Area of Trapezia and Circles

Year 8

#MathsEveryoneCan

White Rose Maths

	Week 1	Week 2	Week 3	Week 4	Week 5	Week 6	Week 7	Week 8	Week 9	Week 10	Week 11	Week 12
Autumn	**Proportional Reasoning**						**Representations**					
	Ratio and scale		Multiplicative change		Multiplying and dividing fractions		Working in the Cartesian plane			Representing data		Tables & Probability
Spring	**Algebraic Techniques**						**Developing Number**					
	Brackets, equations and inequalities				Sequences	Indices	Fractions and percentages		Standard index form		Number sense	
Summer	**Developing Geometry**						**Reasoning with Data**					
	Angles in parallel lines and polygons		Area of trapezia and circles		Line symmetry and reflection		The data handling cycle				Measures of location	

Summer 1: Developing Geometry

Weeks 1 and 2: Angles in parallel lines and polygons

This block builds on KS2 and Year 7 understanding of angle notation and relationships, extending all students to explore angles in parallel lines and thus solve increasingly complex missing angle problems. Links are then made to the closely connected properties of polygons and quadrilaterals. The use of dynamic geometry software to illustrate results is highly recommended, and students following the Higher strand will also develop their understanding of the idea of proof. They will also look start to explore constructions with rulers and pairs of compasses. This key block may take slightly longer than two weeks and the following blocks may need to be adjusted accordingly.
National Curriculum content covered includes:

- apply the properties of angles at a point, angles at a point on a straight line, vertically opposite angles
- understand and use the relationship between parallel lines and alternate and corresponding angles
- derive and use the sum of angles in a triangle and use it to deduce the angle sum in any polygon, and to derive properties of regular polygons
- use the standard conventions for labelling the sides and angles of triangle ABC
- derive and illustrate properties of triangles, quadrilaterals, circles, and other plane figures [for example, equal lengths and angles] using appropriate language and technologies
- derive and use the standard ruler and compass constructions (H only)

Weeks 3 and 4: Area of trapezia and circles

Students following the Higher strand will have met the formulae for the area of a trapezium in Year 7; this knowledge is now extended to all students, along with the formula for the area of a circle.

A key aspect of the unit is choosing and using the correct formula for the correct shape, reinforcing recognising the shapes, their properties and names, and looking explicitly at compound shapes.
National Curriculum content covered includes:

- derive and apply formulae to calculate and solve problems involving: perimeter and area of triangles, parallelograms, trapezia
- calculate and solve problems involving: perimeters of 2-D shapes (including circles), areas of circles and composite shapes

Weeks 5 and 6: Line symmetry and reflection

The teaching of reflection is split from that of rotation and translation to try and ensure students attain a deeper understanding and avoid mixing up the different concepts. Although there is comparatively little content in this block, it is worth investing time to build confidence with shapes and lines in different orientations. Students can revisit and enhance their knowledge of special triangles and quadrilaterals, and focus on key vocabulary such as object, image, congruent etc.
Rotation and translations will be explored in Year 9
National Curriculum content covered includes:

- describe, sketch and draw using conventional terms and notations: points, lines, parallel lines, perpendicular lines, right angles, regular polygons, and other polygons that are reflectively and rotationally symmetric
- identify properties of, and describe the results of, reflections applied to given figures

