Show that...

Year 11

#MathsEveryoneCan

White Rose Maths

	Week 1	Week 2	Week 3	Week 4	Week 5	Week 6	Week 7	Week 8	Week 9	Week 10	Week 11	Week 12
Autumn	**Graphs**						**Algebra**					
	Gradients & lines		Non-linear graphs		Using graphs		Expanding & Factorising		Changing the subject		Functions	
Spring	**Reasoning**						**Revision and Communication**					
	Multiplicative		Geometric		Algebraic		Transforming & Constructing		Listing & describing		Show that...	
Summer	**Revision**						**Examinations**					

Spring 2 : Revision & Communication

Weeks 1 and 2: Transforming & Constructing

Students revise and extend their learning from Key Stage 3, exploring all the transformations and constructions, relating these to symmetry and properties of shapes when appropriate. There is an emphasis on describing as well as performing transformations as using the language promotes deeper thinking and understanding. Higher tier students extend their learning to explore the idea of invariance and look at trigonometric graphs as a vehicle for exploring graph transformations.

National Curriculum content covered includes:

- describe translations as 2D vectors
- reason deductively in geometry, number and algebra, including using geometrical constructions
- interpret and use fractional **{and negative}** scale factors for enlargements
- **{describe the changes and invariance achieved by combinations of rotations, reflections and translations}**
- recognise, sketch and interpret graphs of **{the trigonometric functions (with arguments in degrees) for angles of any size}**
- **{sketch translations and reflections of the graph of a given function}**

Weeks 3 and 4: Listing & Describing

This block is another vehicle for revision as the examinations draw closer. Students look at organisation information, with Higher tier students extending this to include the product rule for counting. Links are made to probability and other aspects of Data Handling such as describing and comparing distributions and scatter diagrams. Plans and elevations are also revised. You can adapt the exact content to suit the needs of your class.

National Curriculum content covered includes:

- explore what can and cannot be inferred in statistical and probabilistic settings, and express their arguments formally
- calculate the probability of independent and dependent combined events, including using tree diagrams and other representations, and know the underlying assumptions
- **{calculate and interpret conditional probabilities through representation using expected frequencies with two-way tables, tree diagrams and Venn diagrams}**
- apply systematic listing strategies, **{including use of the product rule for counting}**
- construct and interpret plans and elevations of 3D shapes

Weeks 5 and 6: Show that

This is another block designed to be adapted to suit the needs of your class. Examples of communication in various areas of mathematics are provided in order to highlight gaps in knowledge that need addressing in the run up to the examinations. “Show that” is used to encourage students to communicate in a clear mathematical fashion, and this skill should be transferred to their writing of solutions to any type of question.

National Curriculum content covered includes:

- know the difference between an equation and an identity; argue mathematically to show algebraic expressions are equivalent, and use algebra to support and construct arguments **{and proofs}**
- apply the concepts of congruence and similarity
- make and use connections between different parts of mathematics to solve problems
- **{change recurring decimals into their corresponding fractions and vice versa}**
- apply addition and subtraction of vectors, multiplication of vectors by a scalar, and diagrammatic and column representations of vectors; **{use vectors to construct geometric arguments and proofs}**

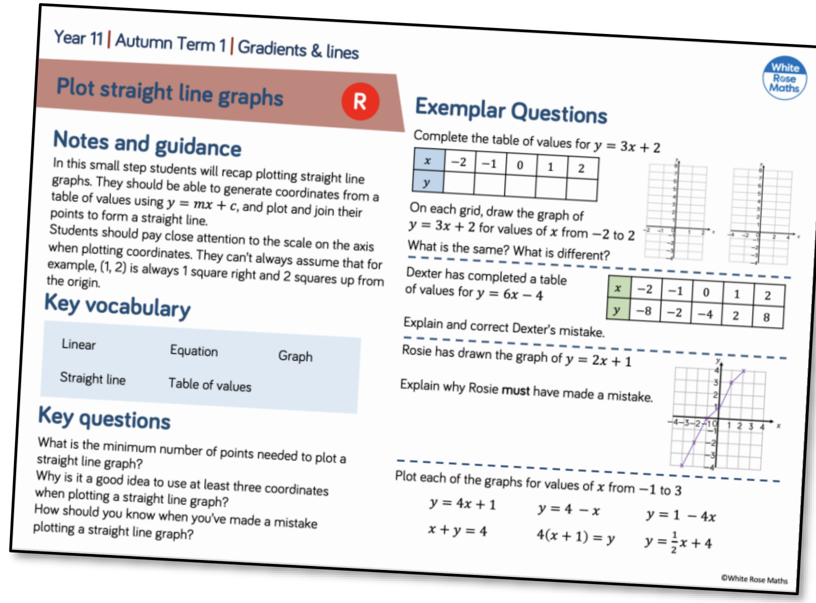
Why Small Steps?

