Functions

Year 11

#MathsEveryoneCan

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WRM – Year 11 Scheme of Learning

	Week 1	Week 2	Week 3	Week 4	Week 5	Week 6	Week 7	Week 8	Week 9	Week 10	Week 11	Week 12
Autumn	**Graphs**						**Algebra**					
	Gradients & lines		Non-linear graphs		Using graphs		Expanding & Factorising		Changing the subject		Functions	
Spring	**Reasoning**						**Revision and Communication**					
	Multiplicative		Geometric		Algebraic		Transforming & Constructing		Listing & describing		Show that...	
Summer	**Revision**						**Examinations**					

Autumn 2: Algebra

Weeks 1 and 2: Expanding and factorising

This block reviews expanding and factorising with a single bracket before moving on to quadratics. The use of algebra tiles to develop conceptual understanding is encouraged throughout. Context questions are included to revisit e.g. area and Pythagoras' theorem.

National Curriculum content covered includes:

- know the difference between an equation and an identity; argue mathematically to show algebraic expressions are equivalent, and use algebra to support and construct arguments **{and proofs}**
- simplify and manipulate algebraic expressions by: factorising quadratic expressions of the form $x^2 + bx + c$, including the difference of two squares; **{factorising quadratic expressions of the form $ax^2 + bx + c$}**
- know the difference between an equation and an identity; solve quadratic equations **{including those that require rearrangement}** algebraically by factorising, **{by completing the square and by using the quadratic formula}**
- identify and interpret roots; deduce roots algebraically **{and turning points by completing the square}**
- solve two simultaneous equations in two variables (linear/linear **{or linear/quadratic}**) algebraically; find approximate solutions using a graph

Weeks 3 and 4: Changing the subject

Students consolidate and build on their study of changing the subject in Year 9. The block begins with a review of solving equations and inequalities before moving on to rearrangement of both familiar and unfamiliar formulae. Checking by substitution is encouraged throughout. Higher tier students also study solving equations by iteration.

National Curriculum content covered includes:

- solve linear inequalities in one variable
- know the difference between an equation and an identity; argue mathematically to show algebraic expressions are equivalent, and use algebra to support and construct arguments **{and proofs}**
- translate simple situations or procedures into algebraic expressions or formulae; derive an equation (or two simultaneous equations), solve the equation(s) and interpret the solution
- **{find approximate solutions to equations numerically using iteration}**

Weeks 5 and 6: Functions

As well as introducing formal function notation, this block brings together and builds on recent study of quadratic functions and graphs. This is also an opportunity to revisit trigonometric functions, first studied at the start of Year 10.

National Curriculum content covered includes:

- where appropriate, interpret simple expressions as functions with inputs and outputs; **{interpret the reverse process as the 'inverse function'; interpret the succession of two functions as a 'composite function'}**
- solve two simultaneous equations in two variables (linear/linear **{or linear/quadratic}**) algebraically; find approximate solutions using a graph
- identify and interpret roots; deduce roots algebraically **{and turning points by completing the square}**
- solve linear inequalities in one **{or two}** variable**{s}**, **{and quadratic inequalities in one variable}**; represent the solution set on a number line, **{using set notation and on a graph}**
- recognise, sketch and interpret graphs of quadratic functions
- apply Pythagoras' Theorem and trigonometric ratios to find angles and lengths in right-angled triangles **{and, where possible, general triangles}** in two **{and three}** dimensional figures

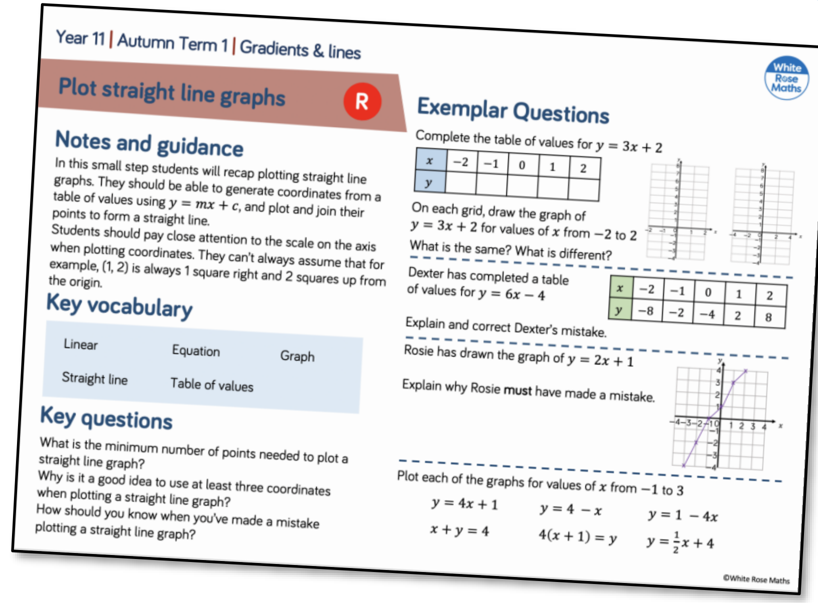
Why Small Steps?