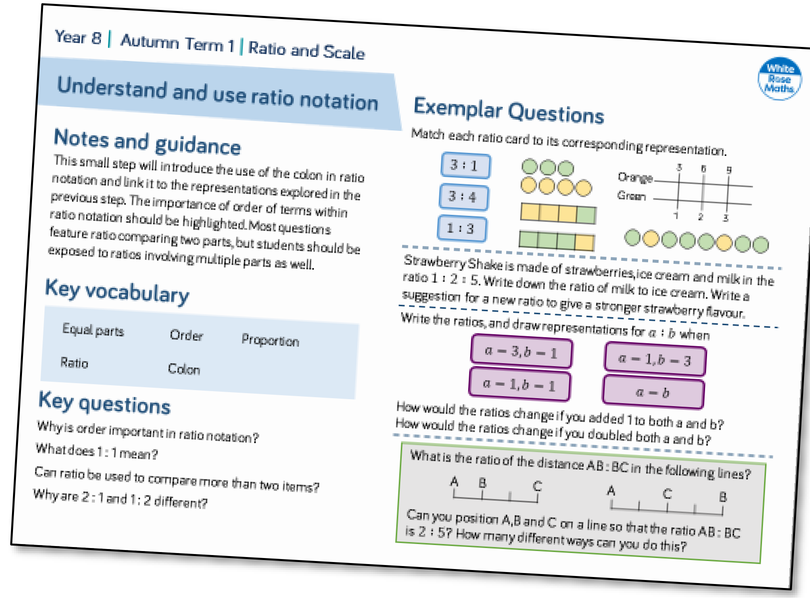
Why Small Steps?

We know that breaking the curriculum down into small manageable steps should help students to understand concepts better. Too often, we have noticed that teachers will try and cover too many concepts at once and this can lead to cognitive overload. We believe it is better to follow a "small steps" approach.

As a result, for each block of content in the scheme of learning we will provide a "small step" breakdown. ***It is not the intention that each small step should last a lesson – some will be a short step within a lesson, some will take longer than a lesson.*** We would encourage teachers to spend the appropriate amount of time on each step for their group, and to teach some of the steps alongside each other if necessary.

What We Provide

- Some ***brief guidance*** notes to help identify key teaching and learning points
- A list of ***key vocabulary*** that we would expect teachers to draw to students' attention when teaching the small step,
- A series of ***key questions*** to incorporate in lessons to aid mathematical thinking.
- A set of questions to help ***exemplify*** the small step concept that needs to be focussed on.


Year 8 | Autumn Term 1 | Ratio and Scale

Understand and use ratio notation

Notes and guidance

This small step will introduce the use of the colon in ratio notation and link it to the representations explored in the previous step. The importance of order of terms within ratio notation should be highlighted. Most questions feature ratio comparing two parts, but students should be exposed to ratios involving multiple parts as well.

Key vocabulary

Equal parts, Order, Proportion, Ratio, Colon

Key questions

Why is order important in ratio notation?
What does 1 : 1 mean?
Can ratio be used to compare more than two items?
Why are 2 : 1 and 1 : 2 different?

Exemplar Questions

Match each ratio card to its corresponding representation.

3 : 1
3 : 4
1 : 3

Orange
Green

Strawberry Shake is made of strawberries, ice cream and milk in the ratio 1 : 2 : 5. Write down the ratio of milk to ice cream. Write a suggestion for a new ratio to give a stronger strawberry flavour.

Write the ratios, and draw representations for $a : b$ when

$a = 3, b = 1$
$a = 1, b = 3$
$a = 1, b = 1$
$a = b$

How would the ratios change if you added 1 to both a and b?
How would the ratios change if you doubled both a and b?

What is the ratio of the distance AB : BC in the following lines?

Can you position A, B and C on a line so that the ratio AB : BC is 2 : 5? How many different ways can you do this?

- These include reasoning and problem-solving questions that are fully integrated into the scheme of learning. Depending on the attainment of your students, you many wish to use some or all of these exemplars, which are in approximate order of difficulty. Particularly challenging questions are indicated with the symbol .
- For each block, we also provide ideas for key representations that will be useful for all students.

In many of the blocks of material, some of the small steps are in **bold**. These are content aimed at higher attaining students, but we would encourage teachers to use these with as many students as possible – if you feel your class can access any particular small step, then please include it in your planning.

