to 1 and below. Time should be spent discussing and also investigating 10^0 and explore misconceptions such as $10^0 = 0$ and $10^{-2} = -100$ Students should also be confident working between standard form, decimals and fraction equivalences.

Key vocabulary

Base	Index/Indices	Power
Exponent	Negative	Place value

Key questions

How many different ways can you write 0.001?
How could you show 10^{-2} on a place value grid?
What is the value of 10^0? What is the value of 8^0?
What is x^0 for any value of x?

Exemplar Questions

Fill in the blanks.

$10^3 = \square$

$10^2 = 100$

$\square = 10$

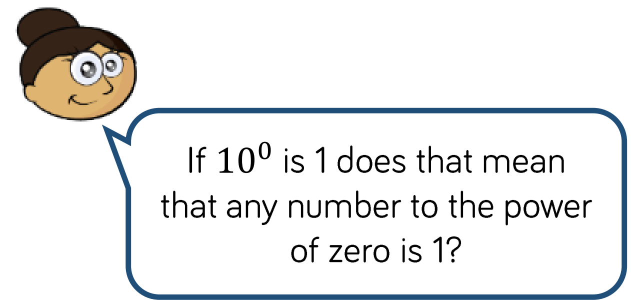
$10^0 = \square$

$\square = \frac{1}{10^1} = \frac{1}{10} = 0.1$

$10^{-2} = \frac{1}{10^2} = \square = 0.01$

$10^{-3} = \frac{1}{10^3} = \frac{1}{1000} = \square$

Investigate Dora's theory.
Is she correct?



Match the equivalent cards and write a different card for each.

$\frac{1}{10^3 \times 10^0}$	$\frac{1}{10000}$	$\frac{1}{10}$	$\frac{1}{10^4}$
0.01	10^{-1}	$10^{-5} \times 10^3$	10^{-3}

Complete the statements using <, > or =

$10^3 \square 10^4$	$10^{-3} \square 10^4$	$10^3 \square 10^{-4}$
$10^{-3} \square 10^{-4}$	$10^1 \square 10^{-2}$	$10^1 \square 10^2$

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Numbers between 0 and 1

Notes and guidance

Once negative powers are understood, students can explore the patterns and connections between decimal numbers and standard form. They should also be exposed to similar questions that have different answers to deepen their understanding e.g. comparing 8.9 and 8.09 written in standard form. Misconception such as $3 \times 10^{-1} = -0.3$ should also be explored and addressed.

Key vocabulary

Base	Index/Indices	Power
Exponent	Negative	Place value

Key questions

What is the same and what is different about (e.g.) 3×10^{-4} and 3×10^{4}?

Explain why (e.g.) 4×10^{-3} is greater than 5×10^{-4}.

Are negative powers of 10 always, sometimes or never negative numbers?

Exemplar Questions

Fill in the blanks.

$5 = 5 \times 1 = 5 \times 10^{0}$

$0.5 = 5 \times 0.1 = \square$

$0.05 = 5 \times 0.01 = \square$

$0.006 = \square = \square$

$4.5 = \square = 4.5 \times 10^{0}$

$\square = 4.5 \times 0.1 = 4.5 \times 10^{-1}$

$0.045 = 4.5 \times 0.01 = \square$

$0.0045 = \square = \square$

Which of the following cards correctly represents 0.0003?

3×10^{4}	$\frac{1}{3000}$	$\frac{1}{3^4}$	$\frac{1}{30000}$	3×10^{-4}

0.0302 is the same as 3.2×10^{-2}

I think 0.0302 is 3.02×10^{-2}

- Discuss which statement is correct?
- Can you represent 0.504 on a place value grid?

Match the cards of equal value.

4.05×10^{-3}	5.4×10^{-2}	0.045	0.00405
0.054	4.05×10^{-2}	5.4×10^{-1}	5.04×10^{-2}
0.54	0.0504	0.0405	4.5×10^{-2}

Give three examples of ordinary numbers that could round to 5.6×10^{-3}. What degree of accuracy are you rounding to?

