Geometric Reasoning

Year 7

#MathsEveryoneCan

2019-20

White Rose Maths

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Summer 1: Lines and Angles

Weeks 1 to 3: Construction, measurement and notation

Students will build on their KS2 skills using rulers, protractors and other measuring equipment to construct and measure increasingly complex diagrams using correct mathematical notation. This will include three letter notation for angles, the use of hatch marks to indicate equality and the use of arrows to indicate parallel lines. Pie charts will be studied here to gain further practice at drawing and measuring angles.
National curriculum content covered:

- use language and properties precisely to analyse 2-D shapes
- begin to reason deductively in geometry including using geometrical constructions
- draw and measure line segments and angles in geometric figures, including interpreting scale drawings
- describe, sketch and draw using conventional terms and notations: points, lines, parallel lines, perpendicular lines, right-angles, regular polygons, and other polygons that are reflectively and rotationally symmetric
- use the standard conventions for labelling sides and angles
- construct and interpret pie charts for categorical, ungrouped and grouped numerical data
- identify and construct congruent triangles.

Interleaving/Extension of previous work

- revisit four operations

Weeks 4 to 6: Geometric reasoning

This block covers basic geometric language, names and properties of types of triangles and quadrilaterals, and the names of other polygons. Angles rules will be introduced and used to form short chains of reasoning . The higher strand will take this further, investigating and using parallel line rules.
National curriculum content covered:

- use language and properties precisely to analyse 2-D shapes,
- begin to reason deductively in geometry including using geometrical constructions
- describe, sketch and draw using conventional terms and notations: points, lines, parallel lines, perpendicular lines, right-angles, regular polygons, and other polygons that are reflectively and rotationally symmetric
- use the standard conventions for labelling sides and angles
- derive and illustrate properties of triangles, quadrilaterals, circles, and other plane figures [for example, equal lengths and angles] using appropriate language and technologies
- apply the properties of angles at a point, angles at a point on a straight line, vertically opposite angles
- apply angle facts, triangle similarity and properties of quadrilaterals to derive results about angles and sides, and use known results to obtain simple proofs
- understand and use the relationship between parallel lines and alternate and corresponding angles **(H)**
- derive and use the sum of angles in a triangle and use it to deduce the angle sum in any polygon, and to derive properties of regular polygons **(H)**

Interleaving/Extension of previous work

- forming and solving linear equations
- revisiting addition and subtraction, including decimals

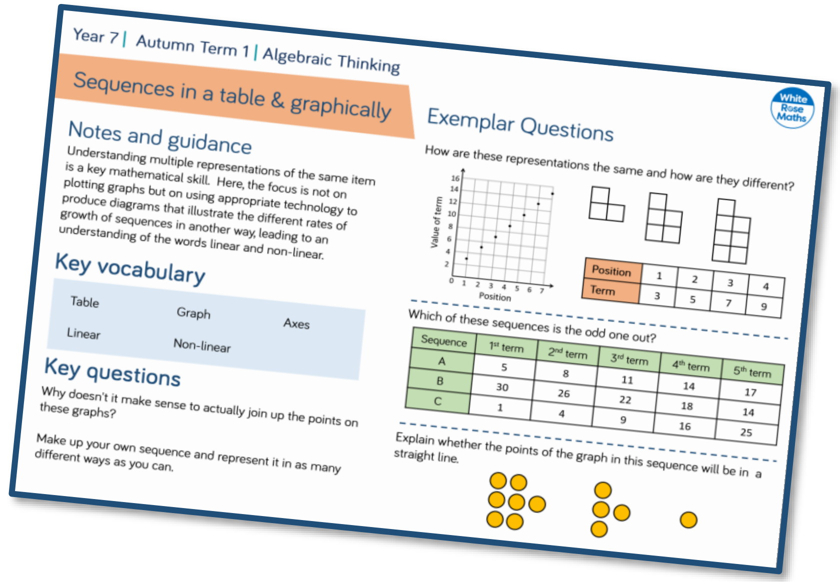
Why Small Steps?

We know that breaking the curriculum down into small manageable steps should help students to understand concepts better. Too often, we have noticed that teachers will try and cover too many concepts at once and this can lead to cognitive overload. We believe it is better to follow a "small steps" approach.