We know that breaking the curriculum down into small manageable steps should help students to understand concepts better. Too often, we have noticed that teachers will try and cover too many concepts at once and this can lead to cognitive overload. We believe it is better to follow a “small steps” approach.

As a result, for each block of content in the scheme of learning we will provide a “small step” breakdown. ***It is not the intention that each small step should last a lesson – some will be a short step within a lesson, some will take longer than a lesson.*** We would encourage teachers to spend the appropriate amount of time on each step for their group, and to teach some of the steps alongside each other if necessary.

What We Provide

- Some ***brief guidance*** notes to help identify key teaching and learning points.
- A list of ***key vocabulary*** that we would expect teachers to draw to students’ attention when teaching the small step.
- A series of ***key questions*** to incorporate in lessons to aid mathematical thinking.
- A set of questions to help ***exemplify*** the small step concept that needs to be focussed on.



Year 11 | Autumn Term 1 | Gradients & lines

Plot straight line graphs R

Notes and guidance

In this small step students will recap plotting straight line graphs. They should be able to generate coordinates from a table of values using $y = mx + c$, and plot and join their points to form a straight line.
Students should pay close attention to the scale on the axis when plotting coordinates. They can't always assume that for example, (1, 2) is always 1 square right and 2 squares up from the origin.

Key vocabulary

Linear, Equation, Graph, Straight line, Table of values

Key questions

What is the minimum number of points needed to plot a straight line graph?
Why is it a good idea to use at least three coordinates when plotting a straight line graph?
How should you know when you've made a mistake plotting a straight line graph?

Exemplar Questions

Complete the table of values for $y = 3x + 2$

x	-2	-1	0	1	2
y					

On each grid, draw the graph of $y = 3x + 2$ for values of x from -2 to 2
What is the same? What is different?

Dexter has completed a table of values for $y = 6x - 4$

x	-2	-1	0	1	2
y	-8	-2	-4	2	8

Explain and correct Dexter's mistake.

Rosie has drawn the graph of $y = 2x + 1$

Explain why Rosie **must** have made a mistake.

Plot each of the graphs for values of x from -1 to 3

$y = 4x + 1$ $y = 4 - x$ $y = 1 - 4x$

$x + y = 4$ $4(x + 1) = y$ $y = \frac{1}{2}x + 4$

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- These include reasoning and problem-solving questions that are fully integrated into the scheme of learning. Depending on the attainment of your students, you may wish to use some or all of these exemplars, which are in approximate order of difficulty. Particularly challenging questions are indicated with the symbol .
- For each block, we also provide ideas for key representations that will be useful for all students.

Some of the small steps are in **bold** and labelled with H to indicate this is Higher tier GCSE content. We would encourage teachers to use these with as many students as possible – if you feel your class can access any particular small step, then please include it in your planning. Steps that review content covered at Key Stage 3 are labelled R .

Show that

Small Steps

- "Show that" with number
- "Show that" with algebra
- "Show that" with shape
- "Show that" with angles
- "Show that" with data
- **"Show that" with vectors** H
- "Show that" with congruent triangles
- **Formal proof with congruent triangles** H

H denotes Higher Tier GCSE content

R denotes 'review step' – content should have been covered at KS3

"Show that" with number

Notes and guidance

As well as developing students' reasoning skills, this step provides an opportunity for students to revise arithmetical techniques. The suggestions provided can be adapted as suits your class to practice any aspect of number work, including fractions and directed number, the equivalence of fractions, decimals and percentages and, for Higher tier students, fluency with converting recurring decimals and surds.

Key vocabulary

Equivalent	Sum	Product
Difference	Simplest form	Surd

Key questions

How can you determine whether two fractions are equivalent?

How do you find the sum/difference/product/quotient of a pair of fractions? What models could you use to help you?

