Working in the Cartesian Plane

Year 8

#MathsEveryoneCan

White Rose Maths

	Week 1	Week 2	Week 3	Week 4	Week 5	Week 6	Week 7	Week 8	Week 9	Week 10	Week 11	Week 12
Autumn	**Proportional Reasoning**						**Representations**					
	Ratio and scale		Multiplicative change		Multiplying and dividing fractions		Working in the Cartesian plane			Collecting and representing data		Tables
Spring	**Algebraic techniques**						**Developing Number**					
	Brackets, equations and inequalities				Sequences	Indices	Fractions and percentages			Standard index form		Number sense
Summer	**Developing Geometry**						**Reasoning with Data**					
	Angles in parallel lines and polygons		Area of trapezia and circles		Line symmetry and reflection		The data handling cycle				Measures of location	

Autumn 2: Representations

Weeks 1 to 3: Working in the Cartesian Plane

Building on their knowledge of coordinates from KS2, students will look formally at algebraic rules for straight lines, starting with lines parallel to the axes and moving on to the more general form. They can explore the notions of gradient and intercepts, but the focus at this stage is using the equations to produce lines rather than interpretation of m and c from a given equation; this will be covered in Year 9. Use of technology to illustrate graphs should be embedded. Appreciating the similarities and differences between sequences, lists of coordinates and lines is another key point. Students following the higher strand may also explore non-linear graphs and mid-points of line segments.

National curriculum content covered:

- move freely between different numerical, algebraic, graphical and diagrammatic representations
- develop algebraic and graphical fluency, including understanding linear (and simple quadratic) functions
- make connections between number relationships, and their algebraic and graphical representations
- substitute numerical values into formulae and expressions
- recognise, sketch and produce graphs of linear functions of one variable with appropriate scaling, using equations in x and y and the Cartesian plane

Weeks 4 and 5: Representing data

Students are introduced formally to bivariate data and the idea of linear correlation. They extend their knowledge of graphs and charts from Key Stage 2 to deal with both discrete and continuous data.

National curriculum content covered:

describe, interpret and compare observed distributions of a single variable through: appropriate graphical representation involving discrete, continuous and grouped data

- construct and interpret appropriate tables, charts, and diagrams, including frequency tables, bar charts, pie charts, and pictograms for categorical data, and vertical line (or bar) charts for ungrouped and grouped numerical data
- describe simple mathematical relationships between two variables (bivariate data) in observational and experimental contexts and illustrate using scatter graphs
- use language and properties precisely to analyse probability and statistics

Weeks 6: Tables and Probability

Building form the Year 7 unit, this short block reminds students of the ideas of probability, in particular looking at sample spaces and the use of tables to represent these.

National curriculum content covered:

- record, describe and analyse the frequency of outcomes of simple probability experiments involving randomness, fairness, equally and unequally likely outcomes, using appropriate language and the 0-1 probability scale
- generate theoretical sample spaces for single and combined events with equally likely, mutually exclusive outcomes and use these to calculate theoretical probabilities
- use language and properties precisely to analyse probability and statistics

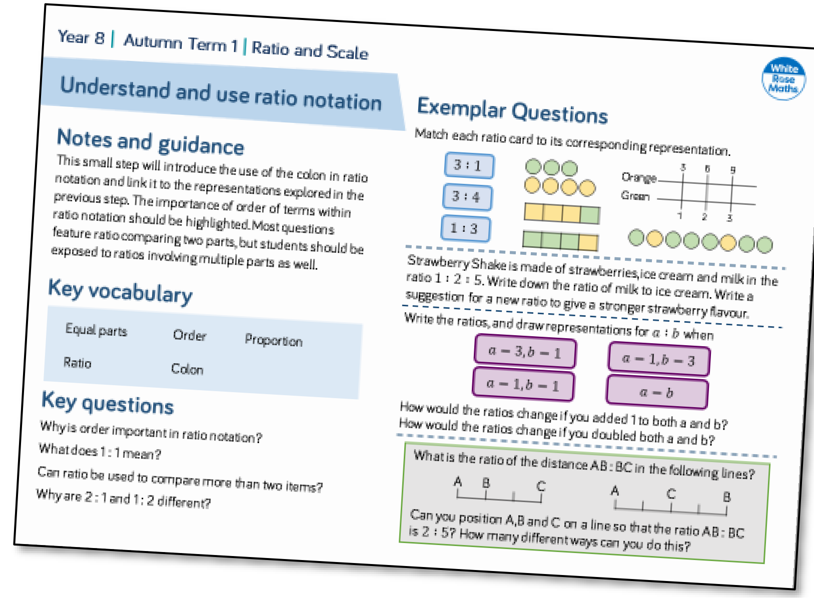
Why Small Steps?

We know that breaking the curriculum down into small manageable steps should help students to understand concepts better. Too often, we have noticed that teachers will try and cover too many concepts at once and this can lead to cognitive overload. We believe it is better to follow a "small steps" approach.

