Geometric Reasoning

# Year 11

#MathsEveryoneCan

White Rose Maths

| | Week 1 | Week 2 | Week 3 | Week 4 | Week 5 | Week 6 | Week 7 | Week 8 | Week 9 | Week 10 | Week 11 | Week 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Autumn | **Graphs** | | | | | | **Algebra** | | | | | |
| | Gradients & lines | | Non-linear graphs | | Using graphs | | Expanding & Factorising | | Changing the subject | | Functions | |
| Spring | **Reasoning** | | | | | | **Revision and Communication** | | | | | |
| | Multiplicative | | Geometric | | Algebraic | | Transforming & Constructing | | Listing & describing | | Show that… | |
| Summer | **Revision** | | | | | | **Examinations** | | | | | |

# Spring 1 : Reasoning

## Weeks 1 and 2: Multiplicative Reasoning

Students develop their multiplicative reasoning in a variety of contexts, from simple scale factors through to complex equations involving direct and inverse proportion. They link inverse proportion with the formulae for pressure and density. There is also the opportunity to review ratio problems.

National Curriculum content covered includes:

- compare lengths, areas and volumes using ratio notation and/or scale factors; make links to similarity
- understand that X is inversely proportional to Y is equivalent to X is proportional to $\frac{1}{Y}$
- **{construct and}** interpret equations that describe direct and inverse proportion
- extend and formalise their knowledge of ratio and proportion, including trigonometric ratios, in working with measures and geometry, and in working with proportional relations algebraically and graphically

## Weeks 3 and 4: Geometric Reasoning

Students consolidate their knowledge of angles facts and develop increasingly complex chains of reasoning to solve geometric problems. Higher tier students revise the first four circle theorems studied in Year 10 and learn the remaining theorems. Students also revisit vectors and the key topics of Pythagoras' theorem and trigonometry.

National Curriculum content covered includes

- reason deductively in geometry, number and algebra, including using geometrical constructions
- **{apply and prove the standard circle theorems concerning angles, radii, tangents and chords, and use them to prove related results}**
- interpret and use bearings
- apply addition and subtraction of vectors, multiplication of vectors by a scalar, and diagrammatic and column representations of vectors; **{use vectors to construct geometric arguments and proofs}**

## Weeks 5 and 6: Algebraic Reasoning

Students develop their algebraic reasoning by looking at more complex situations. They use their knowledge of sequences and rules to made inferences, and Higher tier students move towards formal algebraic proof. Forming and solving complex equations, including simultaneous equations, is revisited. Higher tier students also look at solving inequalities in more than one variable.

National Curriculum content covered includes:

- make and test conjectures about the generalisations that underlie patterns and relationships; look for proofs or counter-examples; begin to use algebra to support and construct arguments **{and proofs}**
- deduce expressions to calculate the $n^{\text{th}}$ term of linear **{and quadratic}** sequences
- solve two simultaneous equations in two variables (linear/linear **{or linear/quadratic}**) algebraically; find approximate solutions using a graph
- solve linear inequalities in one **{or two}** variable**{s}**, **{and quadratic inequalities in one variable}**; represent the solution set on a number line, **{using set notation and on a graph}**

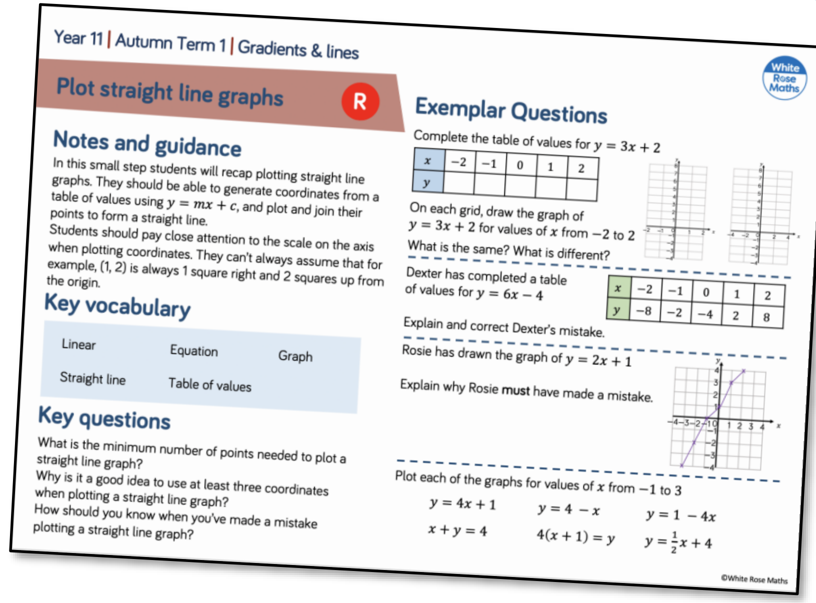
# Why Small Steps?