As a result, for each block of content in the scheme of learning we will provide a "small step" breakdown. ***It is not the intention that each small step should last a lesson – some will be a short step within a lesson, some will take longer than a lesson.*** We would encourage teachers to spend the appropriate amount of time on each step for their group, and to teach some of the steps alongside each other if necessary.

What We Provide

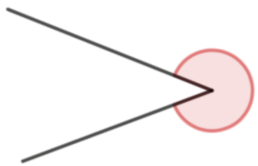
- Some ***brief guidance*** notes to help identify key teaching and learning points.
- A list of ***key vocabulary*** that we would expect teachers to draw to students' attention when teaching the small step.
- A series of ***key questions*** to incorporate in lessons to aid mathematical thinking.
- A set of questions to help ***exemplify*** the small step concept that needs to be focussed on.

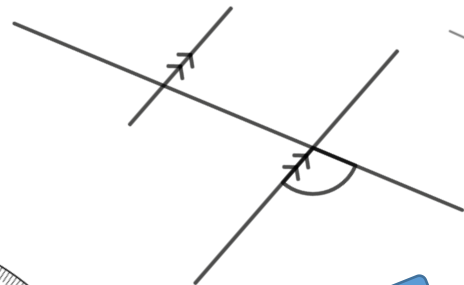


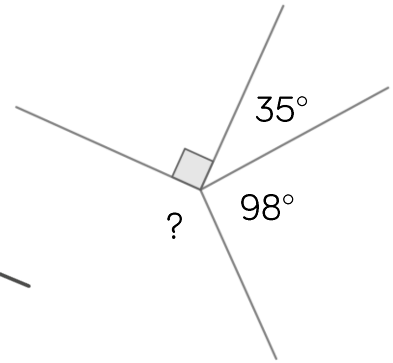
- These include reasoning and problem-solving questions that are fully integrated into the scheme of learning. Depending on the attainment of your students, you many wish to use some or all of these exemplars, which are in approximate order of difficulty. Particularly challenging questions are indicated with the symbol 💡.
- For each block, we also provide ideas for key representations that will be useful for all students.

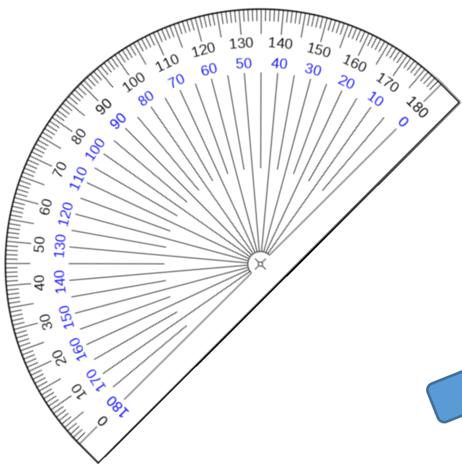
In many of the blocks of material, some of the small steps are in **bold**. These are content aimed at higher attaining students, but we would encourage teachers to use these with as many students as possible – if you feel your class can access any particular small step, then please include it in your planning.

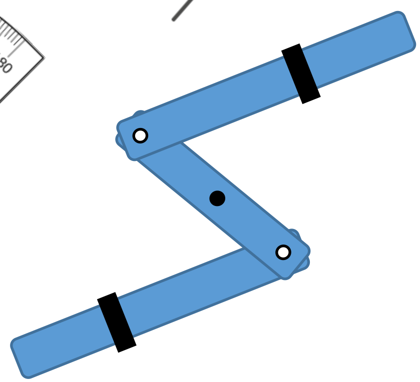
Key Representations

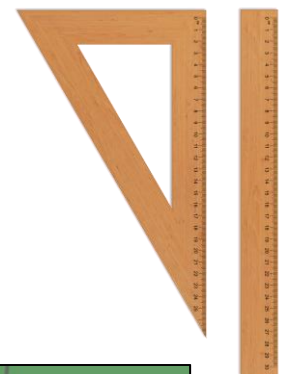


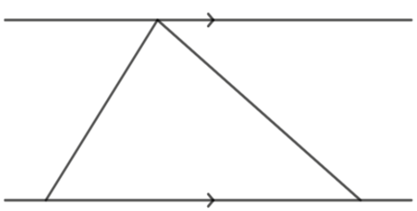












Concrete, pictorial and abstract representations are an important part of developing students' conceptual understanding.

Here are a few ideas of equipment and representations that you might use during Geometric reasoning.

A variety of pictorial models should be shown in a range of orientations.