We know that breaking the curriculum down into small manageable steps should help students to understand concepts better. Too often, we have noticed that teachers will try and cover too many concepts at once and this can lead to cognitive overload. We believe it is better to follow a "small steps" approach.

As a result, for each block of content in the scheme of learning we will provide a "small step" breakdown. ***It is not the intention that each small step should last a lesson – some will be a short step within a lesson, some will take longer than a lesson.*** We would encourage teachers to spend the appropriate amount of time on each step for their group, and to teach some of the steps alongside each other if necessary.

What We Provide

- Some ***brief guidance*** notes to help identify key teaching and learning points
- A list of ***key vocabulary*** that we would expect teachers to draw to students' attention when teaching the small step,
- A series of ***key questions*** to incorporate in lessons to aid mathematical thinking.
- A set of questions to help ***exemplify*** the small step concept that needs to be focussed on.



Year 11 | Autumn Term 1 | Gradients & lines

Plot straight line graphs R

Notes and guidance

In this small step students will recap plotting straight line graphs. They should be able to generate coordinates from a table of values using $y = mx + c$, and plot and join their points to form a straight line.
Students should pay close attention to the scale on the axis when plotting coordinates. They can't always assume that for example, (1, 2) is always 1 square right and 2 squares up from the origin.

Key vocabulary

Linear, Equation, Graph, Straight line, Table of values

Key questions

What is the minimum number of points needed to plot a straight line graph?
Why is it a good idea to use at least three coordinates when plotting a straight line graph?
How should you know when you've made a mistake plotting a straight line graph?

Exemplar Questions

Complete the table of values for $y = 3x + 2$

x	-2	-1	0	1	2
y					

On each grid, draw the graph of $y = 3x + 2$ for values of x from -2 to 2
What is the same? What is different?

Dexter has completed a table of values for $y = 6x - 4$

x	-2	-1	0	1	2
y	-8	-2	-4	2	8

Explain and correct Dexter's mistake.

Rosie has drawn the graph of $y = 2x + 1$

Explain why Rosie **must** have made a mistake.

Plot each of the graphs for values of x from -1 to 3

$y = 4x + 1$ $\quad y = 4 - x \quad y = 1 - 4x$

$x + y = 4 \quad 4(x + 1) = y \quad y = \frac{1}{2}x + 4$

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- These include reasoning and problem-solving questions that are fully integrated into the scheme of learning. Depending on the attainment of your students, you many wish to use some or all of these exemplars, which are in approximate order of difficulty. Particularly challenging questions are indicated with the symbol .
- For each block, we also provide ideas for key representations that will be useful for all students.

Some of the small steps are in **bold** and labelled with H to indicate this is Higher tier GCSE content. We would encourage teachers to use these with as many students as possible – if you feel your class can access any particular small step, then please include it in your planning. Steps that review content covered at Key Stage 3 are labelled R.

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Functions

Small Steps

- Use function machines (R)
- Substitution into expressions and formulae (R)
- Use function notation
- **Work with composite functions** (H)
- **Work with inverse functions** (H)
- Graphs of quadratic functions
- **Solve quadratic inequalities** (H)
- Understand and use trigonometric functions (R)

H	denotes Higher Tier GCSE content
R	denotes 'review step' – content should have been covered at KS3

Use function machines

R

Notes and guidance

Students will recap using function machines in order to aid their understanding when moving onto more abstract functions in later steps. It's important that they can find an output for a given input, and also an input for a given output in both one- and two-step function machines. The link between a numerical input/output and an algebraic input/output of a given function machine is essential knowledge for this block.

