Indices

Year 8

#MathsEveryoneCan

White Rose Maths

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	Week 1	Week 2	Week 3	Week 4	Week 5	Week 6	Week 7	Week 8	Week 9	Week 10	Week 11	Week 12
Autumn	**Proportional Reasoning**						**Representations**					
	Ratio and scale		Multiplicative change		Multiplying and dividing fractions		Working in the Cartesian plane			Representing data		Tables & Probability
Spring	**Algebraic Techniques**						**Developing Number**					
	Brackets, equations and inequalities				Sequences	Indices	Fractions and percentages		Standard index form		Number sense	
Summer	**Developing Geometry**						**Reasoning with Data**					
	Angles in parallel lines and polygons		Area of trapezia and circles		Line symmetry and reflection		The data handling cycle				Measures of location	

Spring 1: Algebraic Techniques

Weeks 1 to 4: Brackets, Equations & Inequalities

Building on their understanding of equivalence from Year 7, students will explore expanding over a single bracket and factorising by taking out common factors. The higher strand will also explore expanding two binomials. All students will revisit and extend their knowledge of solving equations, now to include those with brackets and for the higher strand, with unknowns on both sides. Bar models will be recommended as a tool to help students make sense of the maths. Students will also learn to solve formal inequalities for the first time, learning the meaning of a solution set and exploring the similarities and differences compared to solving equations. Emphasis is placed on both forming and solving equations rather than just looking at procedural methods of finding solutions.

National curriculum content covered:

- identify variables and express relationships between variables algebraically
- begin to model situations mathematically and express the results using a range of formal mathematical representations
- substitute numerical values into formulae and expressions, including scientific formulae
- understand and use the concepts and vocabulary of expressions, equations, inequalities, terms and factors
- simplify and manipulate algebraic expressions to maintain equivalence by:
    - collecting like terms
    - multiplying a single term over a bracket
    - taking out common factors
    - expanding products of two or more binomials
- understand and use standard mathematical formulae
- use algebraic methods to solve linear equations in one variable

Week 5: Sequences

This short block reinforces students' learning from the start of Year 7, extending this to look at sequences with more complex algebraic rules now that students are more familiar with a wider range of notation. The higher strand includes finding a rule for the n^{th} term for a linear sequence, using objects and images to understand the meaning of the rule.

National curriculum content covered:

- generate terms of a sequence from either a term-to-term or a position-to-term rule
- recognise arithmetic sequences and find the n^{th} term
- recognise geometric sequences and appreciate other sequences that arise

Week 6: Indices

Before exploring the ideas behind the addition and subtraction laws of indices (which will be revisited when standard form is studied next term), the groundwork is laid by making sure students are comfortable with expressions involving powers, simplifying e.g. $3x^2y \times 5xy^3$. The higher strand also looks at finding powers of powers.

National curriculum content covered:

- use and interpret algebraic notation, including a^3 in place of $a \times a \times a$; a^2b in place of $a \times a \times b$
- use language and properties precisely to analyse algebraic expressions
- begin to model situations mathematically and express the results using a range of formal mathematical representations
- substitute values in expressions, rearrange and simplify expressions, and solve equations

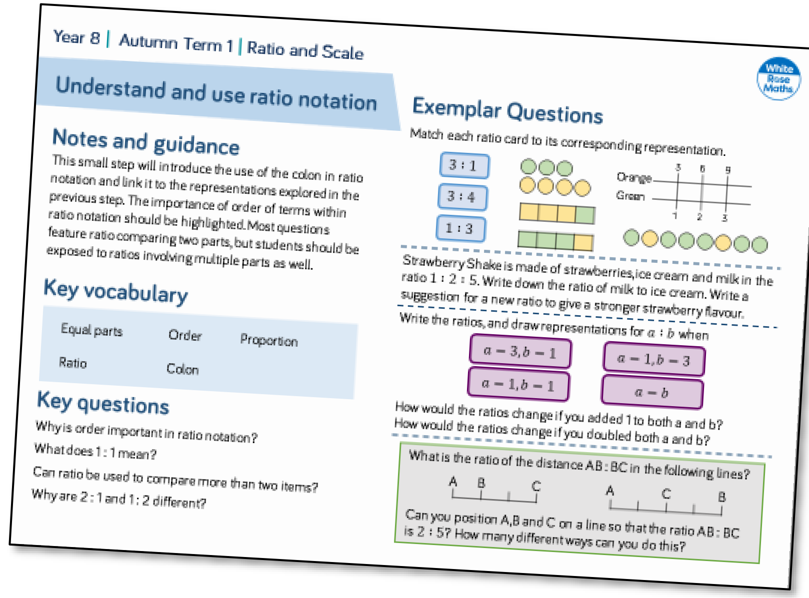
Why Small Steps?