Area of Trapezia and Circles

Small Steps

- Calculate the area of triangles, rectangles and parallelograms R
- Calculate the area of a trapezium
- Calculate the perimeter and area of compound shapes (1)
- Investigate the area of a circle
- Calculate the area of a circle and parts of a circle without a calculator
- Calculate the area of a circle and parts of a circle with a calculator
- Calculate the perimeter and area of compound shapes (2)

H denotes Higher Tier GCSE content

R denotes 'review step' – content should have been covered earlier at KS3

Reviewing area

R

Notes and guidance

This small step revises and extends KS2 and KS3 work. Teachers might first check that students understand the links between the formulae for area of rectangle, triangle and parallelogram. A possible difficulty can be finding the perpendicular height when triangles are not in 'standard' orientations. Ensure students have exposure to these, and include questions that revisit unit conversions.

Key vocabulary

Formula	Area	Triangle
Square	Parallelogram	Rhombus

Key questions

Why is the formula to find the area of a rectangle the same as the formula to find the area of a parallelogram?

Why do we use the perpendicular height when finding the area of a triangle and not the sloping height?

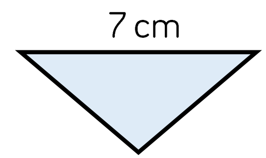
How can you find the area of a rhombus? How do you know?

Exemplar Questions

The large rectangle has been split into four smaller rectangles. Calculate the perimeter of the large rectangle.

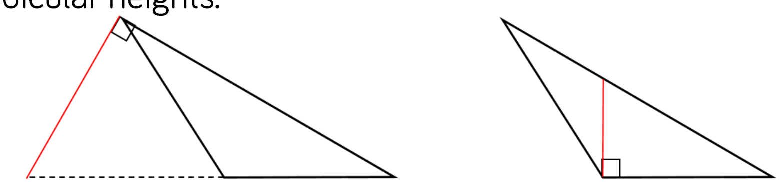
15 cm^2	21 cm^2
30 cm^2	42 cm^2

The area of this triangle is 21 cm^2



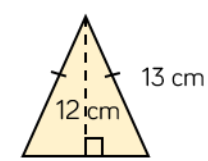
True or False?
The perpendicular height of this triangle = 3 cm
Explain your answer.

The red lines do not show the perpendicular heights of the triangles. Re-draw the diagrams, keeping the same base, to show the correct perpendicular heights.

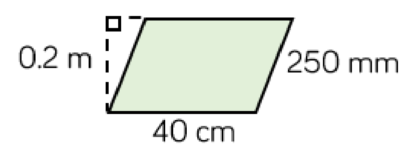


The perimeter of this isosceles triangle is 0.36 m.

Show that the area of the triangle is 60 cm^2.



Calculate the area of the parallelogram.



Calculate the area of a trapezium

Notes and guidance

Teachers might start by ensuring students can identify trapezia using different standard (e.g. isosceles trapezium) and non-standard (e.g. with two right angles) examples. Students can then explore the formula for the area of a trapezium by using congruent trapezia to form a parallelogram. They can then compare the formula with those for the area of other quadrilaterals.

Key vocabulary

Trapezium/Trapezia	Parallel
Perpendicular height	Formula

Key questions

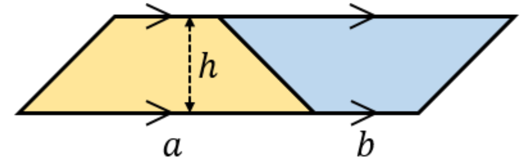
Compare a rectangle, parallelogram and trapezium. What's the same and what's different?

Why does the formula for the area of a trapezium also work if it is applied to parallelograms, rectangles and squares?

Are the parallel sides of a trapezium always horizontal?

Exemplar Questions

Dora places two congruent trapezia next to each other:

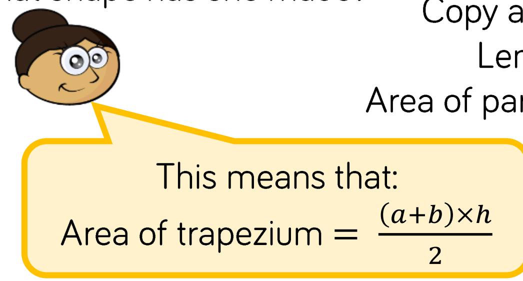


What shape has she made?