We know that breaking the curriculum down into small manageable steps should help students to understand concepts better. Too often, we have noticed that teachers will try and cover too many concepts at once and this can lead to cognitive overload. We believe it is better to follow a "small steps" approach.

As a result, for each block of content in the scheme of learning we will provide a "small step" breakdown. ***It is not the intention that each small step should last a lesson – some will be a short step within a lesson, some will take longer than a lesson.*** We would encourage teachers to spend the appropriate amount of time on each step for their group, and to teach some of the steps alongside each other if necessary.

# What We Provide

- Some ***brief guidance*** notes to help identify key teaching and learning points
- A list of ***key vocabulary*** that we would expect teachers to draw to students' attention when teaching the small step,
- A series of ***key questions*** to incorporate in lessons to aid mathematical thinking.
- A set of questions to help ***exemplify*** the small step concept that needs to be focussed on.


Year 11 | Autumn Term 1 | Gradients & lines

## Plot straight line graphs R

### Notes and guidance

In this small step students will recap plotting straight line graphs. They should be able to generate coordinates from a table of values using $y = mx + c$, and plot and join their points to form a straight line.
Students should pay close attention to the scale on the axis when plotting coordinates. They can't always assume that for example, (1, 2) is always 1 square right and 2 squares up from the origin.

### Key vocabulary

Linear, Equation, Graph, Straight line, Table of values

### Key questions

What is the minimum number of points needed to plot a straight line graph?
Why is it a good idea to use at least three coordinates when plotting a straight line graph?
How should you know when you've made a mistake plotting a straight line graph?

### Exemplar Questions

Complete the table of values for $y = 3x + 2$

| $x$ | $-2$ | $-1$ | $0$ | $1$ | $2$ |
|---|---|---|---|---|---|
| $y$ | | | | | |

On each grid, draw the graph of $y = 3x + 2$ for values of $x$ from $-2$ to $2$
What is the same? What is different?

Dexter has completed a table of values for $y = 6x - 4$

| $x$ | $-2$ | $-1$ | $0$ | $1$ | $2$ |
|---|---|---|---|---|---|
| $y$ | $-8$ | $-2$ | $-4$ | $2$ | $8$ |

Explain and correct Dexter's mistake.

Rosie has drawn the graph of $y = 2x + 1$
Explain why Rosie **must** have made a mistake.

Plot each of the graphs for values of $x$ from $-1$ to $3$

$y = 4x + 1$   $y = 4 - x$   $y = 1 - 4x$

$x + y = 4$   $4(x + 1) = y$   $y = \frac{1}{2}x + 4$



- These include reasoning and problem-solving questions that are fully integrated into the scheme of learning. Depending on the attainment of your students, you many wish to use some or all of these exemplars, which are in approximate order of difficulty. Particularly challenging questions are indicated with the symbol .
- For each block, we also provide ideas for key representations that will be useful for all students.

Some of the small steps are in **bold** and labelled with H to indicate this is Higher tier GCSE content. We would encourage teachers to use these with as many students as possible – if you feel your class can access any particular small step, then please include it in your planning. Steps that review content covered at Key Stage 3 are labelled R .