As a result, for each block of content in the scheme of learning we will provide a "small step" breakdown. ***It is not the intention that each small step should last a lesson – some will be a short step within a lesson, some will take longer than a lesson.*** We would encourage teachers to spend the appropriate amount of time on each step for their group, and to teach some of the steps alongside each other if necessary.

What We Provide

- Some ***brief guidance*** notes to help identify key teaching and learning points
- A list of ***key vocabulary*** that we would expect teachers to draw to students' attention when teaching the small step,
- A series of ***key questions*** to incorporate in lessons to aid mathematical thinking.
- A set of questions to help ***exemplify*** the small step concept that needs to be focussed on.


Year 8 | Autumn Term 1 | Ratio and Scale

Understand and use ratio notation

Notes and guidance

This small step will introduce the use of the colon in ratio notation and link it to the representations explored in the previous step. The importance of order of terms within ratio notation should be highlighted. Most questions feature ratio comparing two parts, but students should be exposed to ratios involving multiple parts as well.

Key vocabulary

Equal parts, Order, Proportion, Ratio, Colon

Key questions

Why is order important in ratio notation?
What does 1 : 1 mean?
Can ratio be used to compare more than two items?
Why are 2 : 1 and 1 : 2 different?

Exemplar Questions

Match each ratio card to its corresponding representation.

3 : 1
3 : 4
1 : 3

Strawberry Shake is made of strawberries, ice cream and milk in the ratio 1 : 2 : 5. Write down the ratio of milk to ice cream. Write a suggestion for a new ratio to give a stronger strawberry flavour.

Write the ratios, and draw representations for $a : b$ when

$a = 3, b = 1$ $a = 1, b = 3$ $a = 1, b = 1$ $a = b$

How would the ratios change if you added 1 to both a and b?
How would the ratios change if you doubled both a and b?

What is the ratio of the distance AB : BC in the following lines?

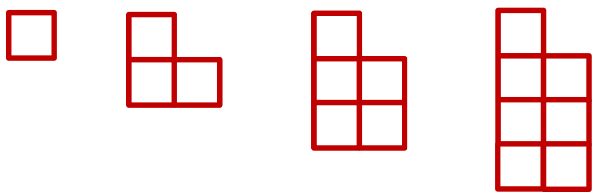
Can you position A, B and C on a line so that the ratio AB : BC is 2 : 5? How many different ways can you do this?

- These include reasoning and problem-solving questions that are fully integrated into the scheme of learning. Depending on the attainment of your students, you many wish to use some or all of these exemplars, which are in approximate order of difficulty. Particularly challenging questions are indicated with the symbol .
- For each block, we also provide ideas for key representations that will be useful for all students.

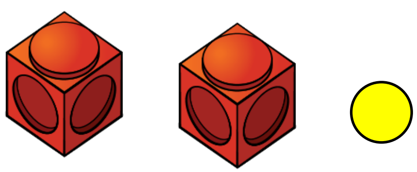
In many of the blocks of material, some of the small steps are in **bold**. These are content aimed at higher attaining students, but we would encourage teachers to use these with as many students as possible – if you feel your class can access any particular small step, then please include it in your planning.

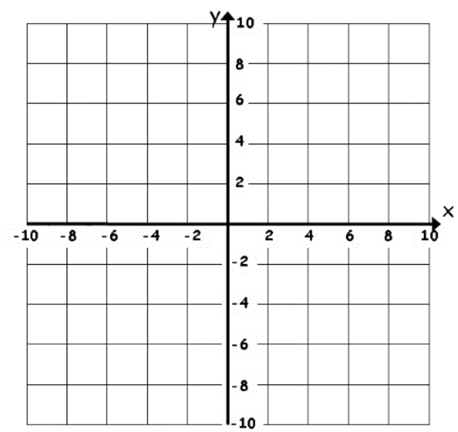
Key Representations

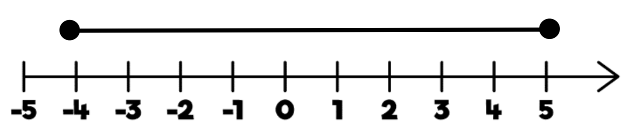
Input → $\times 2$ → -1 → Output



x	1	2	3	4
y	1	3	5	7







In these small steps, axes that allow the use of all four quadrants should be used. It is important that children see a wide range of axes, for example, those with different scales. If appropriate, counters and small cubes can be used to demonstrate a coordinate before it is marked onto the grid.

When introducing the midpoint of a line segment, teachers might start with looking at numbers that are half way along a number line (tape measures may be of use here).

Making links between sequences, co-ordinates and the equation of a straight line help students to make sense of these relationships and therefore children should have opportunities to explore the pictorial representations of these to aid investigation and enhance understanding.