Key vocabulary

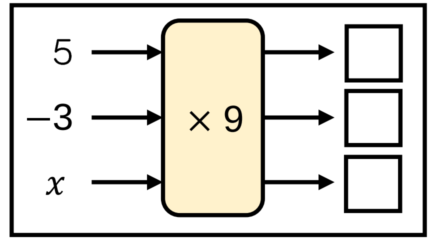
Input	Output	Function
Operation	Inverse	Variable

Key questions

If the input is ___, what is the output? How do you know?
If the output is ___, what is the input? How do you know?
How can you check your answer?
How do you calculate the input given the output?
What is the difference between $2x + 3$ and $2(x + 3)$?

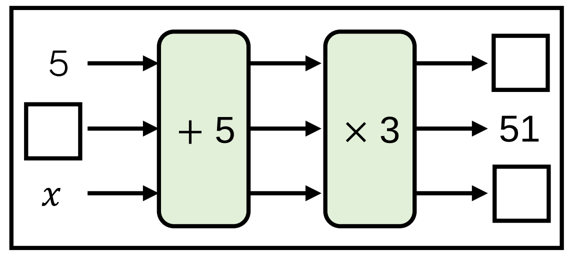
Exemplar Questions

Work out the missing outputs for the function machine.



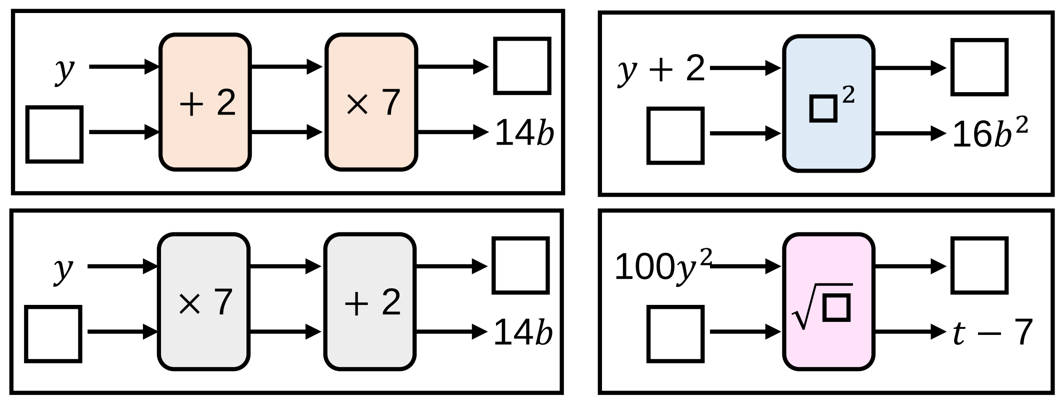
- Work out the value of $9x$ when $x = 5$
- Work out the value of $9x$ when $x = -3$
- What do you notice?

Work out the missing inputs and outputs for the function machine.



- Evaluate $3(x + 5)$ when $x = 5$
- Solve $3(x + 5) = 51$
- What do you notice?

Work out the missing inputs and outputs for the function machines.



Substitution

R

Notes and guidance

This small step provides opportunity for students to revise substituting into expressions and formulae. There is also plenty of opportunity to recap other areas of the curriculum such as fractions, decimals, directed numbers, area and volume etc. as meets your students' needs. It is useful to explore misconceptions such as $2x^2 = (2x)^2$. Students should be exposed to examples involving two or more variables.

Key vocabulary

Evaluate	Substitute	Expression
Formulae	Variable	

Key questions

What does it mean to substitute a value?

What is the difference between the expressions $x - 7$ and $7 - x$?

How can you use substitution to show that $x = 5$ is the solution to the equation $3x - 9 = 6$?

Choose values for x to show that $5x^2 \neq (5x)^2$

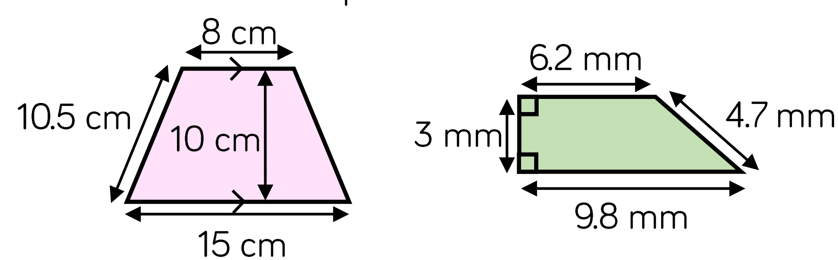
Exemplar Questions

Evaluate each expression when $x = 6$

- $5x + 11$
- $30 - 2x$
- $\frac{x+7}{4}$
- $6(x - 1)$
- x^2
- $2x^2$
- $(2x)^2$
- $x^2 + 5x - 3$

The area A of a trapezium is given by the formula $A = \frac{1}{2}(a + b)h$ where a and b are the lengths of the parallel sides, and h is the perpendicular distance between them.