# Geometric Reasoning

## Small Steps

- Angles at points (R)
- Angles in parallel lines and shapes (R)
- Exterior and interior angles of polygons (R)
- Proving geometric facts
- Solve problems involving vectors (R)
- **Review of circle theorems** (H)
- **Circle theorem: Angle between radius and chord** (H)
- **Circle theorem: Angle between radius and tangent** (H)

H denotes Higher Tier GCSE content

R denotes 'review step' – content should have been covered at KS3

# Geometric Reasoning

## Small Steps

- **Circle theorem: Two tangents from a point** H
- **Circle theorem: Alternate segment theorem** H
- Review Pythagoras' theorem and using trig ratios R

H denotes Higher Tier GCSE content

R denotes 'review step' – content should have been covered at KS3

# Angles at points

R

## Notes and guidance

This small step provides students with opportunity to revise rules of angles at points. They will revisit angles in a full turn, adjacent angles on a straight line and vertically opposite angles. As students have already seen these rules several times, interleaving other topics such as ratio and equations can be used to maintain the level of challenge whilst still securing this essential knowledge.

## Key vocabulary

| | | |
|---|---|---|
| Angle | Adjacent | Vertically opposite |
| Point | Full turn | Straight line |

## Key questions

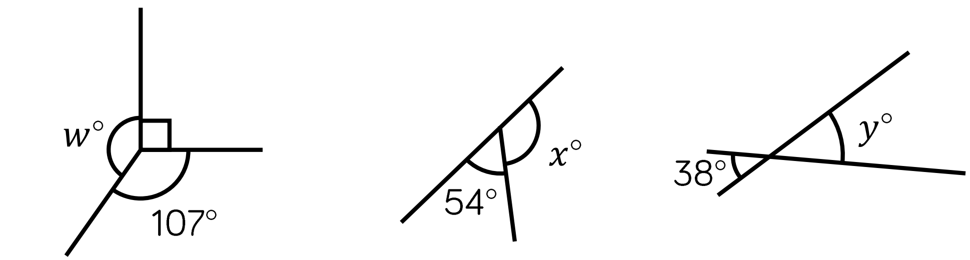
What is the sum of angles around a point?

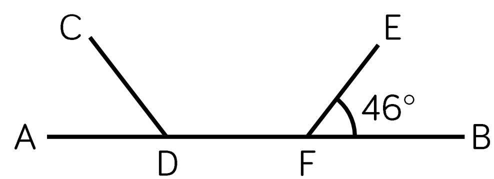
What is important about the word 'adjacent' in 'adjacent angles on a straight line sum to 180 degrees'?

What does it mean for two angles to be vertically opposite?

## Exemplar Questions

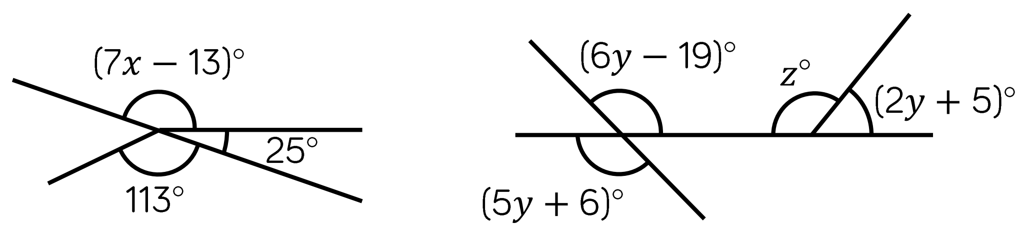
Work out the size of the angles marked with letters.
Give reasons for your answers.




Tommy says "Angle ADC is 134 degrees because angles on a straight line sum to 180 degrees".
Do you agree with Tommy? Explain your answer.

Work out the value of the letters.



# Angles in parallel lines

R

## Notes and guidance

Here students are reminded of rules of angles in parallel lines and shapes. Students will focus on triangles and quadrilaterals as shapes in this step, as polygons will be recapped more formally in the next small step. Students should be confident what is meant by alternate, corresponding and co-interior angles. This small step provides opportunity to revisit other content such as bearings.

## Key vocabulary

| | | |
|---|---|---|
| Angle | Parallel | Corresponding |
| Alternate | Bearing | Co-interior |

## Key questions

What do angles in a triangle/quadrilateral sum to?