Working in the Cartesian Plane

Small Steps

- Work with coordinates in all four quadrants
- Identify and draw lines that are parallel to the axes
- Recognise and use the line $y = x$
- Recognise and use lines of the form $y = kx$
- Link $y = kx$ to direct proportion problems
- **Explore the gradient of the line $y = kx$** H
- Recognise and use lines of the form $y = x + a$
- Explore graphs with negative gradient ($y = -kx, y = a - x, x + y = a$)

H denotes higher strand and not necessarily content for Higher Tier GCSE

Working in the Cartesian Plane

Small Steps

- Link graphs to linear sequences
- Plot graphs of the form $y = mx + c$
- **Explore non-linear graphs** (H)
- **Find the midpoint of a line segment** (H)

(H) denotes higher strand and not necessarily content for Higher Tier GCSE

Coordinates in all four quadrants

Notes and guidance

Students build on their KS2 knowledge of the coordinate plane, created by the intersection of two number lines in 2-D space, developing their understanding of the x-axis, y-axis and the origin. Students should be given the opportunity to draw their own axes and need careful support in labelling these correctly. Students should be able to label each quadrant from 1st to 4th

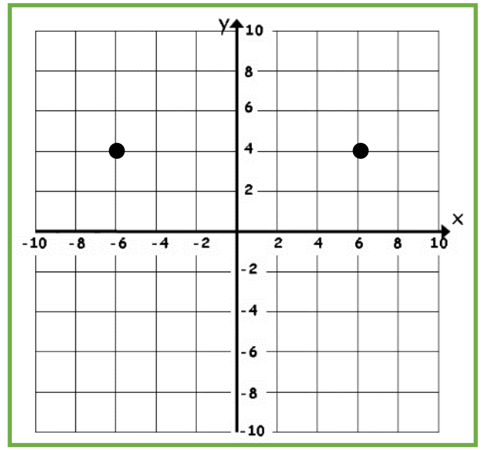
Key vocabulary

Quadrant	Coordinates	Horizontal
Vertical	Axis	Origin

Key questions

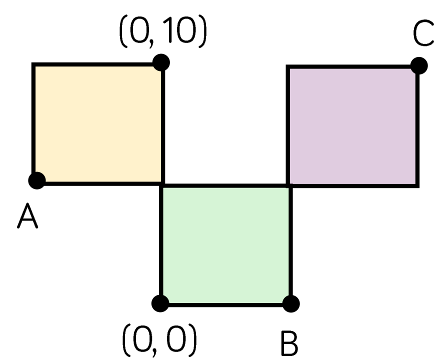
What is the same and what is different about the points with coordinates $(a, 0)$ and $(-a, 0)$?
Why are coordinates $(a, 0)$ and $(0, a)$ different?
Why do the order of the numbers in a coordinate matter?
Describe how you read and plot a coordinate.
Where is the origin?

Exemplar Questions



Write down the coordinates of the points highlighted.
By plotting one further point, create an isosceles triangle and work out its area.
Investigate other types of triangles that can be created by plotting one other point.

Plot the coordinates $(1, -4), (7, 3), (-4, 3), (9, -5)$ on a coordinate grid.
Which two coordinates are on the same line?
Which coordinate is in the second quadrant?



Three identical squares are shown.

Work out the coordinates of the points A, B and C.

Explain your strategy.

Lines parallel to the axes

Notes and guidance

Students are introduced to the idea of a straight line as an infinite set of points with a common feature. Care needs to be taken to ensure they understand that lines parallel to the x-axis have equations of the form $y = a$ and similarly $x = a$ will be parallel to the y-axis. This should be revisited regularly in starter activities to aid retention. The term 'parallel' might need revision here.

Key vocabulary

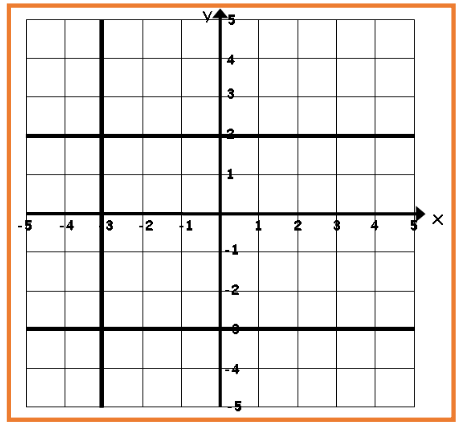
Parallel	Straight line	Vertical
Horizontal	Equation	Graph

Key questions

Give an example of an equation of a line that is parallel to the x-axis/y-axis.
Is the line $3 = x$ the same as the line $x = 3$? What about the line $x - 3 = 0$?
Why is the line $x = 0$ different from the x-axis?
Will the lines $x = \cdots$ and $y = \cdots$ ever meet? Why or why not?

Exemplar Questions

Plot and join the points with coordinates $(4, 1)$, $(4, -3)$ and $(4, 5)$
What do you notice? Write down another coordinate on the line.
How many points are on the line in total? How do you know?
Why do you think we call this line $x = 4$?