Work out the area of each trapezium.



Speed (S), distance (D) and time (T) are connected by the formula $S = \frac{D}{T}$.

A car travels 420 miles in 8 hours. Work out its speed in mph.

Use substitution to show that $y = 5$ and $y = -2$ are solutions to the equation $y^2 - 3y - 10 = 0$

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Use function notation

Notes and guidance

In this small step, students are introduced to formal function notation for the first time. The most common function notation is $f(x)$ which reads 'f of x'. $f(x)$ is a function applied to x, and $f(5)$ for example would be worked out by substituting $x = 5$ into the function. Students should also be aware that other letters can be used, with different letters used to distinguish between different functions within the same question.

Key vocabulary

Function   Variable   Evaluate   Solve

Key questions

What's the difference between $f(x)$ and $f(2)$?
What's the difference between $f(x)$ and $f(a)$?
What's the difference between $f(x)$ and $g(x)$?
If you know that $h(x) = 3x + 7$, how can you work out $h(2x)$?
If you know that $f(x) = 12$, what else do you know?

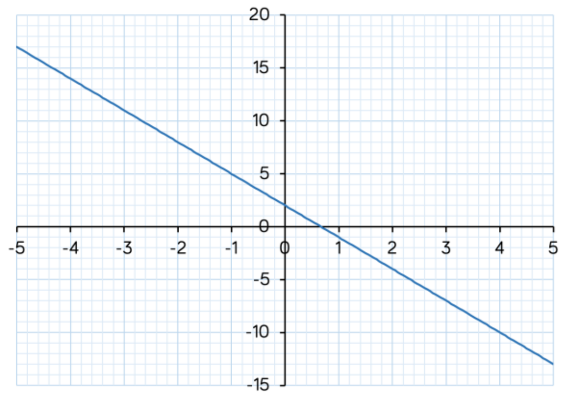
Exemplar Questions

The function $f(x)$ is defined by $f(x) = 5x + 2$
Work out

- $f(3)$
- $f(-4)$
- $f(0.5)$
- $f(\frac{1}{5})$
- $f(2a)$

How does this relate to finding points on the line $y = 5x + 2$?

A function, $g(x)$, is shown on the graph.



- Find $g(4)$
- Find $g(0)$
- Find $g(-2)$
- Solve $g(x) = 11$

The function $h(x)$ is defined by $h(x) = \frac{x+3}{7}$

- Solve $h(x) = \frac{5}{7}$
- Solve $h(x) = -9$

The function $f(x)$ is defined by $f(x) = x^2 - 9$

- Evaluate $f(2x)$
- Evaluate $f(x + 5)$
- Solve $f(x) = 0$

Composite functions

H

Notes and guidance

A composite function is a function made of other functions, where the output of one is the input of the other. For example, $fg(x)$ reads as 'f of g of x' and students will first need to evaluate $g(x)$ before substituting the result into $f(x)$. The order is important, and students should explore the difference between $fg(x)$ and $gf(x)$, knowing they should "work form the middle" when evaluating composite functions.

Key vocabulary

Function	Variable	Evaluate
Solve	Composite	Substitute

Key questions

What does $fg(x)$ mean?
What does $gf(x)$ mean?
What does $ff(x)$ mean?
Calculate $fg(5)$ and $gf(5)$. What's the same? What's different?
If $f(2) = 6$ and $g(6) = 17$, what is $gf(2)$?

Exemplar Questions

Given that $f(x) = 7x + 11$ and $g(x) = 10 - x$, work out

- $f(2)$
- $g(2)$
- $fg(2)$
- $gf(2)$
- $ff(2)$
- $gg(2)$

$g(b) = 5 + 3b$ $\qquad$ $h(b) = 2b^2$

Tommy and Eva are working out $hg(b)$ but they have each made a mistake. Spot the mistakes and work out $hg(b)$.

Tommy

$$hg(b) = 5 + 3(2b^2) = 5 + 6b^2$$

Eva

$$hg(b) = 2(5 + 3b)^2 = 2(25 + 9b^2) = 50 + 18b^2$$

$f(x) = 4x - 13$ $\qquad$ $g(x) = 15 - 8x$ $\qquad$ $h(x) = x^2 - 36$

Find expressions for:

- $fg(x)$
- $gf(x)$
- $fh(x)$
- $hf(x)$
- $gh(x)$
- $hg(x)$

Solve the equations:

- $fg(x) = 30$
- $fg(x) = gf(x)$

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Inverse functions

H

Notes and guidance

In this small step students will be introduced to inverse functions, making the link to inverse operations. The inverse of $f(x)$ is denoted $f^{-1}(x)$ and can be confused with the reciprocal x^{-1}. Students need to be secure in rearranging formula before looking at inverse functions. Working backwards through function machines is suitable for simple cases, but not for more complex cases.