The blue diagram shows a reverse motion linkage that students should be familiar with from primary school. The upper and lower parts will always remain parallel. Parallel line constructions can also be demonstrated using a set square and ruler. Real life uses of parallel could be discussed such as "the roads run parallel."

Geometric Reasoning

Small Steps

- Understand and use the sum of angles at a point
- Understand and use the sum of angles on a straight line
- Understand and use the equality of vertically opposite angles
- Know and apply the sum of angles in a triangle
- Know and apply the sum of angles in a quadrilateral
- Solve angle problems using properties of triangles and quadrilaterals
- Solve complex angle problems

Geometric Reasoning

Small Steps

- Find and use the angle sum of any polygon H
- Investigate angles in parallel lines H
- Understand and use parallel line angle rules H
- Use known facts to obtain simple proofs. H

H denotes higher strand and not necessarily content for Higher Tier GCSE

Sum of angles at a point

Notes and guidance

Students should know that angles at a point sum to 360°. They should understand that this is a definition and that it is not possible to prove that angles at a point sum to 360°. Interactive geometry software should be used by teachers and students to demonstrate and explore this step.

Key vocabulary

Sum	Angle	Degrees
Line Segment	Notation	

Key questions

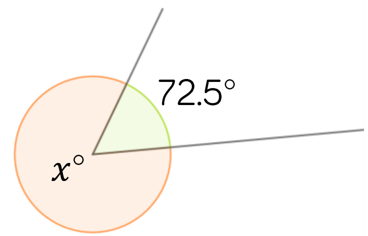
What is the sum of angles at a point?

How many right-angles fit around a point?

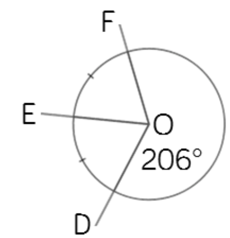
How does 180° compare to the sum of angles at a point?

Exemplar Questions

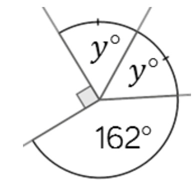
Work out the size of each angle. Diagrams are not drawn accurately.



$x =$ ______



FOE = ______



Form and solve an equation and find the value of y.

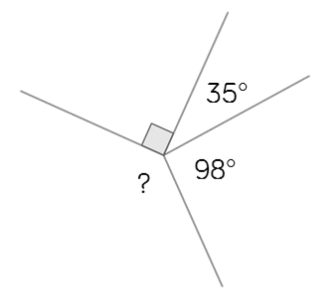
Mo and Rosie are discussing how to find the missing angle in the diagram below. Who do you agree with and why?

Mo

I think you just need to subtract 35 and 98 from 360

Rosie

I think that you need to subtract another 90



Four line segments are drawn from a point O. They are OA, OB, OC and OD. A, B, C and D are points drawn clockwise in order around O. Angle AOB is 54°. Angle BOC is 106°. Angle BOD is 187°.
What is the size of angle AOD? Circle the correct answer.

13° 81° 119° 347°

Sum of angles on a straight line

Notes and guidance

Students should know that adjacent angles at a point on a straight line sum to 180°. They should recognise when this fact can or cannot be applied. Non-examples should be shown where two angles on a straight line do not share a common point. Interactive geometry software can be used to demonstrate this step and allow students to explore when this rule can and cannot be applied.

Key vocabulary

Sum	Angle	Degrees
Line Segment	Notation	Adjacent

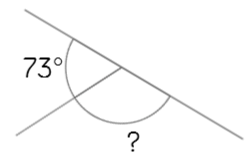
Key questions

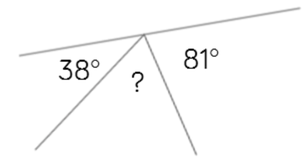
What is the sum of angles at a point on a straight line?
How many right-angles would fit on a straight line?

John measures three angles on a straight line. They are 81°, 47° and 51°. Has John measured the angles correctly? Explain your answer

Exemplar Questions

Calculate the size of the missing angle in each diagram.





Amir and Whitney are discussing a diagram.
Angle WXY is 19°. Angle YXZ is 161°.

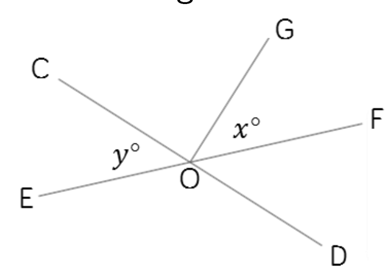
Amir: Y, X and Z all lie on the same line.