Write down the equations of the lines shown.

Label the lines with their equations. Draw the line $x = 4$ onto the grid.

Write down the coordinates of the points where the lines intersect.

The line $y = 3$ is parallel to the y-axis.

The line $y = 3$ is parallel to the x-axis.

Dora

Who do you agree with? Use a diagram to help explain why.

Amir

Recognise and use the line $y = x$

Notes and guidance

This step is to help students understand the line $y = x$, the first 'diagonal' line they will formally study. It should be explicitly covered that the line will only form a 45 degree angle with the axes if the scales are equal on both axes.

If appropriate with students following the Higher strand, compare and contrast with the line $y = -x$

Key vocabulary

Axis	Diagonal	Straight line
Origin	Scale	Graph

Key questions

Is the graph $y = x$ the same as the graph $x = y$?

How many points lie on the line $y = x$? Why?

Why are the scales of the axes important when plotting graphs?

Exemplar Questions

Plot the points given in the table. Give the coordinates of three more points on the line.

x	-4	-1.5	3	$5\frac{1}{2}$
y	-4	-1.5	3	$5\frac{1}{2}$

Discuss as a class what the equation of the line should be.

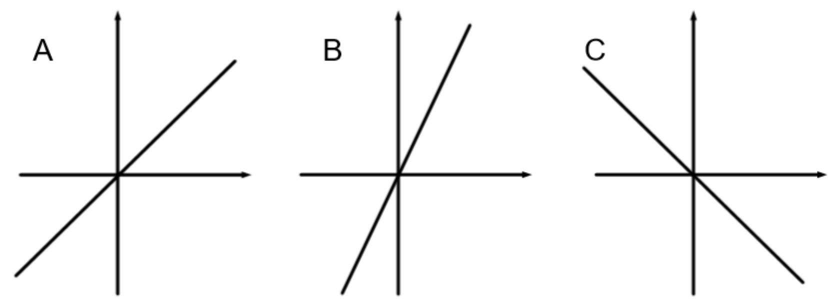
Which of the following points will lie **on** the line $y = x$?
Which of the others lie above the line $y = x$, and which lie below?

$(19, 19)$ $(-10, -9 - 1)$ $(8, 7)$ $(7, 8)$

(a, a) $(0.3, 0.3)$ $(b \times 2 , b + b)$ $(6, -6)$

Whitney

Whitney thinks graph A and graph C could be $y = x$, but graph B can't be because it's not at 45 degrees to the axes. Do you agree? Why?



Recognise and use lines $y = kx$

Notes and guidance

This step builds on the understanding of the line $y = x$ by introducing k and highlighting its affect on the steepness of a line. Lines with equation $y = kx$, make up a family of straight lines through the origin. Students can use graphical software to explore how increasing/decreasing the value of k affects the graph. It is important to vary coordinates and scale values beyond an integer.

Key vocabulary

Multiple	Steep	Linear	Substitute
Table	Slope	Scale	Axes

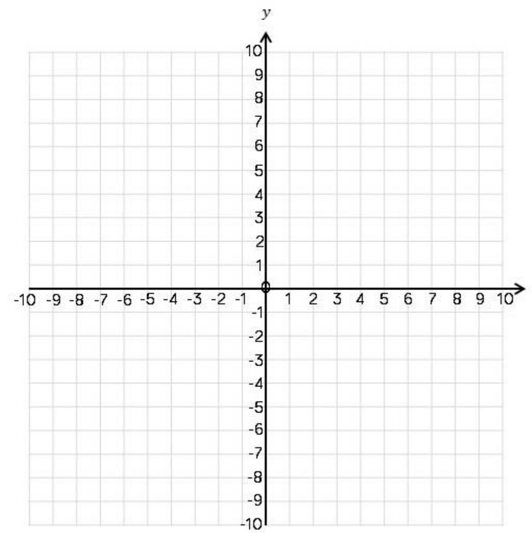
Key questions

How can you recognise a line of the form $y = kx$?
What's the same and what is different about the lines $y = kx$ and $y = x$?
What effect does increasing or decreasing the value of k have on lines with equations in the form $y = kx$?
Do all lines with equations in the form $y = kx$ form a straight line and go through the origin? Why or why not?

Exemplar Questions

Tommy is completing a table of values to plot the graph $y = 2x$
Complete the table and plot the graph using the axes provided.

x	-3	-2	-1	0	1	2
$y = 2x$	-6				2	



Using the same axes, plot the graph of $y = x$

What is the same and what is different about the two lines?

Which graph is steeper and why?

Using the same method as Tommy, plot the following two graphs.

$y = 4x$

$y = \frac{3x}{2}$

Why is it important to consider which x values to use? How will this impact on the scale of your y-axis?
What is the same and what is different about the two lines you have drawn?

Direct proportion using $y = kx$

Notes and guidance

Teachers might introduce this using a familiar context, for example, 'if one apple costs 20p, how much would 2 apples cost, 3 apples, 0 apples etc.' This helps to introduce the idea of a constant multiplier and hence, direct proportion.

