Sets and Probability

Year 7

#MathsEveryoneCan

White Rose Maths

	Week 1	Week 2	Week 3	Week 4	Week 5	Week 6	Week 7	Week 8	Week 9	Week 10	Week 11	Week 12
Autumn	**Algebraic Thinking**						**Place Value and Proportion**					
	Sequences		Understand and use algebraic notation		Equality and equivalence		Place value and ordering integers and decimals			Fraction, decimal and percentage equivalence		
Spring	**Applications of Number**						**Directed Number**			**Fractional Thinking**		
	Solving problems with addition & subtraction		Solving problems with multiplication and division			Fractions & percentages of amounts	Operations and equations with directed number			Addition and subtraction of fractions		
Summer	**Lines and Angles**						**Reasoning with Number**					
	Constructing, measuring and using geometric notation			Developing geometric reasoning			Developing number sense		Sets and probability		Prime numbers and proof	

Summer 2: Reasoning with Number

Weeks 7 to 8: Developing Number Sense

Students will review and extend their mental strategies with a focus on using a known fact to find other facts. Strategies for simplifying complex calculations will also be explored. The skills gained in working with number facts will be extended to known algebraic facts.

National curriculum content covered:

- consolidate their numerical and mathematical capability from key stage 2 and extend their understanding of the number system and place value to include decimals, fractions, powers and roots
- select and use appropriate calculation strategies to solve increasingly complex problems
- begin to reason deductively in number and algebra

<u>Interleaving/Extension of previous work</u>

- Generating and describing sequences
- Substitution into expressions
- Order of operations

Weeks 9 to 10: Sets and Probability

FDP equivalence will be revisited in the study of probability, where students will also learn about sets, set notation and systematic listing strategies.

National curriculum content covered:

- record, describe and analyse the frequency of outcomes of simple probability experiments involving randomness, fairness, equally and unequally likely outcomes, using appropriate language and the 0-1 probability scale
- understand that the probabilities of all possible outcomes sum to 1
- enumerate sets and unions/intersections of sets systematically, using tables, grids and Venn diagrams
- generate theoretical sample spaces for single and combined events with equally likely and mutually exclusive outcomes and use these to calculate theoretical probabilities
- appreciate the infinite nature of the sets of integers, real and rational numbers

<u>Interleaving/Extension of previous work</u>

- FDP equivalence
- Forming and solving equations
- Adding and subtracting fractions

Weeks 11 to 12: Prime Numbers and Proof

Factors and multiples will be revisited to introduce the concept of prime numbers, and the Higher strand will include using Venn diagrams from the previous block to solve more complex HCF and LCM problems. Odd, even, prime, square and triangular numbers will be used as the basis of forming and testing conjectures. The use of counterexamples will also be addressed.

National curriculum content covered:

- use the concepts and vocabulary of prime numbers, factors (or divisors), multiples, common factors, common multiples, highest common factor, lowest common multiple, prime factorisation, including using product notation and the unique factorisation property
- use integer powers and associated real roots (square, cube and higher), recognise powers of 2, 3, 4, 5
- make and test conjectures about patterns and relationships; look for proofs or counterexamples
- begin to reason deductively in number and algebra