Copy and complete the following

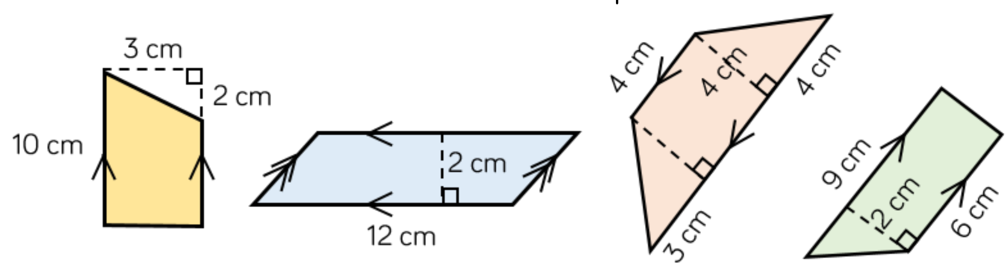
Length of base = $a +$ __

Area of parallelogram = $(a +$ __$) \times$ __



Explain why Dora is correct.

Give the mathematical name of each shape.



Find the areas of the shapes.
Did you use the same method for each one?

Draw a trapezium which has area of 24 cm^2 and

- a base of 10 cm and a height of 3 cm
- parallel sides of lengths 4 cm and 8 cm
- a height of 12 cm

Compound shapes (1)

Notes and guidance

Students sometimes simply multiply all of the dimensions marked in an attempt to find the area. Model splitting up different compound shapes before introducing students to compound shapes with dimensions labelled. Ensure students are aware that there are different methods of splitting compound shapes and that they should aim to be efficient.

Key vocabulary

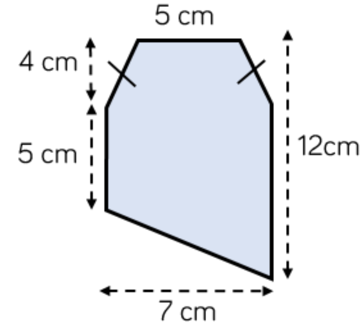
Compound	Component shapes	
Parallelogram	Trapezium	Perpendicular

Key questions

How can you divide this compound shape up into shapes we know how to find the area of? Name each of these shapes.

What length(s) do you need to substitute into your formula? Is this length given, or do you need to calculate it first? What is your strategy for find the missing length(s)?

Exemplar Questions



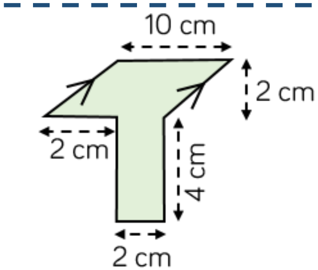
Mo is finding the area of this shape. Draw lines on the shape to show how he could split it into easier shapes. Show that there is more than one way of doing this.

Mo decides to split the shape into two trapezia, A and B. Here are his workings:

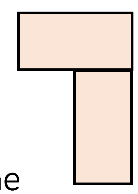
$$\text{Area A} = \frac{7+5}{2} \times 4$$
$$\text{Area B} = \frac{12+5}{2} \times 7$$

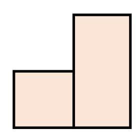
Identify and correct any mistakes that he has made and calculate the total area of the shape.

Show that the area of the shape is 28 cm^2.
What smaller shapes did you split the shape into?
Compare the methods in your class.
Which method was the most efficient?



A square of perimeter of 48 cm is cut in half and the two halves are put together to make this shape. Calculate the perimeter and area of this shape.





A new shape is made using the same two rectangles as shown.
Calculate the area and perimeter of this shape.

Investigate the area of a circle

Notes and guidance

Students explore the area of a circle by cutting up circles into sectors and placing them in an arrangement to resemble a parallelogram or (with more sectors) a rectangle. They might need teacher guidance to notice that as the number of sectors increases, the shape that they can make becomes more rectangular. They then use a known area formula to deduce the area of a circle.