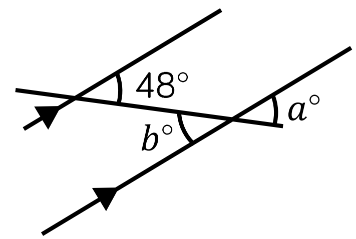
What is meant by alternate/corresponding?

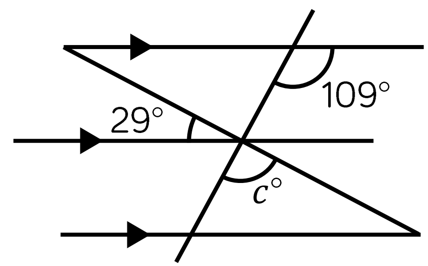
Could you have worked it out another way?

Are the line segments parallel? How do you know?

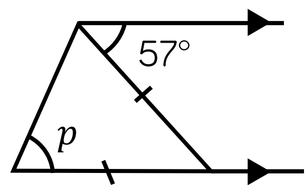
## Exemplar Questions

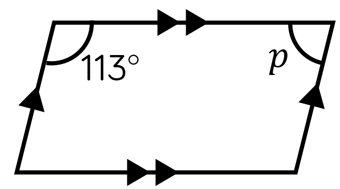
Work out the size of the angles marked with letters.
Give reasons for your answers.




Find the size of the angle marked $p$ in each diagram.




The angles in a triangle are in the ratio 3 : 8 : 4
Work out the size of the smallest angle in the triangle.

The bearing of B from A is 102°
What is the bearing of A from B?

# Angles in polygons

R

## Notes and guidance

This small step provides opportunity for students to recap rules of angles in polygons. It's important that students understand what is meant by a regular polygon and are able to work out the size of both interior and exterior angles in a regular polygon. Students should be familiar with rules of both interior and exterior angles and should be able to work fluently with both in regular and irregular shapes.

## Key vocabulary

| | | |
|---|---|---|
| Angle | Polygon | Regular |
| Interior | Exterior | |

## Key questions

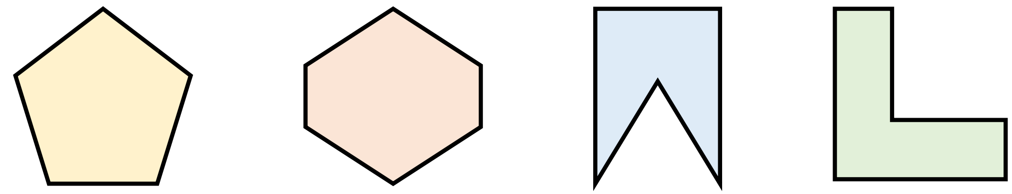
What does it mean for a polygon to be regular?

What is the sum of the angles in this polygon? How do you know?

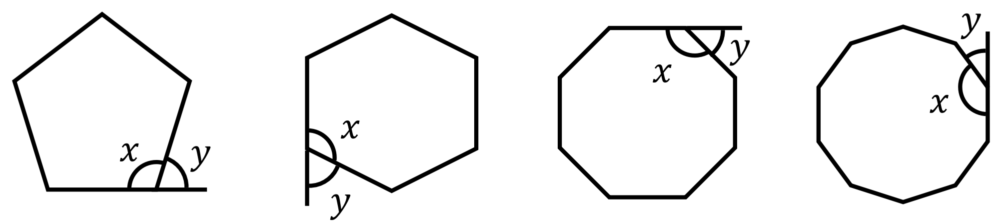
Could you have worked it out a different way?

## Exemplar Questions

Find the sum of the interior angles of each shape.



Each of the polygons is regular.
Work out the size of the angles marked $x$ and $y$.




A regular polygon has 15 sides.
- Work out the size of one interior angle of the polygon.
- Work out the size of one exterior angle of the polygon.

A regular polygon has $n$ sides.
- Write an expression for the size of one interior angle of the polygon.
- Write an expression for the size of one exterior angle of the polygon.

The exterior angle of a regular polygon is 7.2°
How many sides does the polygon have?