<u>Interleaving/Extension of previous work</u>

- Generating and describing sequences
- Factors and multiples

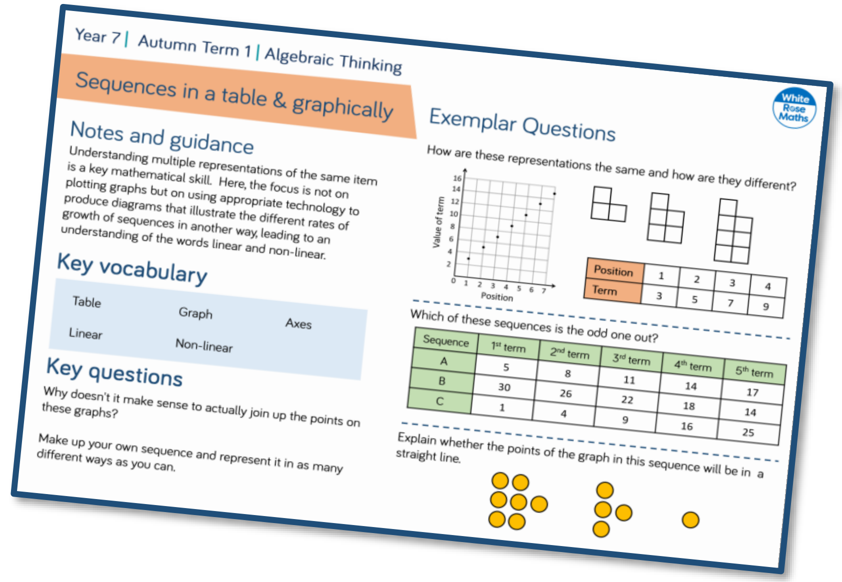
Why Small Steps?

We know that breaking the curriculum down into small manageable steps should help students to understand concepts better. Too often, we have noticed that teachers will try and cover too many concepts at once and this can lead to cognitive overload. We believe it is better to follow a "small steps" approach.

As a result, for each block of content in the scheme of learning we will provide a "small step" breakdown. ***It is not the intention that each small step should last a lesson – some will be a short step within a lesson, some will take longer than a lesson.*** We would encourage teachers to spend the appropriate amount of time on each step for their group, and to teach some of the steps alongside each other if necessary.

What We Provide

- Some ***brief guidance*** notes to help identify key teaching and learning points
- A list of ***key vocabulary*** that we would expect teachers to draw to students' attention when teaching the small step,
- A series of ***key questions*** to incorporate in lessons to aid mathematical thinking.
- A set of questions to help ***exemplify*** the small step concept that needs to be focussed on.



- These include reasoning and problem-solving questions that are fully integrated into the scheme of learning. Depending on the attainment of your students, you many wish to use some or all of these exemplars, which are in approximate order of difficulty. Particularly challenging questions are indicated with the symbol .
- For each block, we also provide ideas for key representations that will be useful for all students.

In many of the blocks of material, some of the small steps are in **bold**. These are content aimed at higher attaining students, but we would encourage teachers to use these with as many students as possible – if you feel your class can access any particular small step, then please include it in your planning.

Sets and Probability

Small Steps

- Identify and represent sets
- Interpret and create Venn diagrams
- Understand and use the intersection of sets
- Understand and use the union of sets
- **Understand and use the complement of a set** H
- Know and use the vocabulary of probability

H denotes higher strand and not necessarily content for Higher Tier GCSE

Sets and Probability

Small Steps

- Generate sample spaces for single events
- Calculate the probability of a single event
- Understand and use the probability scale
- Know that the sum of probabilities of all possible outcomes is 1

H denotes higher strand and not necessarily content for Higher Tier GCSE

Identify and represent sets

Notes and guidance

In this small step, students begin to systematically organise information into sets using set notation. They can identify members of sets given a description, and describe simple sets given the elements. All should find the idea of a set familiar, much of the language will be unfamiliar and will need revisiting regularly to aid retention.

Key vocabulary

Universal Set	Inclusive	Element
Member	Set	

Key questions

What makes a group of objects a set?

Do sets just have to be numerical?

Can you have a set with an infinite number of elements?

Exemplar Questions

In each case, are the sets A and B the same or different?

- A { 2, 4, 6, 8 } — B { 8, 4, 6, 2 }
- A { -2, -4, -6, -8, -10 } — B { 2, 4, 6, 8, 10 }
- A { names of girls in your class }
- B { names of girls in your school }

List the element of the sets.

Set A: Types of triangle

Set B: Quadrilaterals with at least one pair of parallel sides

Set C: Factors of 30

Given that ξ = { Integers between 1 and 50 inclusive }

List the following sets.

Set A: Multiples of 9 Set B: Factors of 9 Set C: Factors of 100

How would sets A, B and C change if:

ξ = { Integers between 1 and 100 inclusive } ?

ξ = { Integers between 1 and 10 inclusive } ?