Key vocabulary

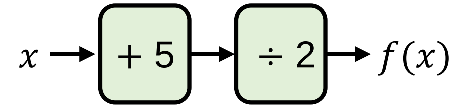
Function	Variable	Evaluate
Solve	Inverse	Rearrange

Key questions

What is an inverse operation?
If $y = x + 9$, how would you work out x given y?
What is an inverse function?
If $f(7) = 19$, what is $f^{-1}(19)$? How do you know?
Work out $ff^{-1}(x)$ and $f^{-1}f(x)$. What do you notice? Will this always happen?

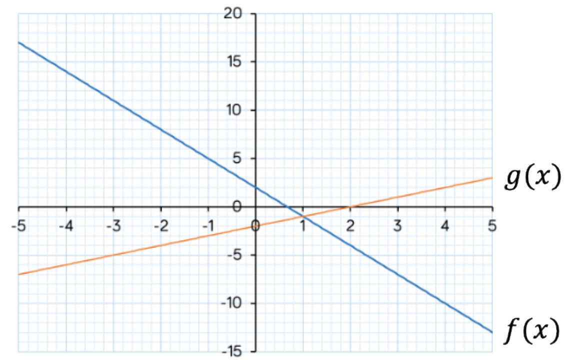
Exemplar Questions

Here is a function machine.



- Write an expression for $f(x)$
- If the output is 19, what was the input?
- Work out $f^{-1}(x)$

Two functions, $f(x)$ and $g(x)$ are shown on the graph.



- Solve $f(x) = g(x)$
- Find an expression for $f^{-1}(x)$
- Find an expression for $g^{-1}(x)$
- Solve $f^{-1}(x) = g^{-1}(x)$

$$g(x) = \frac{7x-1}{2}$$

- Find $g^{-1}(x)$
- Find $gg^{-1}(x)$
- Find $g^{-1}g(x)$

Given that $h(x) = \frac{5x+2}{x+4}$, find an expression for $h^{-1}(x)$

Graphs of quadratic functions

Notes and guidance

This small step consolidates quadratic graphs. All students should be able to recognise and plot the graph of a quadratic function. They need to be able to estimate solutions and identify the coordinates of the turning point. Students sitting Higher tier GCSE should also be able to identify the turning point by completing the square, recognising the turning point of $y = (x + a)^2 + b$ has coordinates $(-a, b)$.

Key vocabulary

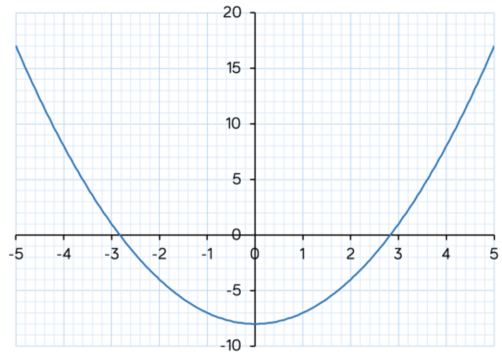
Quadratic	Function	Graph
Intercept	Turning point	Roots

Key questions

How do you recognise the graph of a quadratic function?
How many turning points will it have?
At what point will the graph of $y = x^2 + 5x - 1$ intercept the y-axis?
How do you identify the turning point from the completed square form?

Exemplar Questions

The function $f(x) = x^2 - 8$ is shown on the graph.



- Is the function linear, quadratic, cubic or other? How do you know?
- What are the coordinates of the turning point of the graph?

Complete the table of values for $y = x^2 + 3x - 1$

x	-3	-2	-1	0	1	2	3
y							

- Plot the graph of $y = x^2 + 3x - 1$
- Identify the turning point of the graph of $y = x^2 + 3x - 1$
- Estimate the solutions to $x^2 + 3x - 1 = 0$

By writing each equation in the form $y = (x + a)^2 + b$, identify the coordinates of the turning point of each quadratic function.