XY and XZ are two different lines intersecting at X.

Whitney

Who do you agree with? You must justify your answer with a sketch.

CD and EF are straight lines.
Write expressions for the size of any missing angles.
Use the correct three letter and geometric notation where appropriate.



Vertically opposite angles

Notes and guidance

Students should know that vertically opposite angles are equal. They should understand that vertically opposite angles are formed when two or more lines intersect at a point. They should be able to show that vertically opposite angles are equal by considering angles at a point on a straight line.

Key vocabulary

Angle

Vertically Opposite

Line

Intersect

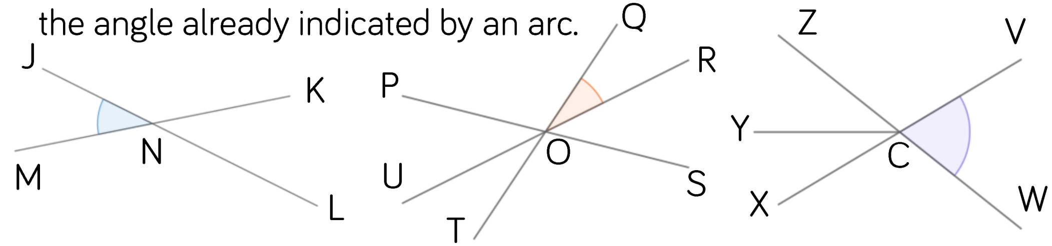
Key questions

When are vertically opposite angles formed?

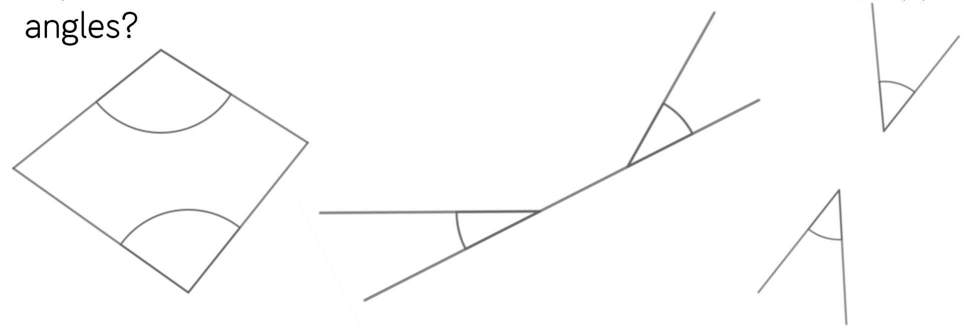
Given an angle formed at the intersection of two straight lines, is it always possible to find all angles at that point?

Exemplar Questions

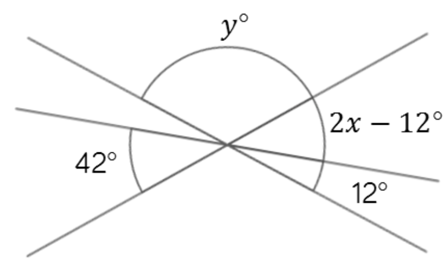
Use letter notation to describe the angle that is vertically opposite to the angle already indicated by an arc.



Explain why each of these diagrams do not show vertically opposite angles?



The following diagram shows three straight lines that intersect at a single point. Work out the value of x and y.



Sum of angles in a triangle

Notes and guidance

Students should know that the interior angles in a triangle sum to 180°. Students may be familiar with and could investigate tearing the corners from a triangle and using them to form 180°.

Demonstrations using interactive geometry software may also aid understanding.

Key vocabulary

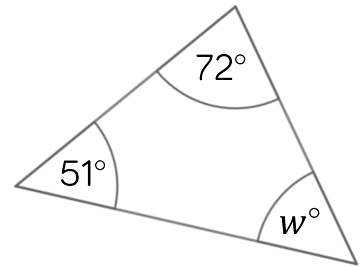
Angle	Isosceles	Equilateral
Scalene	Right-angled	Sum

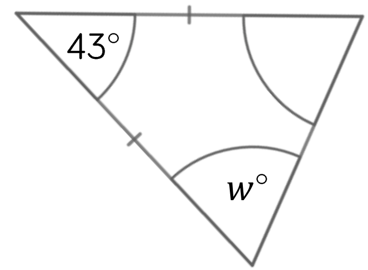
Key questions

What is the sum of the interior angles of a triangle?
How many angles do you need to know to be able to find all of the interior angles of a triangle?
If one angle in an isosceles triangle is 60°, is it an equilateral triangle?
Can a triangle have two right-angles?