Teachers should illustrate direct proportion using different representations, for example, tables, graphs and equations.

Key vocabulary

Linear	Proportion	Scale
Unitary	Multiplier	Direct

Key questions

How would you know if a straight line or a table of values represents direct proportion? What are the key features?
What is a conversion graph and how can information be obtained from it to answer questions?
Why do direct proportion graphs always start at $(0, 0)$?

Exemplar Questions

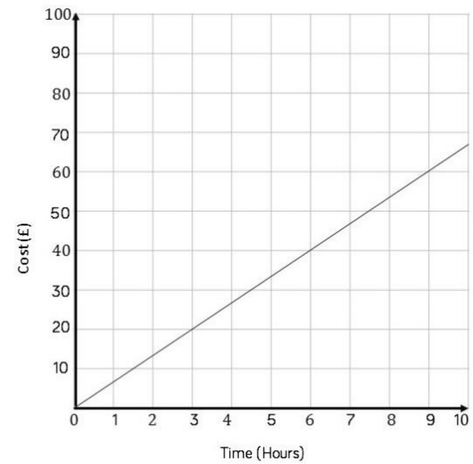
No of Pencils	1	2	3	4	5	6	10	40
Cost		36p						

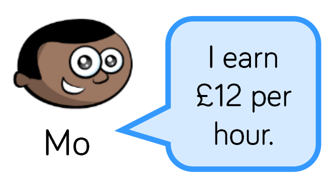
Two pencils cost £0.36, use this to complete the table.
What would the cost of 8 pencils be? What about 100 pencils?
What do you notice with the numbers in the table, what is the link?

This conversion graph shows the cost and time it takes to paint a room.

What is the cost of painting a room that takes 3 hours?

How long would it take to paint room costing £120?





Create a table of values showing the number of hours Mo worked and his earnings.
Draw the straight line graph that represents these values. What is the equation of this line?

Use this equation to work out how much Mo earns in 37.5 hours.

Gradient of the lines $y = kx$ H

Notes and guidance

Teachers might introduce this using real-life examples of gradients (e.g. pictures of hills and mountains). Students are then introduced to finding the gradient using right-angled triangles on the straight line. This can be extended to finding the equation for the gradient given two points (without the straight line being drawn onto a graph). Linking both gradient to k and direct proportion is useful here.

Key vocabulary

Steepness	Difference	Gradient
Straight line	Vertical	Horizontal

Key questions

What does the gradient of a line represent?
How do we know if one line is steeper than another?
Does it matter which right-angled triangle we choose on the straight line when we are calculating the gradient?
What does a gradient of zero mean?
How can working out the gradient of a line help in direct proportion calculations?

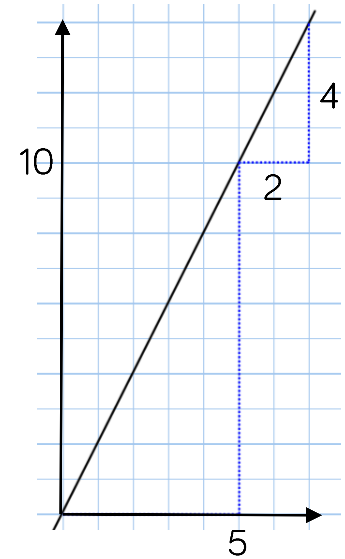
Exemplar Questions

The gradient of this line is $\frac{4}{2}$

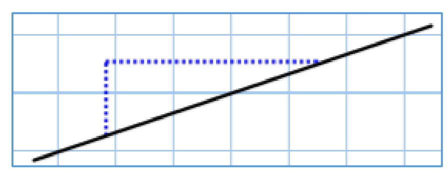
The gradient of this line is $\frac{10}{5}$

The equation of the line is $y = 2x$. The gradient is 2

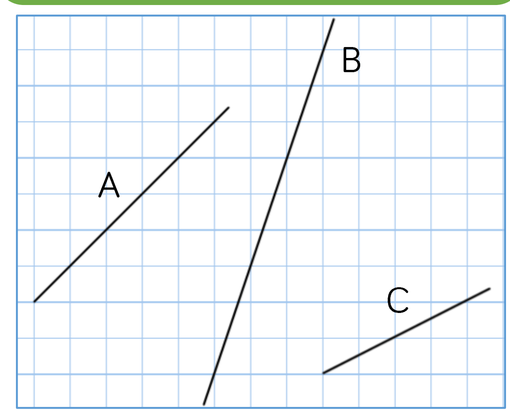
Who is correct and why?



Amir has drawn a right-angled triangle onto the line. What could Amir change about his right-angled triangle to help him work out the gradient?
Work out the gradient of this line.



Using right-angled triangles, calculate the gradients of these lines. Place them in order of steepness.