$y = x^2 + 8x - 7$ $y = x^2 - 12x + 1$ $y = x^2 - 7x$

Solve quadratic inequalities H

Notes and guidance

This topic was previously covered in the Autumn term of year 10. Here it provides opportunities for students to consolidate factorising, and then link their factorisation to the solution set. They need to be able to represent their solutions on a graph, a number line and using set notation. Look out for erroneous statements such as "$x < -3$ and $x > 3$", as x cannot satisfy both conditions at once.

Key vocabulary

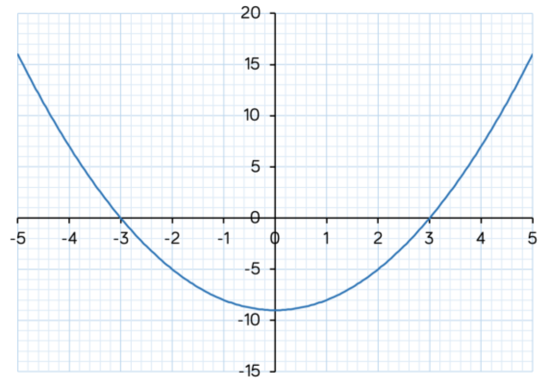
Quadratic	Inequality	Solve
Represent	Set	Solution

Key questions

How do you identify the region on a graph which shows where $y < 0$? So how can you find the values of x for which $y < 0$?
What is the difference between > and ≥?
How do you represent your solutions on a number line?
Why will $x^2 + 23$ be positive for all values of x?

Exemplar Questions

The function $f(x) = x^2 - 9$ is shown on the graph.



- Shade the region on the graph where $f(x) < 0$
- Shade the region on the graph where $f(x) > 0$
- Solve $x^2 - 9 \leq 0$

A function, g, is given by $g(x) = 2x^2 - 7x - 30$

- Sketch $g(x)$ highlighting any roots and intercepts.
- Solve $g(x) > 0$ giving your answer in set notation.

Amir and Mo are solving $12 - x - x^2 > 0$
Mo says "The solution is $-4 < x < 3$"
Amir says "The solution is $x < -4$ and $x > 3$"
They are both incorrect.
Explain any mistakes and work out the correct solution.
Show your solution on a number line.

Solve $3x^2 + 32x + 72 \leq 27$

Trigonometric functions

R

Notes and guidance

This step provides a timely opportunity to remind students how to find missing sides and angles in right-angles triangles, relating the trigonometric ratios to the corresponding functions, last studied in Year 10. Depending on your class' needs, you might also remind them of exact trig values. Higher tier students could also revise using the sine and cosine rule. You could use Year 10 worksheets to support this step.

Key vocabulary

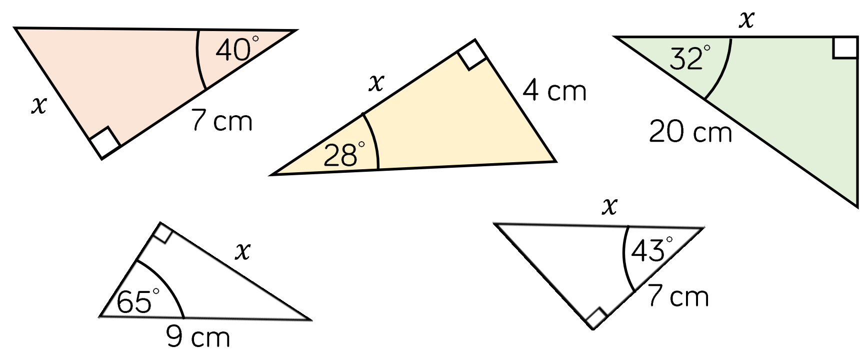
Ratio	Sine	Cosine	Tangent
Opposite	Adjacent	Hypotenuse	

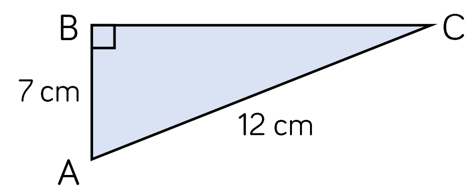
Key questions

How do you decide which ratio to use to find a missing side or angle in a right-angled triangle?
How do you know which side is which?
How can I use trigonometry in a rectangle or a non-right-angled isosceles triangle?
What's the difference between $\sin x$ and $\sin^{-1} x$?

Exemplar Questions

Which trigonometric ratio would you use to find the sides labelled x in each right-angled triangle?





- Calculate the size of angle BAC
- How can you calculate the length BC without using trigonometry?

Work out the size of:

- angle BAD
- angle ABC

Find the area of the trapezium.