

Small Steps Guidance - Probability

Year 10

#MathsEveryoneCan

	Week 1	Week 2	Week 3	Week 4	Week 5	Week 6	Week 7	Week 8	Week 9	Week 10	Week 11	Week 12
Autumn	Similarity						Developing Algebra					
	Congruence, similarity and enlargement			Trigonometry			Representing solutions of equations and inequalities			Simultaneous equations		
Spring	Geometry						Proportions and Proportional Change					
	Angles & bearings		Working with circles		Vectors		Ratios & fractions		Percentages and Interest		Probability	
Summer	Delving into data						Using number					
	Collecting, representing and interpreting data						Non-calculator methods		Types of number and sequences		Indices and Roots	

Spring 2: Proportions and Proportional Change

Weeks 1 and 2: Ratios and Fractions

This block builds on KS3 work on ratio and fractions, highlighting similarities and differences and links to other areas of mathematics including both algebra and geometry. The focus is on reasoning and understanding notation to support the solution of increasing complex problems that include information presented in a variety of forms. The bar model is a key tool used to support representing and solving these problems.

National curriculum content covered:

- consolidating subject content from key stage 3:
- use ratio notation, including reduction to simplest form
- divide a given quantity into two parts in a given part : part or part : whole ratio; express the division of a quantity into two parts as a ratio
- relate the language of ratios and the associated calculations to the arithmetic of fractions and to linear functions
- use compound units such as speed, unit pricing and density to solve problems
- compare lengths, areas and volumes using ratio notation and/or scale factors; make links to similarity
- apply the concepts of congruence and similarity, including the relationships between lengths, **{areas and volumes}** in similar figures

Weeks 4 and 5: Percentages and Interest

Although percentages are not specifically mentioned in the KS4 national curriculum, they feature heavily in GCSE papers and this block builds on the understanding gained in KS3. Calculator methods are encouraged throughout and are essential for repeated percentage change/growth and decay problems. Use of financial contexts is central to this block, helping students to maintain familiarity with the vocabulary there are unlikely to use outside school.

National curriculum content covered:

- consolidating subject content from key stage 3:
- interpret these multiplicatively, express one quantity as a percentage of another, compare two quantities using percentages, and work with percentages greater than 100%
- solve problems involving percentage change, including: percentage increase, decrease and original value problems and simple interest in financial mathematics
- set up, solve and interpret the answers in growth and decay problems, including compound interest **{and work with general iterative processes}**

Weeks 5 and 6: Probability

This block also builds on KS3 and provides a good context in which to revisit fraction arithmetic and conversion between fractions, decimals and percentages. Tables and Venn diagrams are revisited and understanding and use of tree diagrams is developed at both tiers, with conditional probability being a key focus for Higher tier students.

National curriculum content covered:

- apply the property that the probabilities of an exhaustive set of mutually exclusive events sum to one
- use a probability model to predict the outcomes of future experiments; understand that empirical unbiased samples tend towards theoretical probability distributions, with increasing sample size
- calculate the probability of independent and dependent combined events, including using tree diagrams and other representations, and know the underlying assumptions
- **{calculate and interpret conditional probabilities through representation using expected frequencies with two-way tables, tree diagrams and Venn diagrams}**

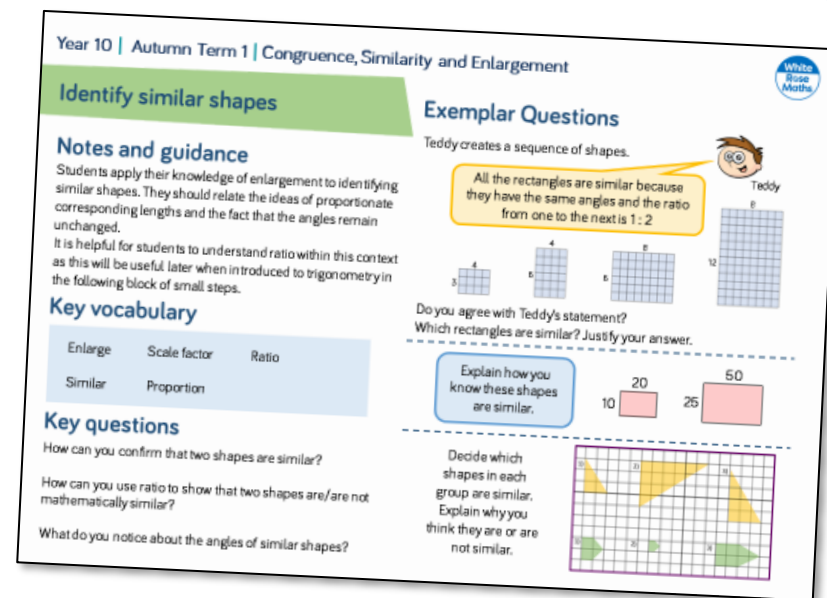
Why Small Steps?