White Rose Maths

Lines of the form $y = x + a$

Notes and guidance

Students now consider the impact of adding a constant to the line with equation $y = x$. Students should be encouraged to explore the effect this has on the straight line by generating tables of values and plotting these.

Using a dynamic geometry package supports whole class discussion on this.

Key vocabulary

Equation	Input	Output
Intercept	Linear	Straight line

Key questions

What is the same and what is different about the line $y = x$ and the line $y = x - a$?

What is the gradient of the line $y = a + x$?

What about $y = x + a$?

Is $a - x = y$ the same line as $x + y = a$? Explain.

Explain how you could check that you have plotted the line $y = x + a$ correctly. What could you look for?

Exemplar Questions

Complete the table of values to generate a set of coordinates and plot the graph of $y = x + 2$

x	-3	-2	-1	0	1	2
y						

Generate further tables of values and draw the graph of $y = x - 2$ and $y = x + 5$ on the same axis.
What is the same and what is different about the graphs?

Match each graph with its equation. Explain your strategy.

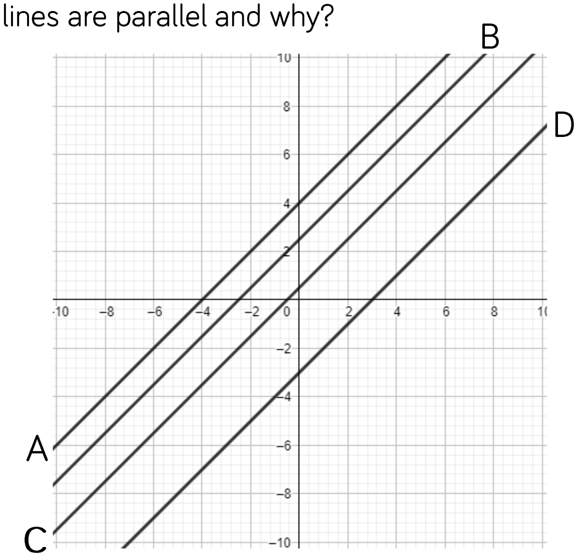
- What do these lines have in common?
- Where do these lines cross the x-axis and the y-axis?
- Which lines are parallel and why?

$y = x + 4$

$y = 2.5 + x$

$y = x - 3$

$x + \frac{1}{2} = y$



Graphs with negative gradients

Notes and guidance

Teachers might start by checking student confidence in using a negative multiplier. The concept of a negative gradient could be introduced by discussing what the gradient of a ski slope might be. Students can then draw straight line graphs where the line has a negative gradient. Equations should be given in different forms (e.g. $y = a - x,\ x + y = a$). Calculating the gradient when it is negative is not expected at this stage.

Key vocabulary

Negative	Gradient	Steepness
Incline	Ratio	Slope

Key questions

What's the same and what's different about the straight lines represented by the equations $y = kx$ and $y = -kx$?

How can you identify whether a straight line, plotted on a graph, has a negative or positive gradient?

How can you identify the type of gradient (positive or negative) of a line by just looking at the equation of the line?

Exemplar Questions

Complete the table of values to generate coordinates and plot the graph $y = -x$

x	-3	-2	-1	0	1	2
y		2				

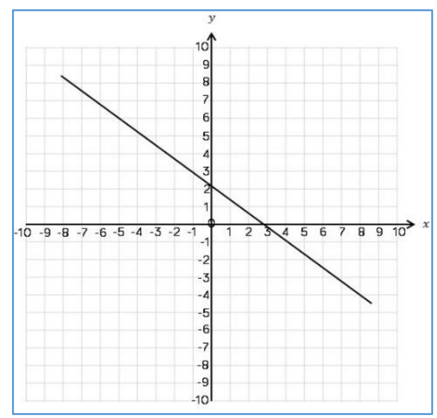
What is the same and what is different about the straight lines represented by the equations $y = x$ and $y = -x$?
Now plot the graph of $y = -3x$ and $y = -\frac{3x}{2}$
What do you notice?

For each of the following equations:

- Use x values from -3 to 3, generate a table of values for x and y
- Plot each straight line onto a separate graph.
- Label each line with the equation and then state whether the gradient is positive or negative. How can you see this from your graph?

$y = 3 - x$ $y + x = 5$ $y - x = 2$

True or False?
The line has a positive gradient.
When x is 0, y is 2
When y is 3, x is 0
When x is a negative number, y is a positive number.
x is always less than y
As x increases, y decreases.



White Rose Maths

Linking graphs to sequences

Notes and guidance

Students link prior knowledge of sequences with linear equations and their respective graphs. Using pictorial sequences, function machines, coordinates and tables of values, children can explore relationships between these, establishing important connections.
Teachers should highlight how coordinates on a straight line relate to the position and term in a sequence.