# Proving geometric facts

## Notes and guidance

This small step provides opportunity for students to use all the angle facts they have covered to prove simple geometric facts. This is also a good opportunity to revisit properties of shape covered earlier in the curriculum. There should be a focus placed on the explanations used throughout each proof. Students should know that etc 'angles in a triangle' is not sufficient but 'angle in a triangle sum to 180 degrees' is.

## Key vocabulary

| | | |
|---|---|---|
| Angle | Parallel | Equilateral |
| Isosceles | Right-angle | Trapezium |

## Key questions

What do you know? What can you find out?

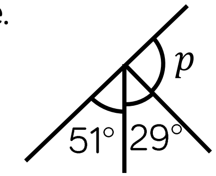
How do you know if a triangle is isosceles?

If the lines are parallel, what must be true?

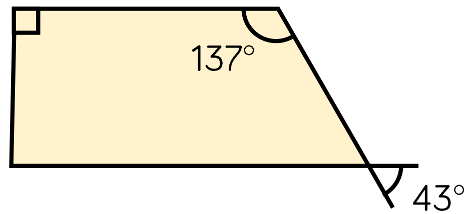
What are the properties of the shape? How does that help?

## Exemplar Questions

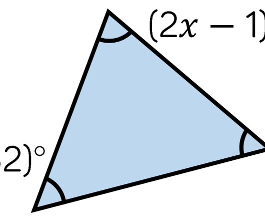
Show that the angle marked $p$ is a right angle.



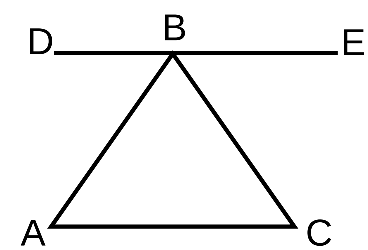

Prove that the quadrilateral is a trapezium.




Prove that this triangle is isosceles.




Line segments AC and DE are parallel.
ABC is a triangle such that point B lies on the line segment DE.
Using rules of angles in parallel lines, prove that the sum of the angles in a triangle is 180 degrees.



# Solve problems involving vectors

## Notes and guidance

Students should be able to find a column vector given a diagram and vice versa. They need to be able to calculate with vectors using addition and subtraction, and multiply a vector by a scalar. Students should know the conditions that make two or more vectors parallel and be able to prove this. They need to be particularly careful with directions of arrows and calculations involving negative numbers.

## Key vocabulary

| | | |
|---|---|---|
| Vector | Column | Horizontal |
| Vertical | Position | Parallel |

## Key questions

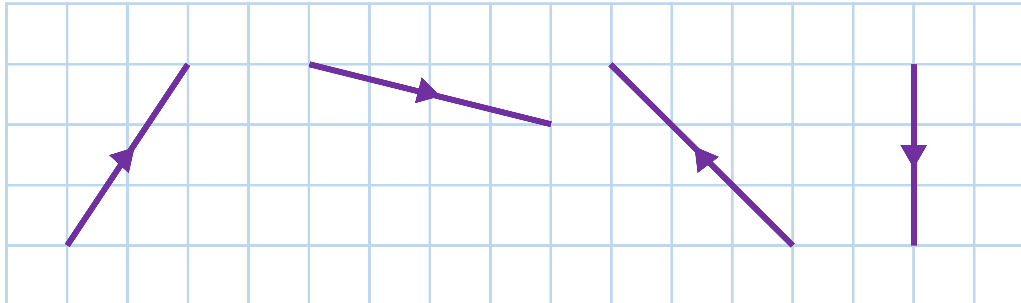
In the column vector $\begin{pmatrix} a \\ b \end{pmatrix}$ what do $a$ and $b$ represent?

How does this link to vectors when performing a translation?

What does it mean for two or more vectors to be parallel?

## Exemplar Questions

Write each vector in column form.



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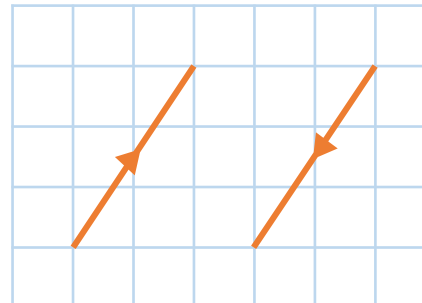
Two vectors are shown on the grid.