We know that breaking the curriculum down into small manageable steps should help students to understand concepts better. Too often, we have noticed that teachers will try and cover too many concepts at once and this can lead to cognitive overload. We believe it is better to follow a “small steps” approach.

As a result, for each block of content in the scheme of learning we will provide a “small step” breakdown. ***It is not the intention that each small step should last a lesson – some will be a short step within a lesson, some will take longer than a lesson.*** We would encourage teachers to spend the appropriate amount of time on each step for their group, and to teach some of the steps alongside each other if necessary.

What We Provide

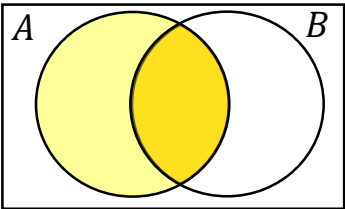
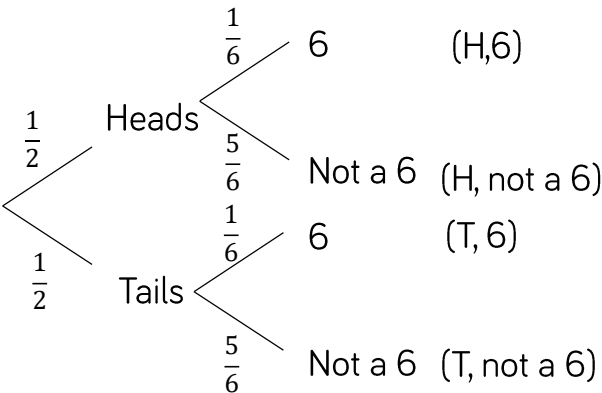
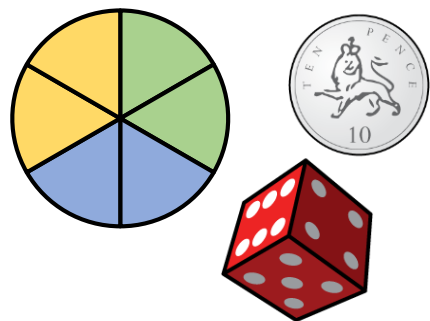
- Some **brief guidance** notes to help identify key teaching and learning points
- A list of **key vocabulary** that we would expect teachers to draw to students' attention when teaching the small step,
- A series of **key questions** to incorporate in lessons to aid mathematical thinking.
- A set of questions to help **exemplify** the small step concept that needs to be focussed on.



- These include reasoning and problem-solving questions that are fully integrated into the scheme of learning. Depending on the attainment of your students, you may wish to use some or all of these exemplars, which are in approximate order of difficulty. Particularly challenging questions are indicated with the symbol .
- For each block, we also provide ideas for key representations that will be useful for all students.

Some of the small steps are in **bold** and labelled with **H** to indicate this is Higher tier GCSE content. We would encourage teachers to use these with as many students as possible – if you feel your class can access any particular small step, then please include it in your planning. Steps that review content covered at Key Stage 3 are labelled **R**.

Key Representations



	Mustard	Chutney	Total
Ham	28	22	50
Cheese	10	49	59
Total	38	71	109

Sport	Netball	Swimming	Running	Football	Hockey
Probability	0.3	0.05		0.35	

Product	1	2	3	4
1				
2				
3				
4				
5				

Bread	Filling	Topping
Whi	Che	Chu
Bro	Che	Chu
Whi	Che	May
Bro	Che	May

Concrete resources such as coins, dice and spinners can be used to aid understanding of probability. Using a bag of coloured counters to demonstrate conditional probability is particularly effective.