Key vocabulary

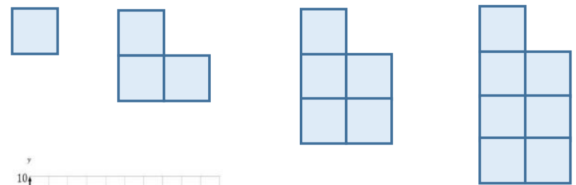
Difference	Sequence	Descending	Linear
Equation	Multiple	Ascending	

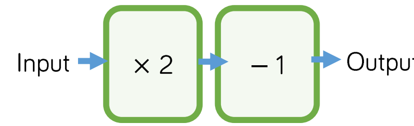
Key questions

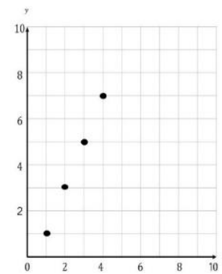
What's the same and what's different about linear graphs and linear sequences? How could we label the axis on a the graph to show the position of a term in the sequence?
Will the gradient of the straight line representing a descending linear sequence be positive or negative? Explain your answer.

Exemplar Questions

What is the same and what is different about these representations?







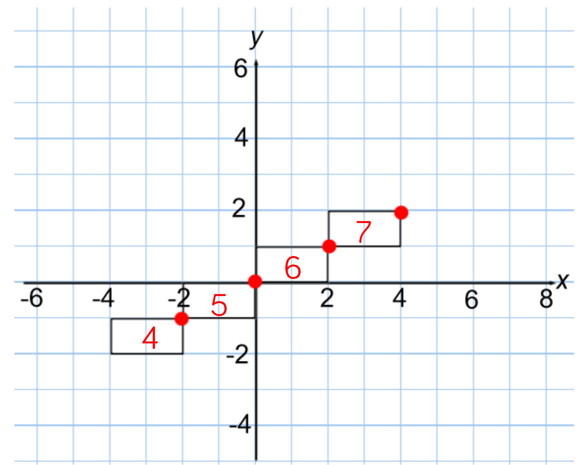
$y = 2x - 1$

(1, 1) (2, 3) (3, 5) (4, 7)

Position	1	2	3	4
Term	1	3	5	7

Write down the coordinates of the points highlighted in each rectangle. What patterns do you notice?

Write down the coordinates of the top right hand vertex of the 3rd and 8th rectangle in the pattern.



The equation of the line which goes through the marked coordinates is $y = \frac{1}{2}x$. How does this relate to the marked coordinates and the patterns you have noticed?

Plotting $y = mx + c$ graphs

Notes and guidance

Students further develop their understanding of equations of straight lines using the general form of $y = mx + c$
This step focuses purely on students becoming familiar with plotting graphs and generating coordinates from a table of values using $y = mx + c$
Interpretation of the equation will be covered in later steps.

Key vocabulary

Gradient	Linear	Integer	Input
Output	Substitution	Table of Values	

Key questions

Why is it a good idea to use three coordinates when plotting a straight line graph?
Can you use non-integer x values in your table to generate your set of coordinates?
Can you extend your straight line outside of the range of values in your table? Explain your answer.

Exemplar Questions

On the same axes, draw the graphs of the following equations by completing the table of values. Discuss key features of each graph.

$y = 3x - 1$

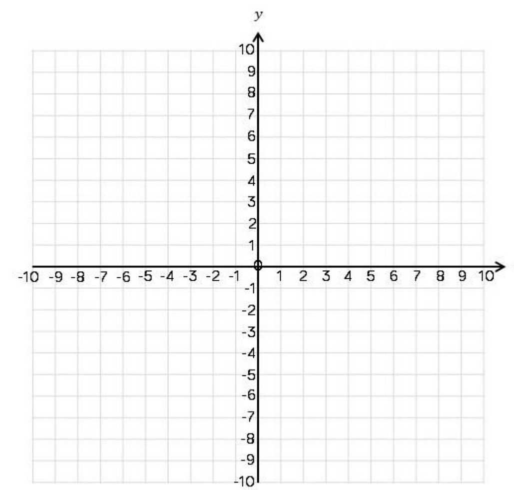
x	−3	0	3
y			

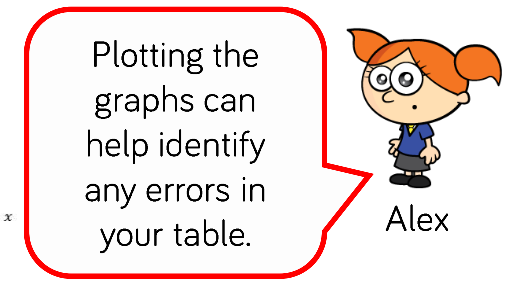
$y = \frac{1}{2}x + 3$

x	−6	0	6
y			

$y = -2x + 6$

x	−2	0	2
y			





Comment on Alex's statement. What does she mean?

Match each coordinate with the equation of the line it could lie on.