How can you find a percentage of a number without a calculator?

How can you show that a number isn't prime?

Exemplar Questions

Amir says, "$\frac{5}{6}$ is greater than $\frac{3}{4}$"

Show that Amir is correct

- by drawing a diagram.
- by converting both numbers to decimals.
- by converting both numbers to fractions.

Can you find any other ways to show that Amir is correct?

Show that

- $\frac{330}{440}$ is equivalent to $\frac{450}{600}$
- $\frac{2}{5}$ of $60 = \frac{4}{5}$ of 30
- 60% of $80 = \frac{4}{7}$ of 84
- $\frac{7}{12}$ of $600 > \frac{9}{20}$ of 600

Can you show any of these results without performing calculations?

Show that the product of 0.8 and 5 is less than the sum of 0.8 and 5

Show that the product of $5\frac{3}{4}$ and $\frac{7}{8}$ is greater than the difference between $5\frac{3}{4}$ and $\frac{7}{8}$

Determine which of the statements on the cards are true.

$0.\dot{4}\dot{5} = \frac{5}{11}$	$0.\dot{7} - 0.2 = \frac{26}{45}$	$3\sqrt{20} = 2\sqrt{45}$

"Show that" with algebra

Notes and guidance

This step can be used to revise solving linear equations and inequalities, sequences, substitution, expanding brackets and factorisation as appropriate. You could also link to the equation of a straight line or identifying lines and curves. Higher tier students can build on the fraction calculations in the last step to manipulate algebraic fractions. You could also revisit completing the square and identifying turning points.

Key vocabulary

Term	Expression	Equation
Identity	Solve/solution	Sequence

Key questions

What does the symbol ≡ mean?
How can you show that two straight lines are parallel?
What's the same and what's different about solving an equation and an inequality?
How do you use a rule to find a term in a sequence?
How do you factorise a quadratic expression? Is this the same as or different from completing the square?

Exemplar Questions

Show that $x = 4$ is a solution to all of the equations on the cards.

$8x - 3 = 29$	$2 = 10 - 2x$	$\frac{3x}{2} = 6$	$\frac{x + 10}{2} = 7$

Here are the equations of three straight lines.

$y = 2x + 5$	$y + 2x = 5$	$10 = 4x - 2y$

Show that
- Two of the lines have the same gradient.
- Two of the lines meet the y-axis at the same point.

Show that
- $4(3x + 5) \equiv 3(2x + 8) + 2(3x - 2)$
- $(y + 5)^2 \equiv y^2 + 10y + 25$
- $(p + 3)(p - 3) \equiv p^2 - 9$
- $(2k^2)^3 \equiv 8k^6$

The nth term of sequence A is given by $7n - 2$
The nth term of sequence B is given by $2n^2 + 3n - 1$
Show that the fourth term of sequence A is equal to the third term of sequence B.

Show that
- $\frac{2}{x+1} - \frac{3}{(x+2)} \equiv \frac{1-x}{x^2+3+2}$
- $\frac{5x+10}{3x^2+7x+3} \equiv \frac{5}{x+2}$

"Show that" with shape

Notes and guidance

Here students have the opportunity to revise finding areas and perimeters of rectilinear and other shapes. Revisiting Pythagoras' theorem and similarity are also included. Showing shapes are similar using angles rules is also incorporated in the next step and congruence is dealt with separately. You could also build in forming and solving equations, including quadratic equations if these need more practice.

Key vocabulary

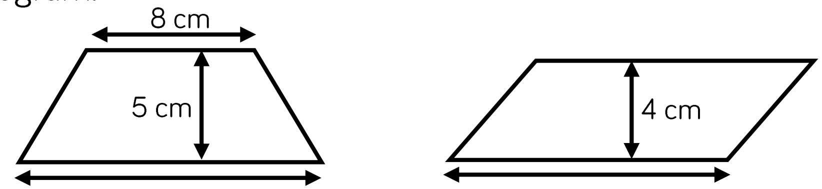
Trapezium	Parallelogram	Similar
Corresponding	Circumference	Area

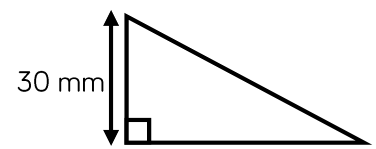
Key questions

What area formulae to you know? What does each letter represent?
How do you work backwards to find the height if given an area? Is it the same or different if there is a fraction in the area formula?
What do you know about the corresponding sides of a pair of similar shapes? What about three similar shapes?