Order numbers in standard form

Notes and guidance

Students will order numbers given in words, standard form and ordinary form. Strategies for comparing numbers, such as considering the exponent of 10 as an initial check should be discussed. Students should revisit ordering decimal numbers to tease out any misconceptions that may be a barrier to ordering numbers in standard from. Access to place value grids would be useful here.

Key vocabulary

Base	Index/Indices	Power
Exponent	Negative	Place value

Key questions

How can we compare a fraction, a decimal and a number written in standard form? What could you do to make it easier?

What do you look at first when comparing numbers written in standard form? Why?

Exemplar Questions

3×10^4 is less than 4×10^3

Show that Tommy is wrong.

A: $6 \times 10^7, 6.3 \times 10^7, 6.03 \times 10^7$ and 6.18×10^7

B: $6.13, 7.31, 6.31, 6.301$ and 6.013

C: $6 \times 10^4, 6.9 \times 10^3, 6.8 \times 10^{-1}$ and 6.7×1^6

- Order the numbers on each card from smallest to largest.
- Which card is easier to write in size order A, B or C?

Complete the statements using <, > or =

5.6×10^6 ◯ 6×10^5	1×10^0 ◯ 1×0
30×300 ◯ 9.5×10^{-3}	0.4 ◯ 4×10^{-1}
71000 ◯ 7.1×10^5	4.1×10^{-4} ◯ $\frac{1}{4100}$

Arrange the following cards in ascending order.

$\frac{1}{7200}$	7.1×10^{-4}	One seventh	$\frac{7}{7000}$
70×10^{-3}	7.2×10^1	7.5×10^0	0.73

Standard form: Mental calculations

Notes and guidance

Using simple numbers, students will complete mental calculations where one number in standard form is multiplied or divided by an integer. The result may no longer be in standard form. A key focus of this step is correcting answers such as 24×10^8 by using the fact that $24 = 2.4 \times 10^1$ and completing the calculation using index laws $2.4 \times 10^1 \times 10^8 = 2.4 \times 10^9$

Key vocabulary

Base	Index/Indices	Power
Exponent	Negative	Commutative

Key questions

Why isn't (e.g.) 200×10^6 in standard form? How could rewriting 200 help us?

Explain how 0.2×10^4 and 0.2×10^{-4} can be written in standard form. What is the same and what is different?

Why is $6 \times (5 \times 10^3)$ more difficult than $4 \times (2 \times 10^3)$?

Exemplar Questions

Fill in the blanks to complete the calculations.

$3 \times (2 \times 10^6)$
$= (3 \times \square) \times 10^6$
$= 6 \times \square$

$(7 \times 10^{-4}) \div 2$
$= (7 \div \square) \times 10^{-4}$
$= 3.5 \times \square$

Use similar strategies to work out the other cards.

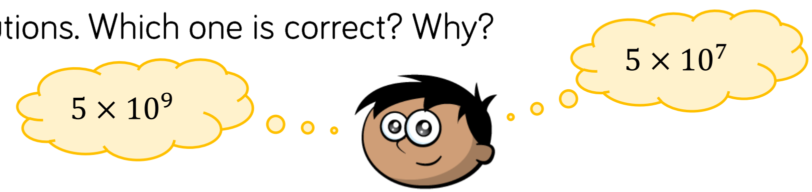
$2 \times (2.5 \times 10^6)$

$(9.3 \times 10^{-4}) \div 3$

Find and correct Alex's mistake.

$(8 \times 10^7) \times 2 = 16 \times 10^7$
$= 1.6 \times 10^1 \times 10^7 = 1.6 \times 10^{17}$ ✗

Amir works out $2 \times 10^8 \div 4$ and gets the answer 0.5×10^8.
He realises his answer is not in standard form and thinks of two possible solutions. Which one is correct? Why?



Complete the calculations, giving your answers in standard form.

$3 \times (6 \times 10^6)$
$= 3 \times 6 \times 10^6$
$= 18 \times 10^6$
$= 1.8 \times 10^1 \times 10^6$
$= 1.8 \times \square$

$(7 \times 10^4) \times 1000$
$= 7 \times 10^4 \times 10^{\square}$
$= 7 \times \square$

$(2 \times 10^{-5}) \div 5$
$= 0.4 \times 10^{-5}$
$= 4 \times 10^{\square} \times 10^{-5}$
$= 4 \times 10^{\square}$

$+$ and – numbers in standard form

Notes and guidance

Students will compare strategies for addition and subtraction without a calculator. There is a risk of just adding the numbers and adding the powers separately and students may prefer to always convert to ordinary numbers. Even when the powers of 10 are the same, there can be problems such as $3 \times 10^4 + 8 \times 10^4 = 11 \times 10^4$ where the answer needs changing as covered last step.