Representations of sample spaces can take the form of lists, two-way tables, grids, Venn diagrams and tree diagrams. Shading activities with Venn diagrams can help to reinforce the idea of conditional probability. Tree diagrams should include those where there are 3 possible outcomes in at least one of the trials.

Students should experience probabilities in the form of fractions, decimals and percentages. At all times, concrete and pictorial models should be linked to abstract calculations, so that students understand how these relate to each other.

Probability

Small Steps

- ▶ Know how to add, subtract and multiply fractions R
- ▶ Find probabilities using equally likely outcomes R
- ▶ Use the property that probabilities sum to 1 R
- ▶ Using experimental data to estimate probabilities
- ▶ Find probabilities from tables, Venn diagrams and frequency trees
- ▶ Construct and interpret sample spaces for more than one event R
- ▶ Calculate probability with independent events
- ▶ Use tree diagrams for independent events

H Denotes Higher Tier GCSE content

R Denotes 'review step' – content should have been covered at KS3

Probability

Small Steps

- ▶ Use tree diagrams for dependent events
- ▶ Construct and interpret conditional probabilities (Tree diagrams) H
- ▶ Construct and interpret conditional probabilities (Venn diagrams and two-way tables) H

H Denotes Higher Tier GCSE content

R Denotes 'review step' – content should have been covered at KS3

Add, subtract and multiply fractions R

Notes and guidance

Students need a conceptual understanding of adding, subtracting and multiplying fractions before exploring probability. Returning to pictorial representations may be necessary. There is then an opportunity to interleave many previously taught topics such as order of operations, area of polygons, volume, similar shapes, sequences (linear and geometric) and algebra.

Key vocabulary

Numerator	Denominator	Exact value
LCM	Simplest form	

Key questions

Why do we ensure fractions have a common denominator before adding or subtracting?

How do we multiply together two fractions? Explain why this procedure works.

What is meant by 'exact value'?

Exemplar Questions

Here is part of Dexter's homework:

$$\frac{1}{5} + \frac{1}{15} = \frac{15}{75} + \frac{5}{75} = \frac{20}{75} = \frac{4}{15}$$

Write down a more efficient method to calculate $\frac{1}{5} + \frac{1}{15}$

Use the diagram to justify why $\frac{1}{2} \times \frac{3}{4} = \frac{3}{8}$

Which of the following is larger:

$$\frac{1}{2} \times \frac{3}{4}$$

$$\frac{8}{9} \times \frac{15}{32}$$

Show your working and then explain your answer.



The following is a linear sequence:

$$\frac{3}{10}, \frac{2}{5}, \frac{1}{2}, \frac{3}{5}, \boxed{}, \boxed{}, \frac{9}{10}$$

Calculate the missing terms in the sequence.

Simplify the following:

$$(x^7)^{\frac{3}{8}}$$

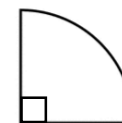
$$g^{-\frac{1}{3}} \times g^{\frac{5}{6}}$$

$$h^{\frac{3}{4}} \div h^{-\frac{1}{2}}$$

$$\frac{d^{\frac{4}{5}} \times d^{\frac{2}{3}}}{d^{\frac{1}{5}}}$$

The radius of a circle is $\frac{3}{10}$ cm.

Find the area and circumference a quadrant of this circle, leaving your answers in terms of π .



Equally likely outcomes

R

Notes and guidance

This step supports students to become conceptually fluent in using equally likely outcomes to find probabilities.

Misconceptions should be highlighted here, particularly considering factors such as 'size' of spinner and whether this impacts on probability of outcomes. Reminding students that they can write probability as a fraction, decimal or percentage is also useful.

Key vocabulary

Equally likely	Outcome	Event
Denominator	Numerator	

Key questions

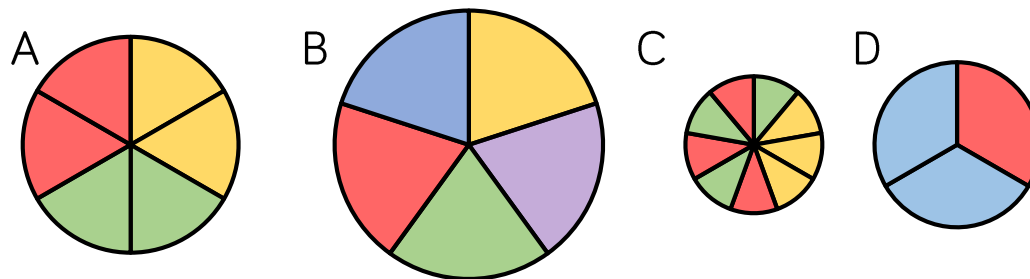
What makes events equally likely to occur?