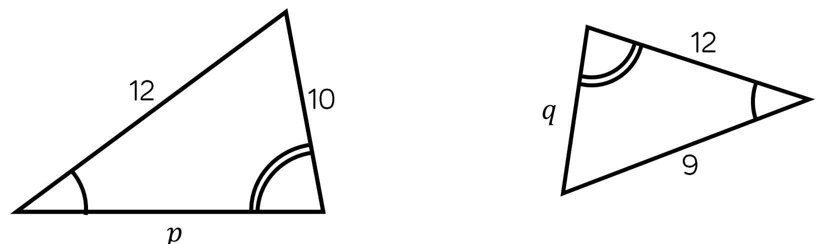
Exemplar Questions

Show that the area of the trapezium is greater than the area of the parallelogram.





The area of the triangle is 600 mm^2.
Show that the perimeter of the triangle is 120 mm.



The triangles are similar.
Show that $p = 16$ and work out the value of q.

Show that the area of a circle of radius x cm is smaller than the area of a square of side $2x$ cm.

Show that the formula for the area of a trapezium can be used to find the area of a parallelogram, a rectangle and a square.

"Show that" with angles

Notes and guidance

Students should be familiar with basic angles rules (parallel lines, isosceles triangles etc.) from earlier years, but may need reminding of the precise wording and how to 'give reasons for your answer.' Model and encourage clear detailed solutions. You could also use this step to revisit trigonometrical ratios, including the exact values that students need to know. Higher students can also revisit circle theorems here.

Key vocabulary

Alternate	Corresponding	Co-interior
Adjacent	Circle Theorems	Trigonometry

Key questions

What's the difference between corresponding and corresponding-interior angles?

What sides in a triangles do you need to know to find an angle's cosine? What do you know about the sides if the cosine of one of the angles is $\frac{1}{2}$?

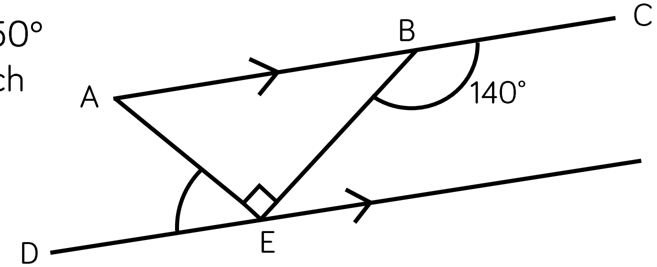
How many circle theorems do you know?

Why are isosceles triangles important?

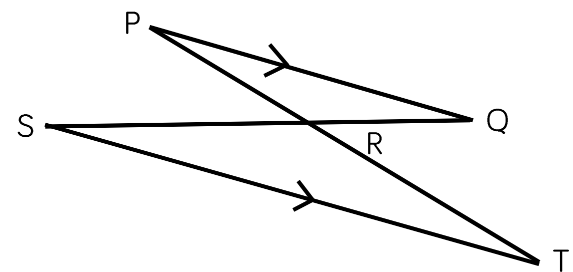
Exemplar Questions

Show that ∠AED = 50°

Give a reason for each step of your working.



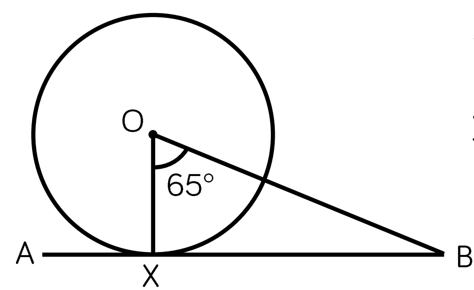
Show that triangles PQR and RST are similar.



- The angles in a triangle are in the ratio 3 : 4 : 5
  Show that all the angles in the triangle are acute.
- The angles in a triangle are in the ratio 1 : 2 : 3
  Show that the triangle is right-angled.

Show that ∠OBX = 25°

Give a reason for each step of your working.



"Show that" with data

Notes and guidance

As with the previous steps, feel free to adapt this step to suit the needs of your class. Students need to be comfortable with the vocabulary surrounding data and in interpreting as well as constructing charts and calculating measures. Probability is also included within this step and you can extend by asking "show that" questions involving more than one event through tree and/or Venn diagrams.