Key vocabulary

Base	Index/Indices	Power
Exponent	Negative	Commutative

Key questions

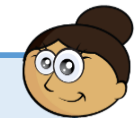
Is it easier to add the numbers as they are or convert them to ordinary numbers first?
What do we do if the total isn't in standard form?
What is (e.g.) $10^{-3} + 10^3$ as an ordinary number?

Exemplar Questions

Mo and Dora are working out $7 \times 10^3 + 2 \times 10^3$
Decide which method you prefer and discuss why.

$= 7000 + 2000$
$= 9000$
$= 9 \times 1000$
$= 9 \times 10^3$

$= (7 + 2) \times 10^3$
$= 9 \times 10^3$



Decide whose method you would use for the following questions.

$6 \times 10^{-3} + 1.5 \times 10^3$	$7 \times 10^5 - 5 \times 10^5$
$8.2 \times 10^{-6} - 1.2 \times 10^{-6}$	$9.6 \times 10^5 - 3.2 \times 10^4$

Dora is working out $8 \times 10^4 + 9 \times 10^4$ and gets 17×10^4
She says her answer is not in standard form.
What could she do?

Fill in the blanks.

$6 \times 10^{\square} + 2.3 \times 10^3 = 6.23 \times 10^4$

$6 \times 10^4 \square 2.3 \times 10^{\square} = 2.9 \times 10^5$

$6 \times 10^{\square} + 2.3 \times 10^{\square} = 8.3 \times 10^{-6}$

× and ÷ numbers in standard form

Notes and guidance

Students will explore the use of commutativity to multiply and divide numbers given in standard form. Their earlier work on indices and dealing with answers like 30×10^7 should have prepared them for this step. It is helpful to include various forms of the same questions such as

$(4 \times 10^5) \div (2 \times 10^6)$ and $\frac{4\times10^5}{2\times10^6}$

Key vocabulary

Base	Index/Indices	Power
Exponent	Negative	Commutative

Key questions

How many different ways can you write (e.g.) $(3 \times 10^4) \times (2 \times 10^4)$?

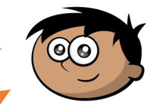
Describe the steps you need to take to multiply/divide a pair of numbers in standard form.

When can we write a division as a fraction?

Exemplar Questions

Amir is working out $(3.2 \times 10^2) \times (2 \times 10^4)$

Because multiplication is commutative I can write the question like this to help me $(3.2 \times 2) \times (10^2 \times 10^4)$



He then works out $(8 \times 10^6) \div (4 \times 10^3)$

I can write this as $(8 \div 4) \times (10^6 \div 10^3)$

Complete Amir's calculations.

Rosie and Whitney are calculating $(5 \times 10^6) \div (2 \times 10^3)$

Here are their answers. Can you spot what mistakes they have made?

2.5×10^2

3×10^3

Which question does not belong?

$(2 \times 10^8) \times (1.6 \times 10^{-12})$	$(2.8 \times 10^{-4}) + (4 \times 10^{-5})$
$(4 \times 10^{-4}) - (8 \times 10^{-3})$	$(9.6 \times 10^5) \div (3 \times 10^9)$

Standard form using a calculator

Notes and guidance

All four operations will be explored, and students can further their knowledge of calculators to use the memory and exponent functions. Alternative methods to the answer to the same question could be shown on the calculator, such as using the fraction button for division. The use of emulator software, if available, would be helpful.

Key vocabulary

Standard form	Exponent	Power
Scientific notation	SCI/EXP	

Key questions

Explain how to input (e.g.) 2.4×10^5 on a calculator. What would be different inputting 2.4×10^{-5}?
What button on your calculator converts an answer into standard form.
How do you round a number in standard form to 1/2/3 significant figures?