Describe these sets in words. Compare your answers to a partner's. Which ones can be described in more than one way?

{ 1, 3, 5, 7, 9 } { 3, 6, 9, 12 } { 1, 2, 5, 10 }

{ a, b, c, d, e, f } $\{ -2, -1, 0, 1, 2, 3, 4, 5 \}$

Interpret and create Venn diagrams

Notes and guidance

By understanding the structure of a Venn Diagram, students will be able to sort information efficiently, seeing whether sets intersect, or whether they are mutually exclusive. Linking this to probability can help students to understand how Venn diagrams can be used as a strategy in working out answers to other problems.

Key vocabulary

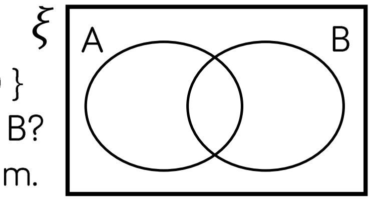
Venn diagram	Intersection
Mutually Exclusive	Union

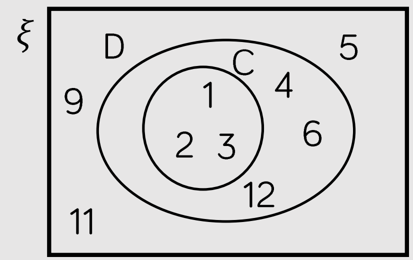
Key questions

How many circles or ellipses are needed in a Venn diagram?
Do we always need a box around the circles/ellipses? Why or why not?
Do the circles/ellipses always need to overlap? Why or why not?

Exemplar Questions

ξ = { 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 }
A = { even numbers } and B = { factors of 10 }
Which numbers are elements of both A and B?
Represent this information on a Venn diagram.

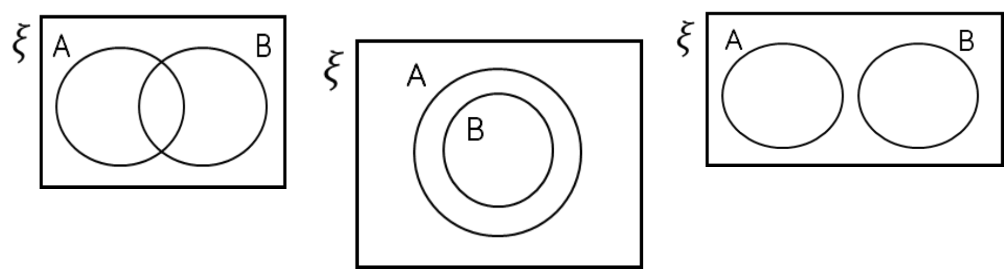




The Venn diagram shows set C is a subset of set D. All its elements are members of D as well as C.
List the members of sets C, D and ξ.
Suggest possible descriptions of each set.

Given that ξ = { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 }
Choose and complete a Venn diagram to show:

- A = { even numbers } and B = { odd numbers less than 8 }
- A = { even numbers } and B = { even numbers less than 7 }
- A = { even numbers } and B = { square numbers }



Intersection of sets

Notes and guidance

Having already explored the structure of a Venn diagram, students should be able to identify and interpret the part that represents the intersection of two or more sets. Using colour to highlight these areas is an effective way of finding this. Students need to be explicitly taught to associate the word 'and' with intersecting sets. Exploring the intersection of areas where both sets are not true (e.g. doesn't like singing or guitar) extends thinking on this topic.

Key vocabulary

Venn Diagram	Intersection	Element
Intersect	Complement	And

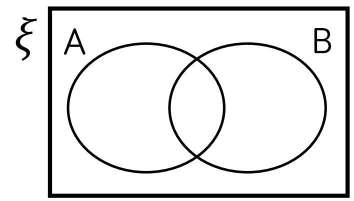
Key questions

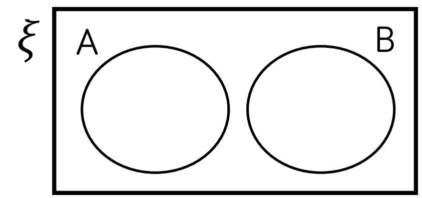
What's the same and what's different about the following Venn diagrams?
Why do you think we use different Venn diagrams for different problems? Do all sets intersect? What does the overlapping region represent?