Key vocabulary

Mean	Median	Mode
Range	Quartile	Interquartile range

Key questions

What is the purpose of averages? How do they help us to compare data sets? What other measures can you use?
Which averages are useful in different situations? Why?
How do you construct a frequency polygon? How is this different from a cumulative frequency polygon?
Why is a probability of −0.2/1.03 impossible?
How do you find the probability of combined events?

Exemplar Questions

Here are two sets of data.

Set A	6	12	12	9	11	
Set B	8	14	7	18	9	4

Rosie says the median of the numbers in Set A is 12

- Show that Rosie is wrong.
- Show that the means of both sets of data are equal.
- Show that the range of Set B is greater than that of set A.

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12 adults and 15 children took a spelling test.
The mean mark scored by the adults was 80%
The mean mark scored by the children was 90%
Show that the mean mark scored by everyone who took the test is not 85%

The table shows the probabilities that a die will land on each number.

Score	1	2	3	4	5	6
Probability	0.3	0.2	0.1	0.2	x	$3x$

Show that the probability that the die lands on a prime number is 0.35

A set of numbers have lower quartile 47, upper quartile 79
The difference between the median and the upper quartile is three times the difference between the median and the lower quartile.
Show that the median of the set of numbers is 55

"Show that" with vectors H

Notes and guidance

Foundation tier students need to be able to use vectors to perform and describe translations. You could include other transformations here. Students should also find sums and differences of vectors, and the product of a scalar and a vector, understanding when two vectors are parallel. In addition, Higher tier students need to able to construct geometric arguments e.g. proving a set of points are collinear.

Key vocabulary

Vector	Component	Parallel
Translate	Transform	Collinear

Key questions

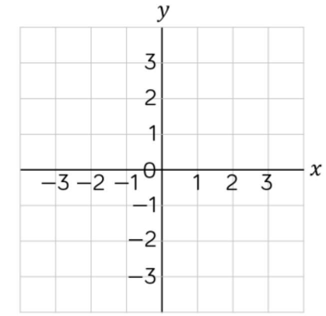
What is a vector?

Can a translation always, sometimes or never be described as a vector? Can a translation always, sometimes or never be described using other transformations?

How do you know when two vectors are parallel when they are written in component form? Can you tell when two vectors are parallel when they are written using letters?

Exemplar Questions

Show that the result of translating the point (1, 1) by the vector $\begin{pmatrix}1\\2\end{pmatrix}$ is the same as the result of translating the point (–1, –1) by the vector $\begin{pmatrix}3\\4\end{pmatrix}$

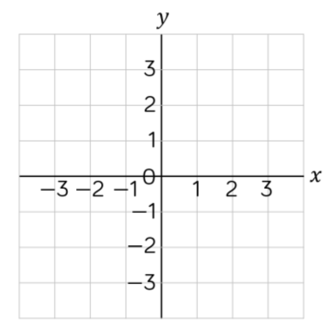


$\boldsymbol{a} = \begin{pmatrix}4\\3\end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix}2\\-1\end{pmatrix}$

- Show that $\boldsymbol{a} + \boldsymbol{b}$ is parallel to $\begin{pmatrix}3\\1\end{pmatrix}$
- Show by calculation that $3\boldsymbol{a} - 2\boldsymbol{b} = \begin{pmatrix}8\\11\end{pmatrix}$

Show that the transformation of shape A to shape B can be described as

- a single translation.
- a single reflection.
- Can the transformation of shape A to shape B be described as a single reflection?



In triangle ABC, X is the midpoint of AC and Y is the point on BC such that BY : YC = 3 : 2

Show that XY is not parallel to AB.

“Show that” with congruent triangles

Notes and guidance

This is the first of two steps on congruent triangles. Concentrate on examples where numerical values are given in this step. Students may need reminding of the four sets of conditions for congruency and the last exemplar illustrates that sometimes more than one option is available. Students needs to take particular care with the difference between AAS and SAS and what is meant by “corresponding side” in the former.