Key vocabulary

Sector	Parallelogram	Rectangle	
Estimate	Infinity	Radius	π

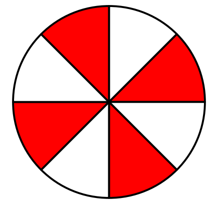
Key questions

Where is the radius of the circle?
How do we find the circumference of a circle?
How do we find the area of a parallelogram?

As the number of sectors increases, is our estimate for the area more or less accurate? Explain why.

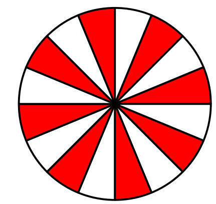
What does this tell you about the area of a circle?

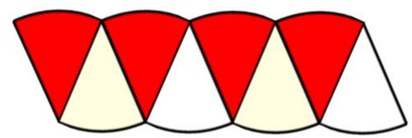
Exemplar Questions

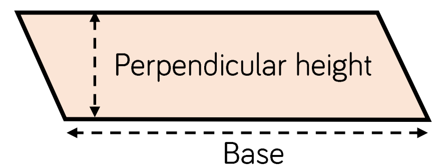


Cut up the circle so that you have 8 separate sectors. Can you place the sectors adjacent to each other to make a shape that it roughly like a parallelogram?

Repeat this using a circle divided into 16 sectors. Compare the two shapes that you have made. What would happen if you used a circle split into 32 sectors, or 3 200 sectors?







Ron compares the diagram he made using sectors of a circle to a parallelogram. He deduces:

Perpendicular height $=$ radius $= r$

Base $=$ half of the circumference of the circle $= \frac{\pi d}{2} = \pi r$

Explain why Ron is correct.

Ron says,

Area of parallelogram $= r \times \pi \times r$
$= \pi r^2$

What does this tell you about the area of a circle? Explain why.

White Rose Maths

Area of a circle without a calculator

Notes and guidance

This small step focusses on how to estimate the area of a circle using the approximations $\pi = 3$ and $\pi = \frac{22}{7}$. It then builds to calculating the exact area of a circle, leaving answers in terms of π. Starting with a recap on squaring and the order of operations can avoid later issues in calculations, so that students can concentrate on new learning.

Key vocabulary

Approximately	Estimate	Diameter
Radius	In terms of π	

Key questions

How do you round a number to 1 significant figure?

Use a calculator to change $\frac{22}{7}$ into a decimal. What do you notice when you compare this to π?

How do I know whether to substitute the radius or the diameter? What mistake do you think people often make?

Exemplar Questions

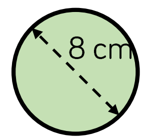
4×3^2

$4 \times 3 = 12$
$12^2 = 144$

$4 \times 9 = 36$

$4 \times 6 = 36$

Which calculation is correct?
What mistakes have been made in the other calculations?



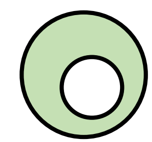
Whitney rounds π to 1 significant figure to estimate the area of the circle.

$\pi = 3.1415\ldots$

Area $\approx 3 \times 8^2 = 192$ cm^2

What mistake has Whitney made?

Whitney then cuts a circular hole out of the circle.
The radius of the circular hole is 3 cm.
Estimate the area of the remaining shape.



Use $\pi \approx \frac{22}{7}$ to estimate the area of a circle of radius 7 cm.

Jack

I can find the exact area of this circle without using a calculator by leaving my answer in terms of π

$$\begin{aligned}\text{Area of this circle} &= \pi \times 7^2 \\ &= \pi \times 49 \\ &= 49\pi \text{ cm}^2\end{aligned}$$

Use Jack's method to work out the exact areas of the shapes.

Area of a circle with a calculator

Notes and guidance

Before considering the area of a circle, students might need to practise entering squares of numbers and π into their calculators. Students need to confidently identify the diameter and radius of a circle and know which to substitute into the formula for area. They also may need a reminder of rounding to an appropriate number of decimal places or significant figures.