Exemplar Questions

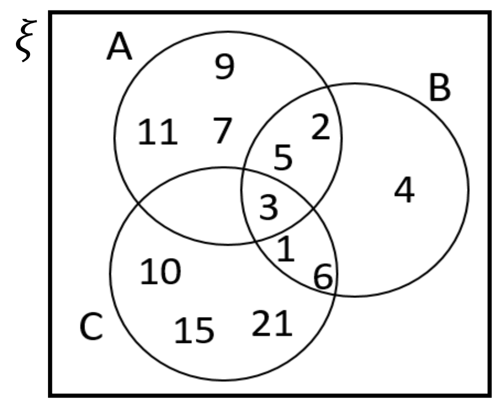
Which Venn diagram would you use to represent sets A and B?

- A = { 5, 10, 15, 20, 25, 30 }
- B = { 1, 2, 3, 5, 6, 10, 15, 30 }





Write down the set A ∩ B.



Write down the elements in the sets:

- A ∩ B
- A ∩ B ∩ C
- B ∩ C
- A ∩ C

Martin says that A ∩ C is exactly the same set as A ∩ B ∩ C. Is he right?
Explain your answer.
Describe in words the sets A, B and C.

In a group of 100 children, 30 like singing, 40 play the guitar, and 20 like singing and play the guitar.
Draw a Venn diagram to represent this information.
How many children don't like singing and don't play the guitar?

Union of sets

Notes and guidance

The step is to support students to distinguish between the union of sets – members belong to A or B or both – from the intersection covered in the previous step.

Labelled Venn diagrams and the use of colour are useful representations to develop understanding. Students need to be explicitly taught to use the words 'and' and 'or'.

Key vocabulary

Union	Or	Element
Intersection	And	Both

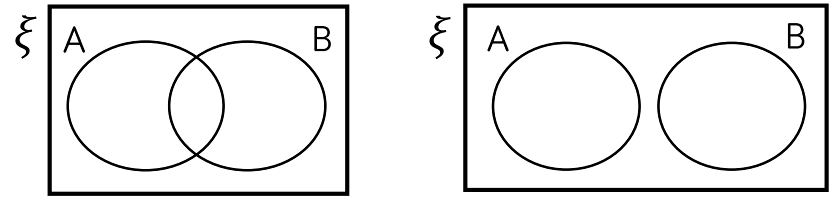
Key questions

What's the same and what's different between the union of sets and the intersection of sets?
What does the union of two sets look like if they have no intersection?
What's the same and what's different about A ∪ B in these situations?

Exemplar Questions

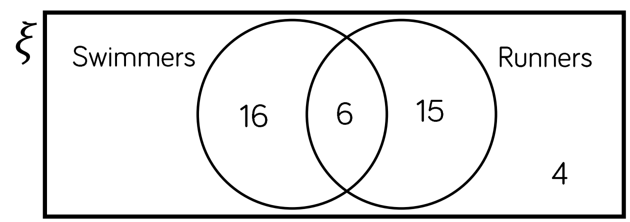
Which Venn diagram would you use to represent sets A and B?

- A = { -10, -8, -6, -5, -4, -2, 0 }
- B = { -2, -1, 0, 1, 2 }



Write down the sets A ∪ B and A ∩ B. Why are they different?

If A = { factors of 36 } , B = { multiples of 3 between 1 and 20 }
Draw a Venn diagram to represent this information.
List:
A ∪ B A ∩ B



The Venn Diagram shows how many students in a class to some sports.

- How many swimmers are there?
- How many students swim or run?
- Using S and R to represent the sets, write the above using set notation.

The complement of a set

H

Notes and guidance

Students need to be taught that the complement of a set is the members of the universal set that are not members of the set. Matching activities where children have to match complements of sets to the pre-shaded Venn diagrams help to embed this understanding. Students could be introduced to the notation A' to represent the complement of set A.