If it (e.g.) might rain, or might not, are these events equally likely?

How can we use the fact that events are equally likely to find probabilities?

Exemplar Questions

Which of these spinners have equally likely outcomes?



Which spinners have an equal chance of landing on any of the colours?

What's the probability of landing on red when using

◆ Spinner B? ◆ Spinner C?

Tickets with the numbers 1 to 9 are put into a bag. One ticket is randomly selected from the bag.

◆ Write down the probability that the number on the ticket is even.

Decide if the probability of selecting an even number

A: Stays the same

B: Increases

C: Decreases

when,

◆ 5 more even and 4 more odd numbers are added to the bag

◆ 8 more even and 10 more odd numbers are added to the bag

◆ 1 odd and 1 even number are removed from the bag

Justify your answers.

Probabilities sum to 1

R

Notes and guidance

Students use the fact that probabilities sum to 1 (or 100%) to calculate missing probabilities. Students should have opportunities to work with percentages, fractions and decimals when finding probabilities. This step is also an opportunity to revisit Venn diagrams, set notation and forming/solving linear equations

Key vocabulary

Event	Complement	Venn diagram
Intersect	Union	

Key questions

What types of number can we use to represent probabilities? Can we use a ratio? Why or why not?

How do we know that for these events probabilities must add up to 1? Why can't they add up to more/less than 1?

What does the word 'complement/union/intersect' mean? Where is this represented on the Venn diagram?

Exemplar Questions

Only one of these statements is true.

Which one is it? Explain why the other statements are false.

It's a million percent certain that I'll sleep this week

In a bag, there are only red, blue and yellow marbles. The probability of getting a red is 0.15, a blue is 0.45, and a yellow is 0.35

In a bag of chocolates, 2 are dark and 8 are milk. The probability of getting a dark chocolate is $\frac{1}{5}$, and a milk chocolate is $\frac{4}{5}$.

A large number of people are asked their favourite of five sports. The table shows the probabilities that certain sports are picked. Given that it is twice as likely that someone chose hockey rather than running, complete the table.

Sport	Netball	Swimming	Running	Football	Hockey
Probability	0.3	0.05		0.35	

Red, blue, yellow and green balls are in a bag in the ratio 7 : 2 : 1 : x . The probability of selecting a red ball is 50%.

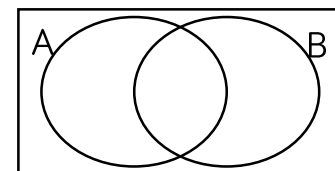
Find the value of x

If there are 42 balls in the bag, how many of each colour are there?

$$P(A \cup B) = 0.6$$

Find the probability that neither A nor B occur.

Given that $P(A) = 0.3$ and $P(A \cap B) = 0.2$, complete the Venn diagram.



Experimental data

Notes and guidance

Students consider why experimental and theoretical probability are different. They learn that the more trials completed, the closer experimental probability is likely to be to the theoretical probability. They consider how they can use relative frequency to predict future events, by calculating expected values. Students could be supported to find experimental probabilities from a variety of sources.

Key vocabulary

Relative frequency

Estimate

Expectation

Expected value

Key questions

Why is experimental probability different from theoretical probability?

What happens to experimental and theoretical probability when a very large number of trials have been completed?

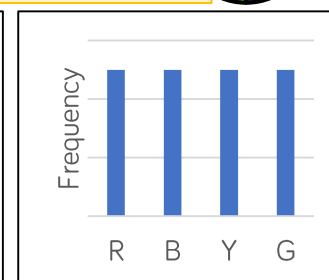
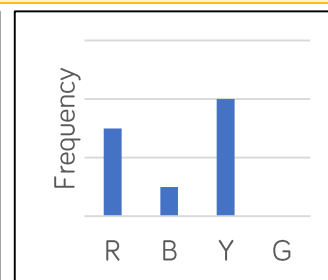
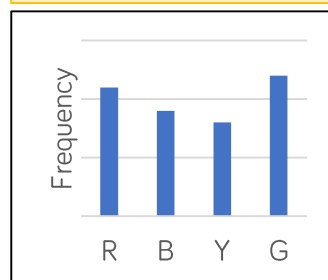
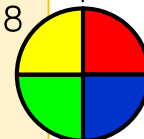
Is the 'expected value' the exact number of times you would expect an event to occur?