Key vocabulary

Decimal place	Estimate	Calculate
Substitute	Significant figures	

Key questions

Where is the π key on your calculator? How do you enter e.g. 3^2 into your calculator? Is there more than one way of doing this?

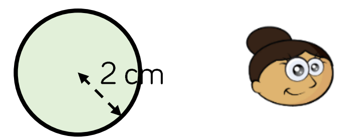
Why is it useful to firstly calculate an estimate of the area?

How many decimal places or significant figures should you round your answer to? Why?

Exemplar Questions

Use your calculator to work out these values.

- 11^2
- $\pi \times 121$
- $\pi \times 11^2$
- $\pi \times r^2$ when $r = 4$



The area of the circle is about 12 cm^2

Explain why Dora's estimate is a good one.

Choose the calculation which will work out the exact area.

Explain your choice.

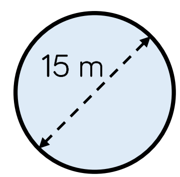
- $\pi \times 4^2$
- $\pi \times 4$
- $\pi \times 2^2$
- $(\pi \times 2)^2$

Use your calculator to work out the area of the circle.

Compare your answer to the estimate.

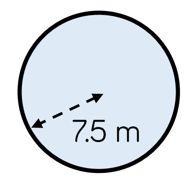
Is your answer likely to be correct?

Calculate the four areas, rounding your answers to 1 decimal place.

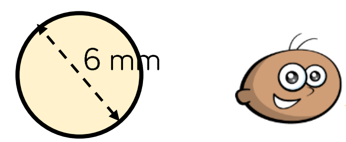


A circle of diameter 12.8 cm

A circle of radius 6.4 cm



What do you notice about your answers?



The area of half the circle is $\pi \times 3^2$

Is Tommy correct? Justify your answer.

Compound shapes (2)

Notes and guidance

In this small step, students are encouraged to identify standard shapes within compound shapes. When two semi-circles are involved, they might discuss whether they can use the formulae for a whole circle, rather than performing separate calculations for each. Students should identify the dimensions required in a formula, and then think about how to calculate these if they are not given.

Key vocabulary

Compound	Component shapes	
Parallelogram	Trapezium	Perpendicular

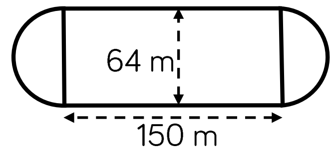
Key questions

Do we need to work out the area/arc length of each semi-circle separately? Why or why not?

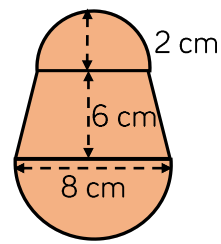
Which standard shapes can you identify in the compound shape?

Identify the dimensions you need to be able to calculate the area. How can you work out the missing ones?

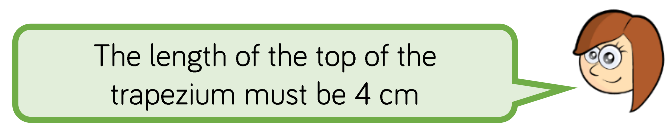
Exemplar Questions



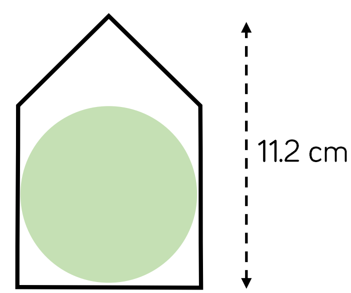
The diagram shows a running track.
To be able to use this for regional school competitions, the perimeter of the track needs to be at least 400 m.
Determine if the track can be used for regional competitions, justifying your answer.



Here is the outline of a logo.
The logo consists of two semi-circles and a trapezium.



Explain why both Rosie and Jack are correct.
Calculate the total area of the logo.



The shape is made up of a square and triangle.
The circle touches the sides of the square and has radius 3.8 cm.

What percentage of total area of the shape is shaded?