We know that breaking the curriculum down into small manageable steps should help students to understand concepts better. Too often, we have noticed that teachers will try and cover too many concepts at once and this can lead to cognitive overload. We believe it is better to follow a "small steps" approach.

As a result, for each block of content in the scheme of learning we will provide a "small step" breakdown. ***It is not the intention that each small step should last a lesson – some will be a short step within a lesson, some will take longer than a lesson.*** We would encourage teachers to spend the appropriate amount of time on each step for their group, and to teach some of the steps alongside each other if necessary.

What We Provide

- Some ***brief guidance*** notes to help identify key teaching and learning points
- A list of ***key vocabulary*** that we would expect teachers to draw to students' attention when teaching the small step,
- A series of ***key questions*** to incorporate in lessons to aid mathematical thinking.
- A set of questions to help ***exemplify*** the small step concept that needs to be focussed on.


Year 8 | Autumn Term 1 | Ratio and Scale

Understand and use ratio notation

Notes and guidance

This small step will introduce the use of the colon in ratio notation and link it to the representations explored in the previous step. The importance of order of terms within ratio notation should be highlighted. Most questions feature ratio comparing two parts, but students should be exposed to ratios involving multiple parts as well.

Key vocabulary

Equal parts, Order, Proportion, Ratio, Colon

Key questions

Why is order important in ratio notation?
What does 1 : 1 mean?
Can ratio be used to compare more than two items?
Why are 2 : 1 and 1 : 2 different?

Exemplar Questions

Match each ratio card to its corresponding representation.

3 : 1
3 : 4
1 : 3

Orange
Green

Strawberry Shake is made of strawberries, ice cream and milk in the ratio 1 : 2 : 5. Write down the ratio of milk to ice cream. Write a suggestion for a new ratio to give a stronger strawberry flavour.

Write the ratios, and draw representations for $a : b$ when

$a = 3, b = 1$
$a = 1, b = 3$
$a = 1, b = 1$
$a = b$

How would the ratios change if you added 1 to both a and b?
How would the ratios change if you doubled both a and b?

What is the ratio of the distance AB : BC in the following lines?

Can you position A, B and C on a line so that the ratio AB : BC is 2 : 5? How many different ways can you do this?

- These include reasoning and problem-solving questions that are fully integrated into the scheme of learning. Depending on the attainment of your students, you many wish to use some or all of these exemplars, which are in approximate order of difficulty. Particularly challenging questions are indicated with the symbol .
- For each block, we also provide ideas for key representations that will be useful for all students.

In many of the blocks of material, some of the small steps are in **bold**. These are content aimed at higher attaining students, but we would encourage teachers to use these with as many students as possible – if you feel your class can access any particular small step, then please include it in your planning.

Indices

Small Steps

- Adding and subtracting expressions with indices
- Simplifying algebraic expressions by multiplying indices
- Simplifying algebraic expressions by dividing indices
- Using the addition law for indices
- Using the addition and subtraction law for indices
- **Exploring powers of powers** H

H denotes higher strand and not necessarily content for Higher Tier GCSE

+/− expressions with indices

Notes and guidance

Students sometimes mix up adding and subtracting when dealing with expressions involving indices; this step is to clarify the need for like terms in order to be able to add and subtract terms. Students may need reminding of the word 'coefficient' and the convention that we don't usually use 1 as a coefficient. Using manipulatives helps to explain why e.g. $2x^2 + 3x^2 \equiv 5x^2$ rather than $5x^4$

Key vocabulary

Expression	Simplify	Term
Coefficient	Index/Indices	Power(s)

Key questions

What is the difference between a term and an expression?