Exemplar Questions

Mo uses a computer simulation of a spinner.

The bar charts show the frequency of each colour after 8 spins, 40 spins and 10 000 spins. Match each graph to the number of spins, explaining your reasoning.

Spinner



Rosie and Eva are flipping a coin.

Rosie says:

My coin must be biased

Explain why she is incorrect.

Teddy estimates the probability of getting a head.

$$P(\text{Head}) = 0.3$$

$$P(\text{Head}) = 53\%$$

$$P(\text{Head}) = \frac{56}{110}$$

Explain how he got each estimate. Which is most accurate? Why? How many times is a head expected if the coin is flipped 220 times?

	Heads	Tails	Total
Rosie	3	7	10
Eva	53	47	100
Total	56	54	110

Dora notes down how many times a netball team wins, loses and draws over 60 games:

Win	Lose	Draw
31	x	y

If the expected number of losses in 900 games is 330, find x and y .

Tables, Venn diagrams, frequency trees

Notes and guidance

This is an opportunity for students to revise key ways of representing information. When working with Venn diagrams, students might need reminding that $P(A)$ includes $P(A \cap B)$. Higher students should also consider more advanced notation such as $P(A \cap B')$. When working with two-way tables, students might need support in choosing the correct cell value for the denominator.

Key vocabulary

Two-way tables

Venn diagrams

Frequency trees

Universal set

Key questions

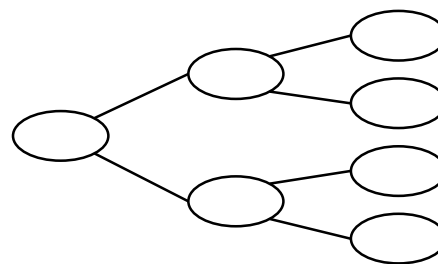
How do we know which cell value is the denominator when calculating a probability from a two-way table?

Where do you start when filling in a Venn diagram?

What's the same and what's different about frequency trees and two-way tables?

Exemplar Questions

A local council believe that girls are more likely to walk to school than boys. They conduct a survey of 800 children to find out.

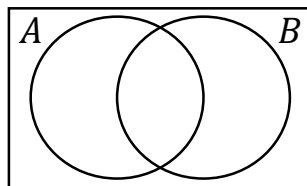


48% of the children are girls

Girls 'walk to school' or 'don't walk' to school in the ratio 7 : 5

104 boys 'don't walk' to school. The rest of the boys 'walk to school'.

Copy and complete the frequency tree. Is the council right that girls are more likely to walk to school than boys? Justify your answer.



$\varepsilon = \{\text{integers from 1 to 12}\}$

$A = \{\text{even numbers}\}$ $B = \{\text{multiples of 3}\}$

Complete the Venn diagram and then find:

a) $P(A)$ b) $P(B)$ c) $P(A')$

d) $P(B')$ e) $P(A \cup B)$ f) $P(A \cap B)$



If 60 is added to the universal set, how does this affect your answers?

The table shows sandwich choices made a group of children.

Find:

	Mustard	Chutney	Total
Ham	28	22	50
Cheese	10	49	59
Total	38	71	109

a) $P(\text{Cheese and chutney})$

b) $P(\text{Chutney})$

c) $P(\text{Ham})$

Sample spaces



Notes and guidance

Students construct sample spaces from a variety of sources so that they know whether a list, or grid is most efficient. Discuss how to be systematic and the different ways of being systematic. A misconception is to add the total number of possible outcomes from each event and use this as the denominator when calculating probabilities (e.g. thinking the total number of possible outcomes when rolling two dice must be 12).

Key vocabulary

Sample Space	Systematic	Outcome
Event	Array	

Key questions

- How can we present the outcomes?
- What does 'systematic' mean?
- How can we be systematic? Is there more than one way?
- How many outcomes will there be in total? How do we know?