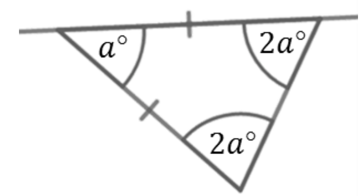
Exemplar Questions

Calculate the size of the angle marked in each diagram.

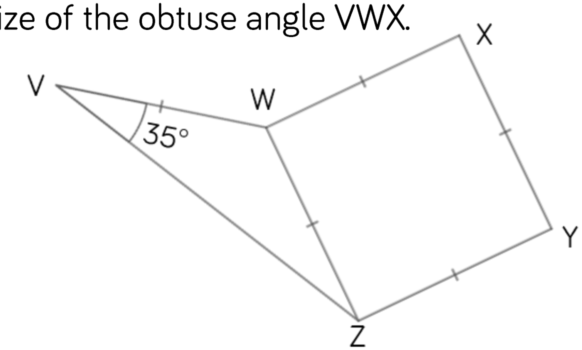




Bella says, "Angle a can be any angle and $2a$ is double that."
Milo says, "Five lots of angle a must be equal to 180°."
Who do you agree with? Why?



Two shapes are connected at their edges as in the diagram below.
Calculate the size of the obtuse angle VWX.



One angle in an isosceles triangle is 30°. What are the other two angles? Give two possible solutions.

Sum of angles in a quadrilateral

Notes and guidance

Students should know and be able to derive that the sum of angles in a quadrilateral is 360°. Both convex and concave quadrilaterals should be considered. Students should derive the angle sum by considering quadrilaterals as two triangles. Interactive geometry software can be used to demonstrate this. Students should revisit the properties of quadrilaterals.

Key vocabulary

Quadrilateral	Convex	Concave
Sum	Parallelogram	Rhombus

Key questions

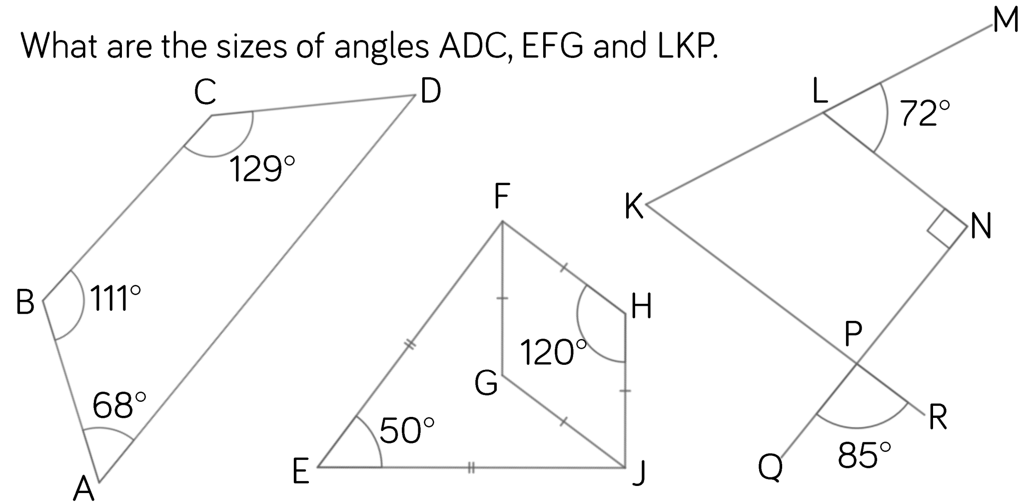
What is the sum of interior angles in a quadrilateral?

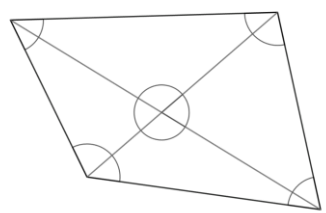
How can you demonstrate that the sum of the interior angles of a quadrilateral is 360°?

If a quadrilateral has four right-angles, is it a square?

Exemplar Questions

What are the sizes of angles ADC, EFG and LKP.