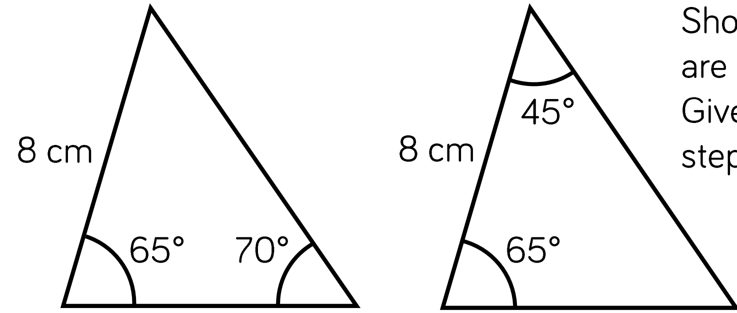
Key vocabulary

Congruent	Condition	Similar
Corresponding	Prove	

Key questions

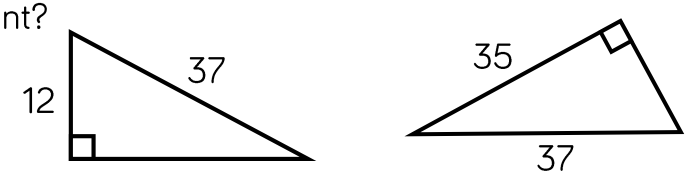
What are the four sets of conditions that show a pair of triangles are congruent? What does each letter stand for in the abbreviated forms?
What is meant by an ‘included’ angle?
What is meant by a ‘corresponding side’? How is this different from corresponding angles?

Exemplar Questions



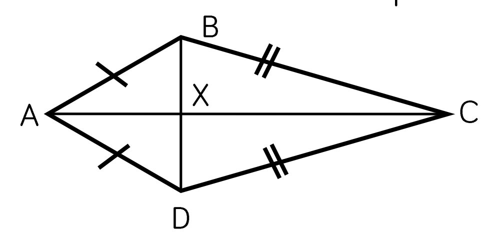
Show that the triangles are congruent.
Give a reason for each step in your working.

What calculation(s) do you need to perform to show that the triangles are congruent?



Which conditions for congruency could you use?

The diagonals of a kite ABCD meet at point X.



Show that three pairs of congruent triangles are formed by joining the diagonals.
Tommy thinks the same result will be true for a parallelogram.
Show that Tommy is wrong.

Proof with congruent triangles

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Notes and guidance

This extends the previous step to look at more formal proof, particularly in cases when no side lengths or angles are given. Properties of special quadrilaterals and circle theorems may again be interleaved here. Model the setting out of step-by-step proofs for students, and encourage them to write down what sides/angles they can identify as equal if they are struggling to see a starting point.

Key vocabulary

Congruent	Condition	Similar
Corresponding	Prove	Common

Key questions

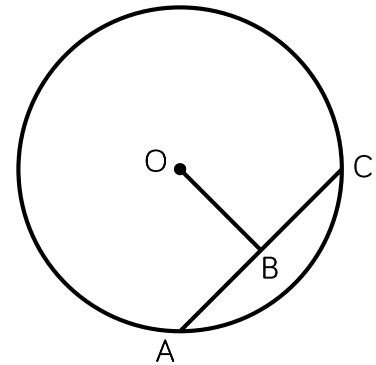
What's the same and what's different about showing that a pair of triangles are similar triangles and showing that a pair of triangles are congruent?
How does (e.g.) AB being common to both triangles help?
Identify a pair of equal sides. How do you know they are equal? Identify a pair of equal angles. How do you know they are equal?

Exemplar Questions

The diagram shows a chord AB drawn through a circle centre O.

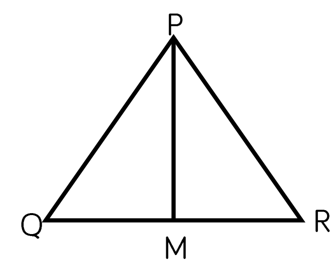
Show that tringles OAB and OCB are congruent

- using the condition RHS.
- using the condition SSS.



Show that a diagonal of a rhombus splits the rhombus into a pair of congruent triangles.
What other quadrilaterals is this true for?

PQR is an equilateral triangle.
M is the midpoint of QR.
How many ways can you find to show that triangles PQM and PRM are congruent?



Triangle ABC is an isosceles triangle with ∠ABC = ∠ACB.
X and Y are points on AB and BC respectively such that AX = AY.
Show that triangle ABY is congruent to triangle ACX.