Exemplar Questions

Rosie uses a calculator to work out some calculations using standard form.

Some of my answers are already written in standard form and some are not…

- Use your calculator to find an example of multiplication, division, addition and subtraction where the answer is given in standard form and one is not.
- Compare your examples with a partner.
- What is the largest number you can type into your calculator before it automatically changes to standard form?

Use the calculator to complete the calculations on the cards where $a = 3.2 \times 10^4$ and $b = 2.1 \times 10^{-3}$
Give your answers in standard form to 3 significant figures.

b^3	$a \div b^3$	$2a \div b^2$
$a^2 - 2b$	$a^2 + 2b$	$(a - 2b)^2$

The average human body can produce 3 million red blood cells every second.
How many red blood cells does the average human body produce in one year?
Give your answer in standard form.

Understand negative indices

H

Notes and guidance

Students will build on their understanding of negative powers of 10 to explore negative indices generally. Common misconceptions around negative powers, such as $5^{-2} \neq -25$ and $5^{-2} \neq -\frac{1}{25}$, should be discussed. It is worth spending time exploring the patterns and linking to inverse operations to deepen conceptual understanding.

Key vocabulary

Power	Negative	Exponent
Reciprocal	Zero	

Key questions

Will a number raised to a negative power always, sometimes or never have a negative value?

How does working out negative powers relate to the subtraction law for dividing indices?

How do you enter negative powers on a calculator?

Exemplar Questions

Teddy is exploring negative indices. Fill in the blanks to complete his investigation.

Every time I decrease the power of 5 by 1 I am dividing the previous answer by 5

$$5^{-1} = 1 \div 5 = \frac{1}{5^1} = \frac{1}{5}$$

$$5^{-2} = 1 \div 5^2 = \frac{1}{\square} = \frac{1}{\square}$$

$$5^{-3} = 1 \div \square = \frac{1}{\square} = \frac{1}{\square}$$

Mo, Alex, Rosie and Tommy are working out 2^{-3}

They all have a different answer.

Mo	Alex	Rosie	Tommy
-6	$\frac{1}{-8}$	$\frac{1}{8}$	$\frac{1}{-6}$

- Who is correct?
- Explain the mistakes that have been made by the others.

Sort the cards in ascending order. Check using a calculator.

1^{-1}	2^{-2}	2^3	3^2
3^{-2}	2^{-3}	1^{-10}	

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Understand fractional indices

H

Notes and guidance

Here students will begin working with fractional indices, finding the square roots and the cube roots of numbers. Powers such as two-thirds are not covered. Misconceptions such as dividing the number by 2 or 3 should be discussed. This step also presents a good opportunity to revisit previous steps of learning by ordering numbers which have fractional and negative powers, as well as revisiting the power of 0.

Key vocabulary

Fraction	Reciprocal	Root
Exponent		

Key questions

How does the addition law for indices help us work out the meaning of "to the power half"?

Give an example to show "to the power half" is not the same as "divide by 2"?

Exemplar Questions

Whitney is working out $16^{\frac{1}{2}} \times 16^{\frac{1}{2}}$

She writes $16^{\frac{1}{2}} \times 16^{\frac{1}{2}} = 16^{\frac{1}{2}+\frac{1}{2}} = 16^1$

If $16^{\frac{1}{2}}$ multiplied by itself is 16^1 then that must mean that a $16^{\frac{1}{2}}$ is equivalent to $\sqrt{16}$

Use Whitney's reasoning to find the values of $25^{\frac{1}{2}}$ and $49^{\frac{1}{2}}$

Annie and Mo are working out $27^{\frac{1}{3}}$

The answer is 9

The answer is 3

Who is correct? How do you know?

Put the following cards in descending order of size.

$64^{\frac{1}{3}}$	10^3	$64^{\frac{1}{2}}$	$100^{\frac{1}{2}}$
$1000^{\frac{1}{3}}$	64^1	10^{-3}	10^0

Dexter says he can represent 8 in 3 different ways.

$64^{\frac{1}{2}}$	2^3	$512^{\frac{1}{3}}$

He does the same again with a different number.

One of his cards is $216^{\frac{1}{3}}$

What could the other cards be?