David draws the diagram to the left. He says the interior angles in a quadrilateral sum to 720° because he has split the shape into four triangles and four lots of 180° is 720°.

Explain why David's diagram does not show this.

Which of the following equations are correct for the diagram?

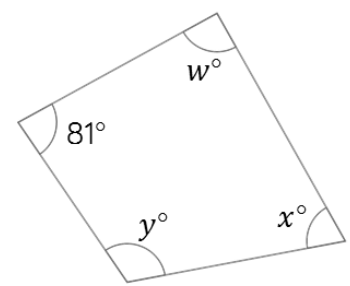
$$81 = w + x + y$$

$$w + x + y + 81 = 360$$

$$279 = w + x + y$$

$$x = 81$$

$$180 = x + 81$$



Angle problems

Notes and guidance

Within this small step, students should use one known angle fact to find a missing angle.

The focus should be on reasoning which angle fact should be applied to each scenario. Justifications using the correct vocabulary and notation should be used throughout.

Key vocabulary

Angle	Sum	Vertically opposite
Point	Straight Line	Polygon

Key questions

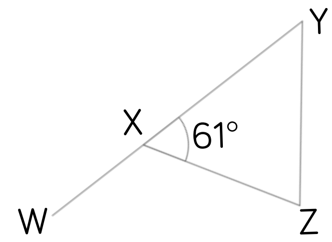
How did you decide which angle fact to use and apply?

Could you have applied a different angle fact?

Which angle facts do you know?

Which angle facts do you think you will need to apply to this question?

Exemplar Questions



Complete the sentences.
Angles on a straight line sum to ______.
Therefore angle WXZ is _______.
The interior angles in a triangle sum to ______ so the other two angles in the triangle must sum to ______.

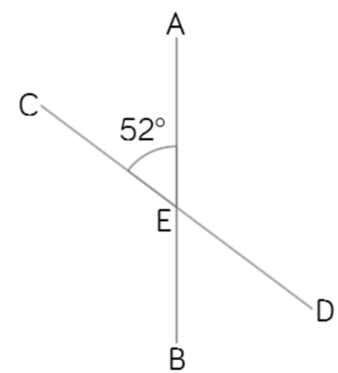
Complete the sentences.

Angle CEB is _____ because __________

______________________________.

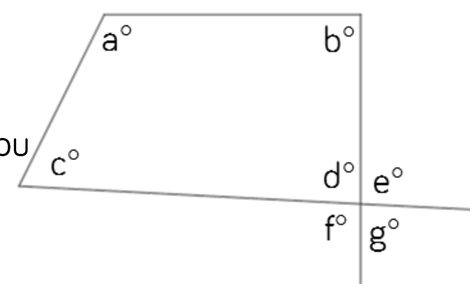
∠BED is _____ because __________

______________________________.



Write as many equations as possible for the diagram to the right.

You must state the angle facts that you have considered to write each equation.



Complex angle problems

Notes and guidance

This step considers angle problems where two or more known angle facts need to be applied to a problem. Students should always give reasons for their solutions, ensuring that they use the correct vocabulary. Different chains of reasoning should be explored alongside discussion of which are the most efficient methods.

Key vocabulary

Angle	Sum	Vertically opposite
Point	Straight Line	Polygon

Key questions

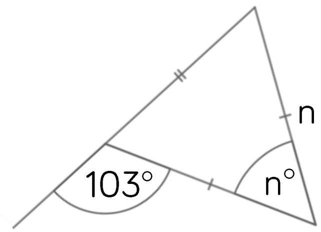
How did you decide which angle facts to apply?

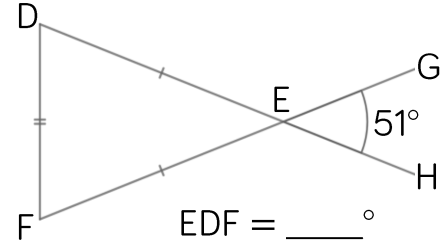
Could you have considered the same angle facts in a different order?

Could you have applied a different angle fact?

Exemplar Questions

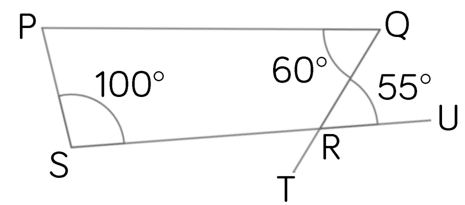
Find the size of the angle specified for each diagram.