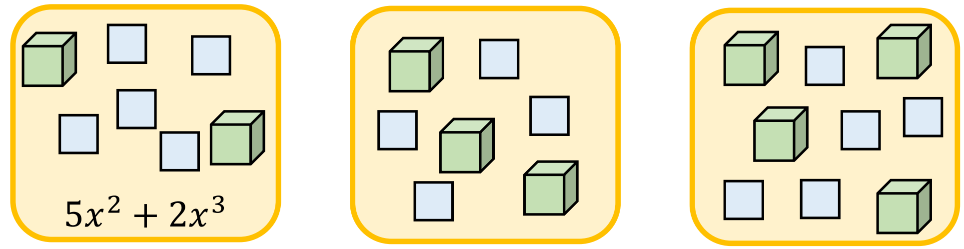
When are terms 'like terms'?

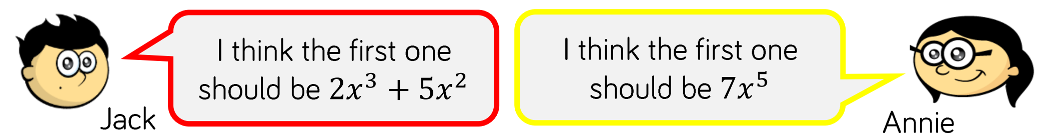
When can/can't an expression be simplified?

Why don't we usually write '$1x$' or '$0x$'?

Exemplar Questions

Each square represents x^2 and each cube represents x^3.
Write expressions for each card. The first one is done for you.





Do you agree with either student? Explain why or why not.

Simplify the expressions on the cards.

$3x^2 + 2x^2$ | $4x^3 + 5x^3$ | $6x^3 - 2x^3$ | $8x^2 - 3x^2$

$2x^3 + 7x + 5x^3 + 3x$ | $5x^3 + 6x^2 - 4x^2 - 2x^3$

Check Dexter's indices homework.
Which ones are definitely right/wrong?
Which ones might people disagree about?

Dexter's homework

$5a^2 + 2a^2 \equiv 7a^2$

$4b^3 - b^3 \equiv 4$

$4c^2 + 2c^2 - 6c^2 \equiv 0c^2$

$9d^3 - 6d^3 - 2d^3 \equiv 1d^3$

$5e^3 + 6e^2 - 2e^3 + e^2 \equiv 3e^3 + 7e^2$

Multiply expressions with indices

Notes and guidance

Students should already be aware of the conventions for simplifying expressions like $3 \times a$, $b \times 4$ and $c \times c$ from their work in Year 7; this step builds on this to include terms with more than one letter and several letters/numbers by considering the factors of each term. The formal rules of indices are dealt with later in this block, but within this step students should deal with squares, cubes and their products.

Key vocabulary

Multiply	Product	Power
Index/Indices	Expand	Simplify

Key questions

What does the word 'index' mean?
What is the result of multiplying x^2 by x? And then multiplying by x again? And again?
What is your strategy for multiplying e.g. $3a^2b$ and $5ab^3$? What do you look at first? Then what?

Exemplar Questions

Complete the working.

$5a \times 3b$
$\equiv 5 \times a \times 3 \times b$
$\equiv 5 \times 3 \times a \times b$
$\equiv$ ________

$7p \times 4q$
$\equiv 7 \times$ ___ $\times 4 \times$ ___
$\equiv 7 \times 4 \times$ ___ $\times$ ___
$\equiv$ ________

Match the expressions on the cards with their simplified forms.

$3a \times 2$	$a \times b$	$a \times 2b$	$3a \times 2b$
$2ab$	$6a$	$6ab$	ab

Correct Tommy's answer.