Exemplar Questions

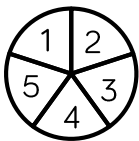
BREAD: White Brown	FILLING: Cheese Ham Chicken	TOPPING: Chutney Mayo Mustard
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A sandwich shop owner needs to work out how many different choices his customers have.

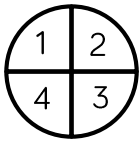
Bread	Filling	Topping
Whi	Che	Chu
Bro	Che	Chu
Whi	Che	May
Bro	Che	May

He systematically begins to write a list. Explain how he is being systematic. What is the benefit of this?
Complete his list.

In a game, the outcome is the product of the score on Spinners A and B.



Spinner A



Spinner B

Complete the sample space

Product	1	2	3	4
1				
2		4	6	
3				
4	4			
5				20

- Show that $P(\text{product is an even number}) \neq 0.5$
- Find $P(\text{product is a prime number})$.
- How many times would you expect a prime number in 100 spins?



There are 6 possible outcomes on each dice, and that makes 12 overall. Therefore $P(\text{total score is 12}) = \frac{1}{12}$

- Use a sample space to show that Whitney is wrong.
- What is the probability of getting a total score of 12?



Independent events

Notes and guidance

Before working with tree diagrams, students need to understand that for independent events, $P(A \text{ and } B) = P(A) \times P(B)$. They also need to be clear that the outcome of one event has no bearing on the outcome of the other. This can be demonstrated using sample spaces. Examples and non-examples of independent events supports understanding of this term.

Key vocabulary

Independent events	Tree diagrams
Outcomes	Product

Key questions

Give an example of a pair of independent events. Give an example of a pair of events that aren't independent.

Do you add or multiply to find the probability of two independent events both happening?

Exemplar Questions

Which of the following events are independent?

Rolling a 6 on a dice and then rolling another 6

Rolling a 6 on a dice and getting a head when flipping a coin

Rolling a 6 on a dice and it raining tomorrow

Selecting a red sweet at random from a bag of sweets, eating it, and then selecting another red sweet.



Getting a tail on a coin happens half the time. So, $P(\text{Tail and a 1 on a dice}) = \text{half of } \frac{1}{6}$
 $P(\text{Tail and a 1 on a dice}) = \frac{1}{2} \times \frac{1}{6} = \frac{1}{12}$

Use a sample space to convince your partner that this is true.

Complete the statement by adding in the correct operation:

For independent events, $P(A \text{ and } B) = P(A) \bigcirc P(B)$

The probability that Dora is late for school is 0.1

The probability that Ron is late for school is 0.2

The probability that Eva is late for school is 0.15

The events that any of the students are late are independent.

Find the probability that

- Dora and Ron are both late for school
- Ron and Eva are both late for school
- All three students are late for school

Tree diagrams for independent events

Notes and guidance

Sample spaces alongside the tree diagram can provide a helpful transitional step. Initially scaffolding by providing students with the tree diagrams to enable all to access this concept. Teachers might include tree diagrams where there are more than two outcomes in each trial. Students may need support in identifying 'pathways' and what final outcome each shows.

Key vocabulary

Independent events Tree diagrams

Outcomes At least one

Key questions

Explain why getting (e.g. a red and a blue) means getting (e.g. colour of the counters) in any order.

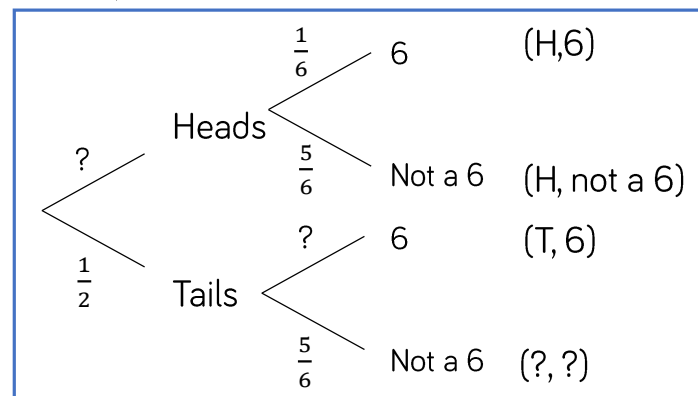
What are the different methods for finding the probability of 'at least one'? Which is the most efficient?

How do you draw a tree diagram if the first event has three possible outcomes and the second event has two?