Below is a diagram and a student explanation of finding the size of angle QPS. Is each stage of reasoning correct? Could the solution have been more efficient?

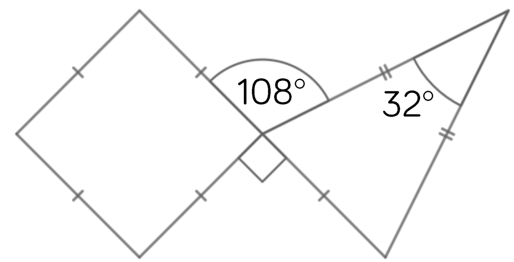
TRU = 125° because angles on a straight line sum to 180°.
QRS = TRU = 125° because vertically opposite angles are equal.
QPS = 75° because angles in a quadrilateral sum to 360°.



The diagram below shows two polygons connected by one of their vertices at a point. Determine if the quadrilateral is a square or not.

How do you know?

The diagram is not drawn accurately.



Angle sum of polygons

H

Notes and guidance

Students should know how to find and use the angle sum of any polygon.

They should investigate interior and exterior angles at vertices.

Students can investigate sums by partitioning polygons into triangles from a single vertex.

Key vocabulary

Polygon	Interior	Sum
Regular	Convex	Concave

Key questions

Explain why the interior angle of any polygon is a multiple of 180°.

How can you calculate the angle sum of any polygon?

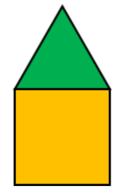
Does your method work for concave polygons?

Exemplar Questions

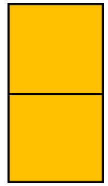
Aliya and Theo are forming polygons by combining pattern blocks.

- Aliyah puts a square and a triangle together to form polygon 1.
- Theo puts two squares together to form polygon 2.

Polygon 1



Polygon 2

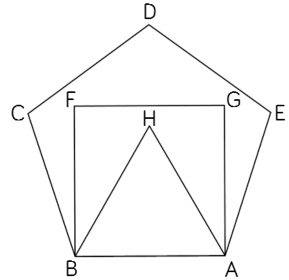


Aliya: The interior angles of a square and triangle sum to 180° and 360° respectively.
The interior angles of the pentagon must sum to 540°.

Theo: The interior angles of polygon 2 must be 720° because the sum of the interior angles in each square is 360°.

Explain whether you agree or disagree with each person.

The diagram to the right shows an equilateral triangle, a square and a regular pentagon all sharing the edge AB.
Find the size of angles:



- CDE
- EAH
- FBH

A regular polygon has an interior angle of 172°.
How many edges does the polygon have?

Angles in parallel lines H

Notes and guidance

Students investigate angles in parallel lines by measuring. They should not formally consider parallel line angle rules during this step. Students should make and test conjectures. They should also be encouraged to use known angle facts to justify any of their conjectures. Both computer software and physical mechanical apparatus should be used to demonstrate to students.

Key vocabulary

Parallel	Perpendicular	Line segment	
Conjecture	Equal	Opposite	Transversal

Key questions

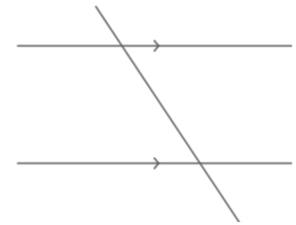
How do you denote that two or more lines are parallel?

What do you notice about the sum of angles ___ and ___ ?

What do you notice about angles ___ and ___ ?

Exemplar Questions

Measure all of the angles below. What do you notice?

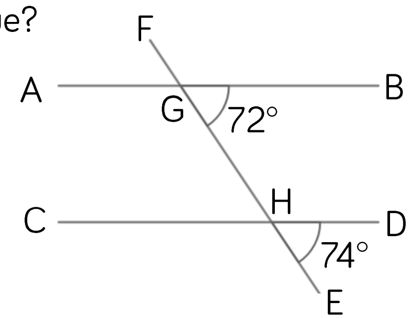


Use dynamic geometry software to see if this is always the case.

All of the odd numbered streets are parallel. All of the even numbered avenues are parallel. Mark on all of the parallel lines with appropriate notation.

Mark any angles that you think will be the same. Does it matter whether the transversal lines are perpendicular to the streets?

Which of the following statements are true?