$3d \times 4d \equiv 12dd$

Tommy

Which of these is the correct simplification of $2t^3 \times 3t^2$?

$5t^5$ $\quad 5t^6$ $\quad 6t^5$ $\quad 6t^6$

Expand the brackets and simplify as far as possible.

$3x(y+z) + 5y(z+2x)$

$5pq(p+q) - 2q^2(p+p^2)$

$6a \times 3b \times 2a + 5ab(3b - 2a)$

Divide expressions with indices

Notes and guidance

This step will reinforce students' understanding of algebraic notation, particularly the use of fractional form to represent division. This is helpful here as the fractional form can help students identify the common factors more easily, and links to writing fractions in simplest form. Students may need to be reminded that it is expected to give answers in the form e.g. $\frac{y}{2}$ rather than involving decimals such as $0.5y$

Key vocabulary

Numerator	Denominator	Factor
Common factor	Coefficient	Simplify

Key questions

What is the difference between a term and an expression?

When can/can't an expression be simplified?

Exemplar Questions

Write these fractions in their simplest form.

$\frac{15}{60}$ $\frac{27}{60}$ $\frac{5 \times 7 \times 11}{7 \times 11 \times 13}$ $\frac{2 \times 3 \times 11}{3 \times 7 \times 11}$

$\frac{24}{60}$ $\frac{80}{360}$ $\frac{2 \times a \times b}{5 \times a}$ $\frac{3 \times c \times d}{3 \times d \times d}$

By thinking about the factors of the numerators and denominators, simplify these fractions.

$\frac{6xy}{2}$ $\frac{6xy}{x}$ $\frac{6xy}{2x}$ $\frac{6xy}{3y}$ $\frac{6x^2}{2x}$ $\frac{6x^2y}{3xy}$

Work out the divisions.

$18 \div 3$ $18a \div 3$ $18a \div 3a$

$18ab \div 3$ $18ab \div 3b$ $36ab \div 3ab$

$24a^2 \div 6a$ $30a^2b \div 6ab$ $30a^2b \div 5ab^2$

Compare the solutions to the calculation $3ab \times 4bc \div 24abc$.

$2c$ $0.5c$ $\frac{c}{2}$

Which one(s) do you agree with? Justify your answer.

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The addition law for indices

Notes and guidance

Through experimentation, students usually quickly see that multiplying terms of the form a^m and a^n gives the result a^{m+n}. Nonetheless, the sight of a multiplication sign often results in errors like $2^6 \times 2^2 = 2^{12}$ and it is helpful to include and discuss examples like this, and also noting the rule does not apply to different bases e.g. $2^3 \times 3^4 \neq 6^7$. Likewise, the convention of writing x rather than x^1 can result in errors.

Key vocabulary

Base	Index/Indices
Power	Exponent

Key questions

What is the difference between a base and an index?
How can you simplify the multiplication of two terms involving indices if they have the same base?
Can you use the same rule if the bases are different?
Why is e.g. $a^6 \times a = a^7$ when there is no index on the second term?

Exemplar Questions

Complete the calculations on the cards.

$$3^2 \times 3^4 \\ = (3 \times 3) \times (3 \times 3 \times 3 \times 3) \\ = 3^?$$

$$5^3 \times 5^5 \\ = (5 \times \cdots) \times (5 \times 5 \times \cdots) \\ = 5^?$$

In the same way, work out $2^5 \times 2^3$ and $10^6 \times 10^6$, giving your answers as single terms.

- What connections do you see between the questions and the answers?
- Predict the answers to $7^4 \times 7^8$ and $6^{11} \times 6^{12}$
- Compare your answers with a partner's & discuss your findings

Explain why Dexter is wrong.

$2^5 \times 2^6 = 4^{11}$

Dexter

Work out the missing values.