Key vocabulary

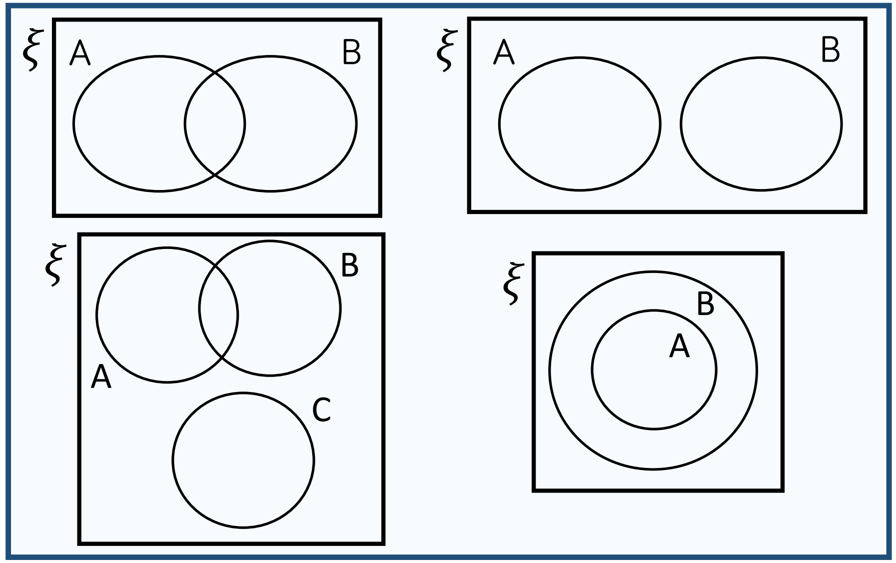
Complement	Member	Element
Universal Set	Not	

Key questions

Do all sets have a complement?
What is the relationship between the complement of a set, the set itself and the universal set?
Can a set whose elements are not numbers have a complement?

Exemplar Questions

Copy each Venn diagram. On each one shade in the area representing the complement of set A.



If ξ = { Integers between 1 and 20 }
Write down the elements in the complements of the following sets.

- A = { factors of 15 }
- B = { square numbers }
- C = { numbers in the sequence where the nth term = 2n + 1 }

ξ = { Integers between 1 and 20 }. If the complement of set X contains the elements { 1, 4, 6, 8, 10,12, 14, 15, 16, 18 }
List the elements in set X. Describe set X

Use the vocabulary of probability

Notes and guidance

Students should be encouraged to think about factors that affect the likelihood of an event happening (such as 'it will rain tomorrow') as this informs their judgements. Sometimes students assume that there is an equal chance of an event happening or not. Exposing these misconceptions by using well chosen examples is crucial (such as scoring a 3 on a die, or not scoring a 3 on a die).

Key vocabulary

Impossible	Likely	Even	Unlikely
Certain	Random	Bias	Event

Key questions

What is the difference between 'almost certain' and 'certain'? Give me examples of events that are 'certain' to happen and those that are 'almost certain'.
Give an example of an experiment with two outcomes that are equally likely. Give an example of an experiment with two outcomes that are not equally likely.

Exemplar Questions

Tommy

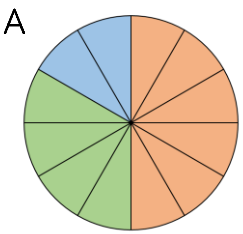
Tomorrow I can go to school, or not go to school. There is an even chance of me going to school tomorrow.

Is Tommy right? Explain your answer.

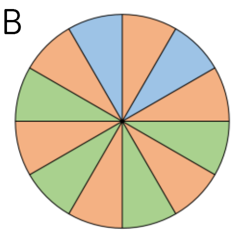
Decide whether these statements are true or false.
Discuss this with your partner. Share your ideas with the class.

- If you flip a coin 4 times, you are more likely to get HTHT than HHHH
- If you throw a dice and get a 6, you've used up some of your luck so will be less likely to get a 6 next time.
- If you flip a coin 8 times and get 8 heads then it is certain that the coin is biased.
- It's impossible that I will ever travel into space.
- If a test has only true or false questions, it's certain that I'll get at least half of the questions right.
- It's certain that I'll watch TV this week.