Mo says the vectors are the same.
Rosie says they're different.

Who do you agree with?
Explain your answer.



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$\boldsymbol{a} = \begin{pmatrix} 5 \\ -1 \end{pmatrix}$ $\boldsymbol{b} = \begin{pmatrix} -3 \\ 2 \end{pmatrix}$

- Draw a diagram to show that $\boldsymbol{a} + \boldsymbol{b} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$
- Work out $\boldsymbol{a} + 2\boldsymbol{b}$

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$\boldsymbol{c} = \begin{pmatrix} 4 \\ 0 \end{pmatrix}$ $\boldsymbol{d} = \begin{pmatrix} -2 \\ 5 \end{pmatrix}$ $\boldsymbol{e} = \begin{pmatrix} -6 \\ 0 \end{pmatrix}$

Show that $\boldsymbol{c} + \boldsymbol{d}$ is parallel to $3\boldsymbol{c} + 6\boldsymbol{d} - 2\boldsymbol{e}$

# Review circle theorems H

## Notes and guidance

Students will review the circle theorems covered in year 10:

- the angle subtended at the circumference is half the angle subtended at the centre
- the angle in a semi-circle is a right-angle
- angles in the same segment are equal
- opposite angles in a cyclic quadrilateral sum to 180°

Students should be able to use and prove each theorem.
Looking for isosceles triangles when solving problems.

## Key vocabulary

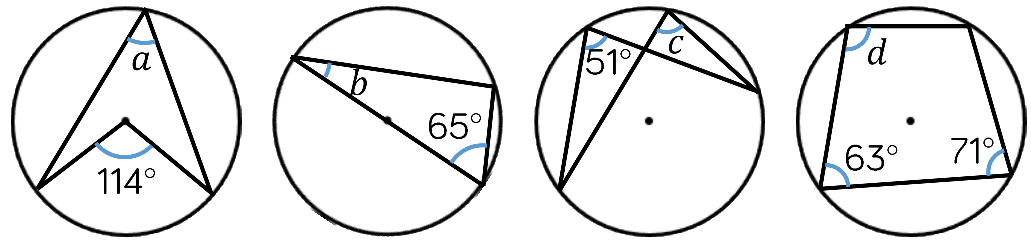
| | | |
|---|---|---|
| Circle | Segment | Circumference |
| Centre | Right-angle | Cyclic quadrilateral |

## Key questions

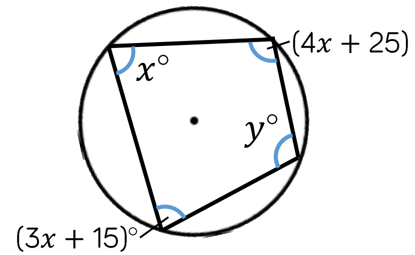
Which is the angle at the circumference?
Is $O$ the centre of the circle? How do you know?
How can you use the fact that the angle at the centre is twice the angle at the circumference to prove that the angle in a semi-circle is 90 degrees?

## Exemplar Questions

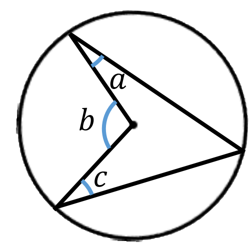
Work out the size of the angles marked with letters.



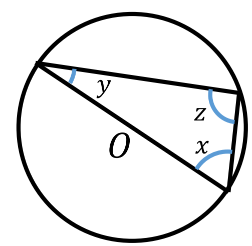

Work out the value of $y$.




$a : c = 5 : 7$
$a + c = 60$
Work out the size of the angle marked $b$.




$x : y : z = 3 : 1 : 5$
Is $O$ the centre of the circle?



# Angle between radius & chord H

## Notes and guidance

Here students are introduced to the circle theorem that the angle between a radius and the midpoint of a chord is a right-angle. Students should be confident in what a chord is and what it isn't. Students should know that a line drawn from the centre of a circle to the midpoint of a chord is a perpendicular bisector of the chord. They then need to be able to apply this in questions.