(−2, 8) | $(5, 4\frac{1}{4})$ | (6, 3.5)

$y = 10 + x$ | $\frac{x+1}{2} = y$ | $y = \frac{x}{4} + 3$

Exploring non-linear graphs H

Notes and guidance

Students are introduced to plotting and identifying non-linear graphs. Students will need guidance on how to draw a smooth curve to join the coordinates. It's helpful to discuss why it's inappropriate to join the coordinates with a straight line. Teachers may want to start with x^n where $n > 1$, e.g. $y = x^2, y = x^3$ etc.
Further investigation of non-linear graphs can be supported using graphical computer software.

Key vocabulary

Equation	Linear	Curve
Non-linear	Symmetrical	

Key questions

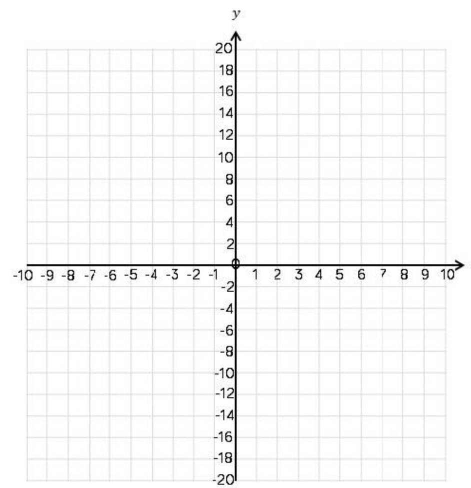
Describe the differences between a linear and a non-linear graph.
How can you use the equation of the graph to determine whether it is linear?
How do we work out the scale for our axes?

Exemplar Questions

Continue completing Dora's table of values to generate a set of coordinates and plot the graph of $y = x^2$ using the axis provided. Describe key features of the graph, including the scale used on the axes.

Dora

x	−3	−2	−1	0	1	2	3	4
y					1			16



Substituting a with numbers, investigate the following graphs using graphical computer software.

$y = ax^2$

$y = ax^3$

$y = \frac{a}{x}$

Which of these equations will produce a non-linear graph. Why?
Check your answer using graphical computer software.

$y = x - \frac{7}{2}$

$y - 4 = 0.5x$

$y = \frac{3}{2}x$

$y = x^2 + 3$

$y = \frac{4}{x}$

$x^3 + y = 5$

Midpoint of a line segment

H

Notes and guidance

Students firstly consider midpoints on number lines. This allows them to consider the most efficient method of finding a midpoint. Students then build on this to find the coordinates of a midpoint of a line segment, finding a general rule. Finally, they might explore finding the starting point or end point of a line segment given the midpoint.

Key vocabulary

Midpoint	Equidistant	Segment
Difference	Mean	

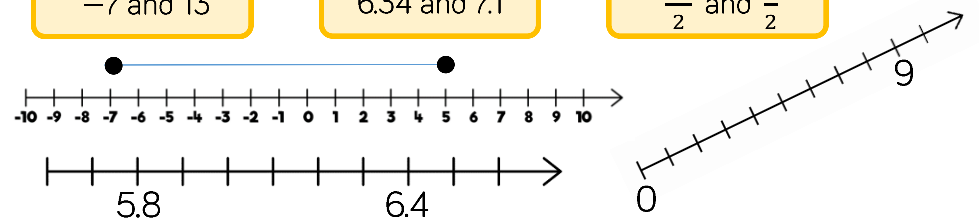
Key questions

What does the word equidistant mean?
How can you work out a midpoint? Is there more than one way?
If given the coordinates of the midpoint, and of the starting point of the line, how can you work out the coordinates of the endpoint of the line?

Exemplar Questions

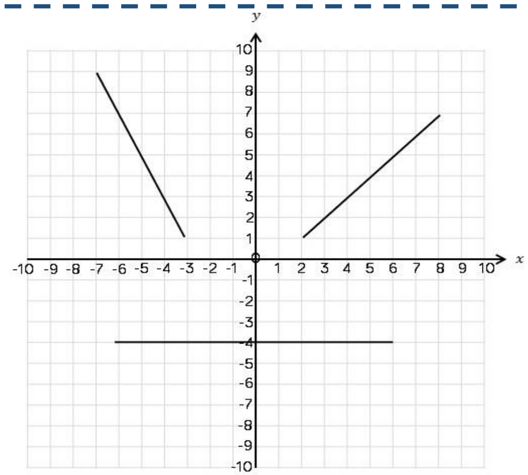
Find the number halfway between each of the pairs shown.
Discuss your strategies.

-7 and 13	6.34 and 7.1	$\frac{15}{2}$ and $\frac{5}{2}$



Calculate the midpoint of each pair of coordinates.

(6, 8) and (6, 20)	(4, 7) and (12, 19)	(2, 7) and (-2, 15)



Work out the midpoint of the following line segments.

Using the same coordinate grid, draw a line with a midpoint of (0, 1)

What are the end points?
How many different pairs of end points could there be?

A line segment starts at (2, 5). It has a midpoint of (6, 1)
What is the end point of the line?