A



B



Spinner A and Spinner B are the same size. Which spinner is more likely to land on red? Why?

Sample spaces

Notes and guidance

Following on from systematically listing outcomes, students now explore writing exhaustive lists for a single event thus defining a sample space. They also need to recognise whether a list is a sample space or whether elements are missing. This step provides opportunities to link with the concepts of sets and set notation.

Key vocabulary

Sample Space	Possibilities	Event
Outcomes	Element	Set

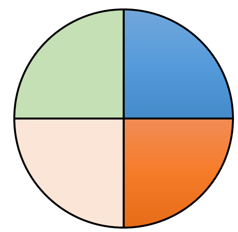
Key questions

How do you know you have a complete sample space?
How does a sample space help you to work out whether something is equally likely to happen or not?
If a sample space has just two possible outcomes, does this mean they are equally likely? If a sample space has 12 outcomes, does this mean they are equally likely?

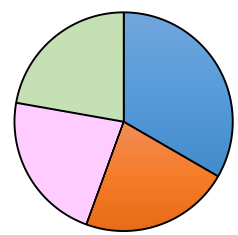
Exemplar Questions

Jack rolls a dice.
The sample space for the possible outcomes is S = { 1, 2, 3, 4, 5, 6 }
Write the sample space for the possible outcomes when:

- A coin is flipped
- An eight-sided dice is rolled
- A letter is picked at random from the word MATHEMATICS
- A letter is picked at random from the word PROBABILITY
- The total score when two six-sided dice are rolled



This spinner is divided into four equal sections.
Write down the sample space of the possible outcomes when this spinner is spun.
Do all the outcomes have the same chance of happening?



What's the same and what's different about this spinner?

Design a spinner with six sections so that,

- All the outcomes have the same chance of happening
- All the outcomes have different chances of happening

Probability of a single event

Notes and guidance

In this small step, students need to be taught how to calculate a single probability giving their answer as a fraction, decimal or percentage but not in ratio notation. Vocabulary such as 'random', 'bias' and 'equally likely' needs to be discussed. This is a good opportunity to practise converting fractions decimals and percentages. The fact that probabilities need to be between 0 and 1 needs to stressed, explicitly discussing why a probability of 60% is possible but 60 isn't.

Key vocabulary

Random	Event	Simplify
Equivalent	Equally Likely	Bias

Key questions

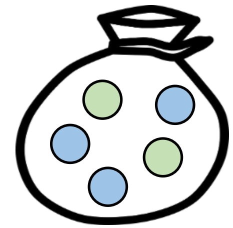
Is $\frac{25}{100}$ a larger probability than $\frac{1}{4}$? Explain your answer.

What does 'random' mean?

Is the probability of rolling a 6 on a dice always $\frac{1}{6}$? Why or why not?

Exemplar Questions

A ball is taken out of this bag at random.
Write down the probability that the ball is blue?
Decide if the probability of selecting a blue
A: stays the same B: increases C: decreases
when,



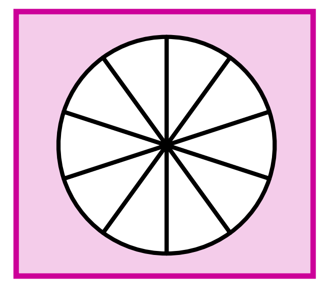
- 5 more blues and 4 more greens are added to the bag
- 6 more blues and 4 more greens are added to the bag
- 1 blue and 1 green are removed from the bag

Justify your answer each time.

Mustafa is making a game using the spinner shown below.

For his game to work, he needs the probability of the spinner landing on:

- an odd number to be $\frac{4}{5}$
- a square number to be 30%
- a number that is 25 or less to be 1



Copy and complete the spinner so that Mustafa's game works.
Make up your own game using an octagon spinner split into 8 sections.