Exemplar Questions

Mo flips a coin and rolls a dice. He is working out $P(\text{Head and a 6})$. The tree diagram shows possible outcomes.

Complete the tree diagram.



Work out the probabilities of all four possible outcomes.

A bag contains 3 blue and 2 red counters. A counter is randomly selected and replaced. A second counter is then randomly selected. Draw a tree diagram to show this.

Alex and Tommy calculate $P(\text{red and a blue counter})$

$$P(R \text{ and } B) = \frac{2}{5} \times \frac{3}{5} = \frac{6}{25}$$



Alex

Tommy



$$P(R \text{ and } B) \text{ or } P(B \text{ and } R) = \left(\frac{2}{5} \times \frac{3}{5}\right) + \left(\frac{3}{5} \times \frac{2}{5}\right) = \frac{6}{25} + \frac{6}{25} = \frac{12}{25}$$

Tommy is correct. What mistake has Alex made?

Alex and Tommy now calculate $P(\text{at least one red})$

Will they both get the same answer?



$$P(\text{at least one red}) = 1 - P(\text{no reds}) = 1 - P(B,B)$$

$$P(\text{at least one red}) = P(R,R) + P(R,B) + P(B,R)$$



Tree diagrams for dependent events

Notes and guidance

Prior to this small step, it is useful to generate examples of dependent events with students, to ensure that they understand what these are. Again, scaffolding by providing incomplete information on a tree diagram or in a method provides a starting point. Working with probability in percentages, decimals and fractions and then discussing which is easier to calculate with can also be helpful.

Key vocabulary

Dependent events

Independent events

Tree diagram

Key questions

Give me an example of two events which are dependent.

Is it easier to calculate mentally with decimals, fractions or percentages?

How do you know which “branch” or “branches” to use in a tree diagram?

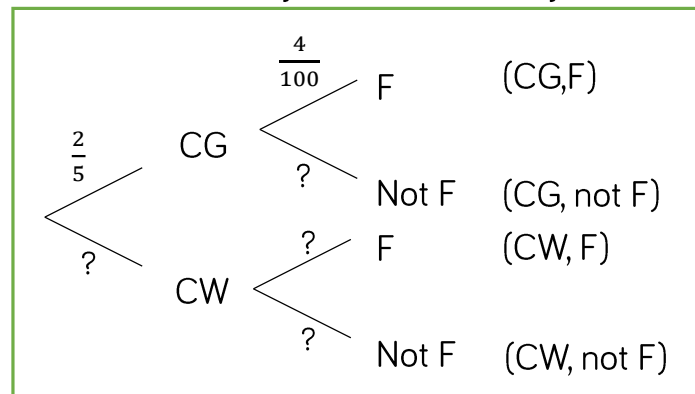
Exemplar Questions

A garage orders two-fifths of its car parts from ‘Car Giant’ (CG) and the remainder from ‘Car World’ (CW).

4% of the parts from ‘Car Giant’ are faulty and 3% are faulty from ‘Car World’.

Complete the tree diagram:

Write down the question that each of these calculations answer.



$$\left(\frac{2}{5} \times \frac{4}{100}\right) + \left(\frac{3}{5} \times \frac{3}{100}\right)$$

$$1 - \left\{\left(\frac{2}{5} \times \frac{4}{100}\right) + \left(\frac{3}{5} \times \frac{3}{100}\right)\right\}$$

In order to test the best revision technique in a group of students:

- 40% are randomly selected to revise on their own (O)
- 30% to revise in a group (G)
- 30% to revise with a teacher (T)

All students then took the same test, which they could either pass (P) or fail (F). The table shows the results:

	O	G	T
Pass	0.7	0.5	0.8

Construct a tree diagram to show all possible outcomes.

Teddy estimates $P(\text{Fail test}) \approx \frac{1}{3}$. Is he right? Justify your answer.

Conditional (Tree diagrams)

H

Notes and guidance

Teachers should introduce this using concrete or pictorial resources, so that students can see how the probabilities in the second trial are affected by the first. Students will practise constructing tree diagrams and assigning probabilities. This should include situations where there are more than two possible outcomes for each trial. Students should also practise expressing probabilities algebraically, and manipulating these.

Key vocabulary

Conditional probability	Given
Show	Outcomes

Key questions

What does 'given' mean? Which part of the tree diagram, does this refer to?