$2^7 \times 2^3 = 2^{\square}$	$a^7 \times a^3 = a^{\square}$	$a^7 \times a^9 = a^{\square}$
$2^4 \times 2^{\square} = 2^8$	$a^3 \times a^{\square} = a^8$	$a^{\square} \times a^3 = a^9$
$2^6 \times 2 = 2^{\square}$	$a^5 \times a = a^{\square}$	$a^x \times a^x = a^{16}$

$+/-$ laws for indices

Notes and guidance

This step develops from the last, illustrating that dividing expressions of the form a^m and a^n gives the result a^{m-n}. Common errors include not realising that a is the same as a^1 and mistakenly treating the exponent as 0. It is worth noting the difference between writing e.g. $6^5 \div 6^3$ as a single power and evaluating the result of $6^5 \div 6^3$. It is useful to mix up questions to include $\times, \div$ and both operations.

Key vocabulary

Base	Index/Indices
Power	Exponent

Key questions

What is the difference between a base and an index?
How can you simplify the multiplication of two terms involving indices if they have the same base?
Can you use the same rule if the bases are different?
Why is (e.g.) $a^6 \div a = a^5$ when there is no index on the second term?

Exemplar Questions

Complete the calculations on the cards.

$$2^6 \div 2^2 = \frac{2^6}{2^2} = \frac{2 \times 2 \times 2 \times 2 \times 2 \times 2}{2 \times 2} = 2^{\square}$$

$$3^7 \div 3^3 = \frac{3^7}{3^3} = \frac{3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3}{3 \times 3 \times 3} = 3^{\square}$$

In the same way, work out $6^5 \div 6^3$ and $9^8 \div 9^4$, giving your answers as single terms.

- What connections do you see between the questions and the answers?
- Predict the answers to $7^{10} \div 7^6$ and $10^{12} \div 10^2$
- Compare your answers with a partner's & discuss your findings

Whitney

$10 \div 5$ is 2, so
$2^{10} \div 2^5 = 2^2$

Explain why Whitney is wrong.

Work out the missing values.

$2^7 \div 2^3 = 2^{\square}$

$a^7 \div a^3 \equiv a^{\square}$

$a^8 \div a^2 \equiv a^{\square}$

$2^4 \times 2^3 \div 2^6 = 2^{\square}$

$b^8 \div b^4 \times b^{\square} \equiv b^4$

White Rose Maths

Exploring powers of powers

H

Notes and guidance

In this higher strand step, it is again quite easy to establish that $\left(a^b\right)^c = a^{bc}$, but this can sometimes then get confused with the addition of indices rules and lead to errors. It is useful for students to look at questions involving all three operations and consider the meaning of the calculations (and what they might look like if written in full) rather than relying on memorisation.

Key vocabulary

Base	Index/Indices	Power
Exponent	Product	

Key questions

How would you start solving an index question that involves more than one operation?

Will $\left(a^b\right)^c$ be the same as, or different from $(a^c)^b$? Why?

Why do we need to be careful with expressions like $(5x^4)^3$?

Exemplar Questions

Complete the calculations on the cards.

$(3^4)^2 = 3^4 \times 3^4 = 3^?$

$(5^6)^3 = 5^6 \times \_\_ \times \_\_ == 5^?$

Mo makes a conjecture.

Mo

When you raise a term with a power to another power, you multiply the indices e.g. $(6^3)^4 = 6^{3\times4} = 6^{12}$

Test Mo's conjecture by checking his example and testing three more of your own. Do you agree with Mo?

Work out the missing values.

$(2^7)^3 = 2^{\square}$

$(2^3)^7 = 2^{\square}$

$(5^5)^5 = 5^{\square}$

$(a^4)^{\square} \equiv a^8$

$(a^4)^{\square} \equiv a^{16}$

$\left(a^{\square}\right)^5 \equiv a^{40}$

Dani thinks $(2x^4)^3 \equiv 6x^{12}$. What mistake has Dani made?

Solve the equations.

$2^x \times 2^4 = 2^{12}$

$2^{12} \div 2^y = 2^3$

$(2^2)^z = 2^{12}$

$3^5 \times 3^6 \div 3^a = 3^{20}$

$3^{14} \div \left(3^b\right)^3 = 3 \times 3^3 \times 3$