The probability scale

Notes and guidance

Here students explore the probability scale understanding that the probability of an impossible event is 0, the probability of a certain event is 1 and that all other probabilities lie between these two extremes. This step allows students to revisit working out intervals on a number line and conversion between fractions, decimals and percentages. Pegging events on a washing line is a good way to encourage class discussion whilst learning about the probability scale.

Key vocabulary

Scale	Certain	Impossible
Likely	Random	Fair

Key questions

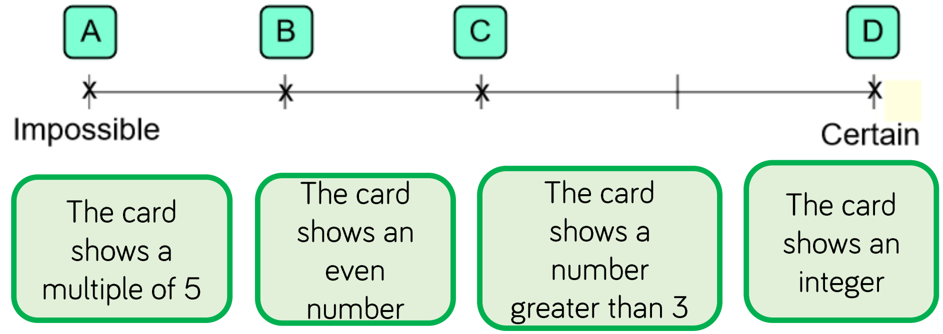
Why does the probability scale end at 1?

What is the probability of an impossible event?

If the probability of two events are marked on a probability scale, how can you tell which is the more likely?

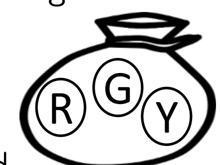
Exemplar Questions

Rosie rolls a fair four-sided spinner labelled with the numbers 1, 2, 3 and 4. Match the statements and the probabilities with the letters shown on the probability scale.

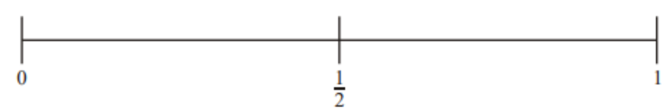


Write the missing percentages on the probability scale.

The are three beads in a bag. One bead is red, one bead is green and one bead is yellow.
Amir takes a bead at random from the bag.
On the probability line,



- mark with R the probability Amir takes a red bead
- mark with B the probability Amir takes a blue bead
- mark with N the probability Amir does not take a white bead



Sum of probabilities

Notes and guidance

This small step requires students to understand that the sum of probabilities for all possible outcomes is 1. They can then calculate unknown probabilities using this fact. Using this same fact, they should also be able to calculate the probability of an event not happening, It is important that students understand that the probability of an event that is certain is 1 Finding unknown probabilities can be linked back to forming and solving equations.

Key vocabulary

Certain	Impossible	Whole
Equivalence	Outcomes	Sum

Key questions

Can a probability be 120%? Why or why not?
Why can't a probability be less than 0?
Why do the sum of probabilities for all possible outcomes add up to 1? Why not 2? or 100?
Why are 100 and 100% different?

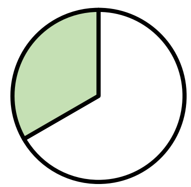
Exemplar Questions

There are some red, blue and green balls in a bag.
The probability of getting a blue ball (when taken at random) is $\frac{3}{10}$

Now think about the probability of getting a red ball and complete the following sentences:

The probability of getting a red ball
- might be……………..
- must be……………..
- cannot be……………..

The probability of a spinner landing on purple is 30%.
The only other two colours on the spinner are yellow and pink.



Lami thinks that the probability of a yellow could be 22.7% and the probability of a pink could be 47.3%
Is he right? Explain your answer.
Are there other possible pairs of answers?

Julie randomly selected chocolates from a box containing dark, milk, white and mint chocolates. There is an equal chance of selecting a white or a mint chocolate. Copy and complete the table below to show the probabilities of selecting each type of chocolate:

Dark	Milk	White	Mint
0.15	0.35		

What's the probability of Julie selecting a chocolate that isn't mint?