Why do the probabilities change between trials? How do they change?

How can you work out the probabilities associated with the second trial if algebra is involved?

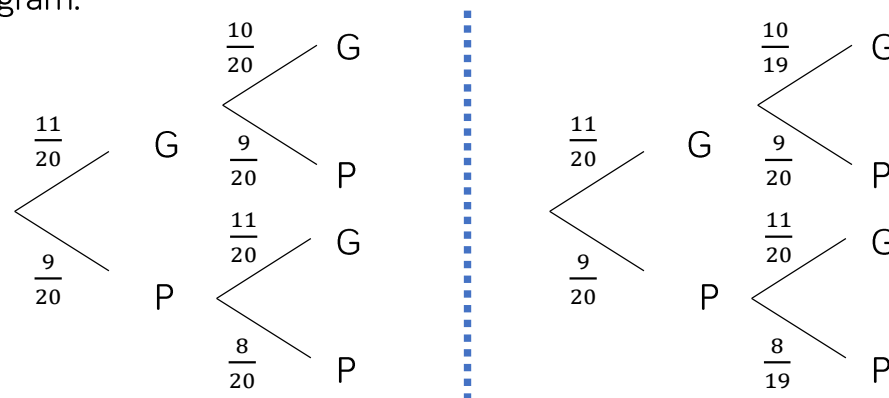
Exemplar Questions

There are 20 sweets in a bag.

11 of the sweets are green. The rest are purple.



Jack randomly takes a sweet and eats it. He repeats this a second time. Both tree diagrams contain errors. Identify these and draw a correct tree diagram.



There are n socks in a drawer.

5 of the socks are red. The rest of the socks are black.

Dexter takes a sock at random from the drawer and puts it on his foot.

He then takes at random another sock from the drawer and puts it on his other foot.

The probability he is wearing two red socks is $\frac{1}{12}$

- Show that $n^2 - n - 240 = 0$
- Solve $n^2 - n - 240 = 0$ to find the value of n
- Calculate the probability that Dexter wears a matching pair of socks.

Conditional (Other)

H

Notes and guidance

The key concept is understanding that the term ‘given’ means that only one set of outcomes are relevant when selecting the event. Students need to identify which set of outcomes they are selecting from. They should be confident in using Venn diagrams and two-way tables to find conditional probabilities. Conditional probability applies to both dependent and independent events.

Key vocabulary

Conditional probability	Given	Intersection
Outcomes	Set	Venn

Key questions

Why do we use the term conditional probability? (e.g. on the condition that...)

What does ‘given’ mean? Which part of the Venn diagram/two-way table does this refer to?

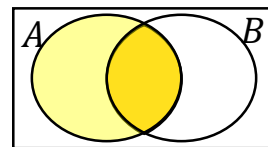
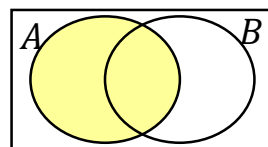
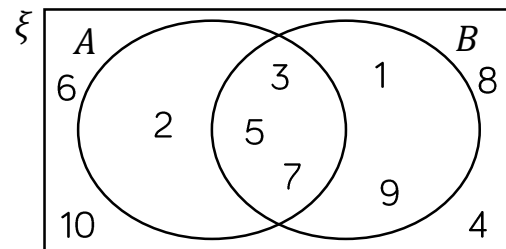
Exemplar Questions

The Venn diagram shows:

$\xi = \{\text{integers from 1 to 10}\}$

$A = \{\text{prime numbers}\}$

$B = \{\text{odd numbers}\}$



How do the two Venn diagrams help to show the probability of a number being odd, given it's prime? Calculate this probability.

Use a similar approach to show that:

The probability of a number being prime, given that it's odd is $\frac{3}{5}$

60 children get to choose a school trip.

The ratio of boys to girls is 2 : 3

$\frac{7}{12}$ of the children pick a city trip, the rest pick a seaside trip.

19 boys picked the city trip.

Construct a two-way table to show this information.

Copy and complete:

P(given the child is a ____, they chose a ____ trip) = $\frac{19}{24}$

P(given the child chose a ____ trip, they are a ____) = $\frac{16}{35}$

Work out

P(the child is a boy, given they chose a seaside trip)

P(the child chose a city trip, given they are a girl)