- AB is parallel to CD.
- Lines through AB and CD will meet exactly once.
- $\angle CHE = \angle FHD$

Parallel line angle rules

H

Notes and guidance

In this step, students should build on the previous step by looking formally at alternate, corresponding and co-interior angles. They should be able to identify these types of angles and use them to find other angles in parallel lines. Students should also be aware of the converses e.g. if a pair of corresponding angles are equal, then the lines are parallel.

Key vocabulary

Parallel	Intersect	Transversal
Co-interior	Corresponding	Alternate

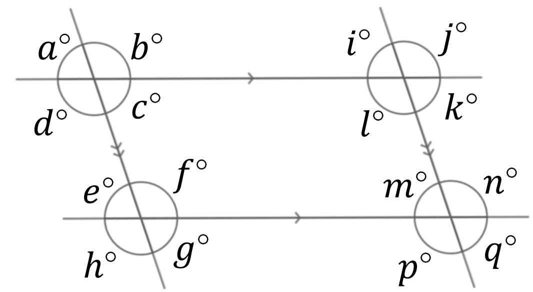
Key questions

How do you identify corresponding/alternate/co-interior angles?

Why are co-interior angles different to corresponding and alternate angles?

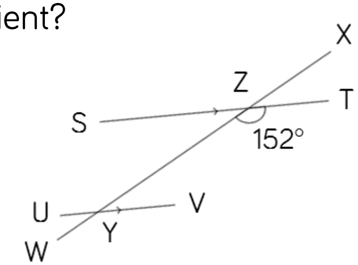
Exemplar Questions

Find as many pairs of angles as possible that are the same.
Find as many pairs of angles as possible that sum to 180°.
State whether each pair is corresponding, alternate or co-interior.



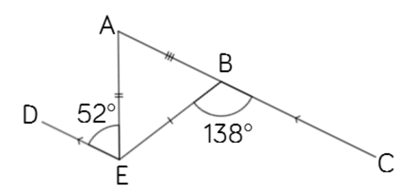
Find the size of the following angles. You must give reasons for your solution. How many different ways can you justify your solution? Which justification is the most efficient?

- UYZ = _____°
- WYV = _____°
- SZX = _____°
- VYZ = _____°
- XZT = _____°



Points A, B and C all lie on a straight line. Find the size of angle BAE in the diagram below.

Give reasons for your solution.



Simple proofs

H

Notes and guidance

Students need to be able to obtain simple proof using known facts from previous steps. They should explore the difference between a demonstration and a proof. The teacher should demonstrate the proof that angles in triangle add up to 180° Writing of proofs should be encouraged following discussions about efficiency and generalisation.

Key vocabulary

Proof	Demonstration	Opposite
Interior	Exterior	Parallel

Key questions

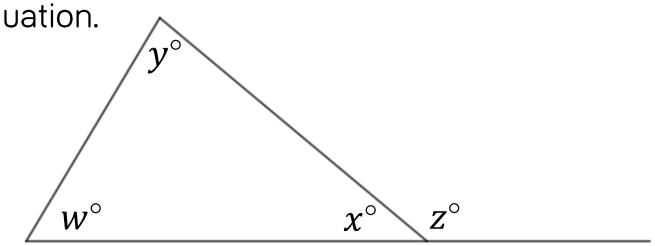
What is the difference between a proof and a demonstration?

Is it possible to prove something in more than one way?

Can you prove that there are 360° in a full turn?

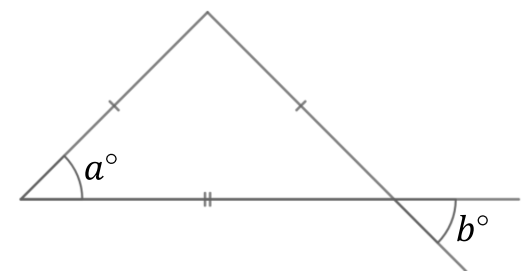
Exemplar Questions

Write down as many equations as possible that you know to be true using the diagram below. You must provide reasons for each equation.



Use your equations to prove that the exterior angle of a triangle is equal to the sum of the two opposite interior angles.

Prove that angle a is equal in size to angle b in the diagram below. Explain your reasoning in full.



By considering a convex kite as two different isosceles triangles. Prove that a convex kite has a pair of opposite angles that are equal.

Does your proof work for a concave kite? If not, can you find a proof that will work for all kites?