## Key vocabulary

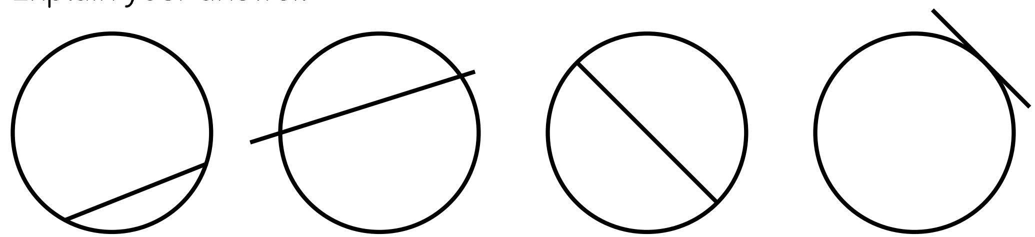
| | | |
|---|---|---|
| Circle | Angle | Radius |
| Chord | Bisects | Right-angle |

## Key questions

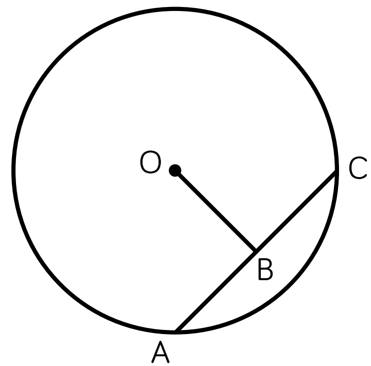
What is a chord?
What is a perpendicular bisector?
Is the diameter of a circle a chord? Why or why not?
If you know the length of the chord and the radius, how can you work out the length of the line from the centre of the circle to the chord?

## Exemplar Questions

Which of the diagrams show a chord?
Explain your answer.

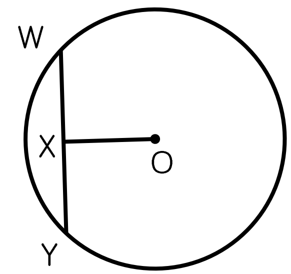


The diagram shows a chord AB drawn through a circle centre O.




- What is the size of ∠OBC?
- If ∠OAB = 29°, work out ∠AOB
- If AC = 18 cm, what is the length of AB?
- If AB = 18 cm, what is the length of AC?

WY is a chord through a circle centre O.
A straight line is drawn from O to point X on WY.
WY = 15 mm, OY = 10 mm.
Work out the length of OX.



# Angle between radius & tangent H

## Notes and guidance

Students are likely to already be confident in identifying a radius of a circle, and it's now important that they fully understand what is meant by tangent as this will be used in the next few theorems. Understanding of a radius meeting a tangent at 90 degrees is essential before going on to look at the alternate segment theorem. As with all the circle theorems, illustrating using dynamic geometry software is useful.

## Key vocabulary

| | | |
|---|---|---|
| Circle | Circumference | Radius |
| Tangent | Right-angle | Angle |

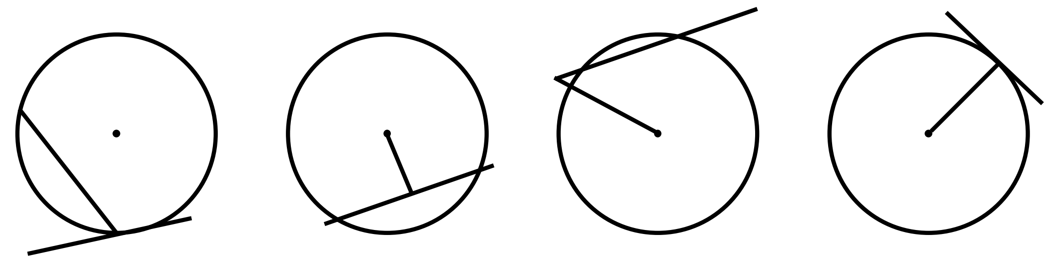
## Key questions

What is a tangent to a circle? Where else do we use the word tangent in mathematics?

Is this line a tangent? How do you know?

## Exemplar Questions

Which diagram shows a radius meeting a tangent?



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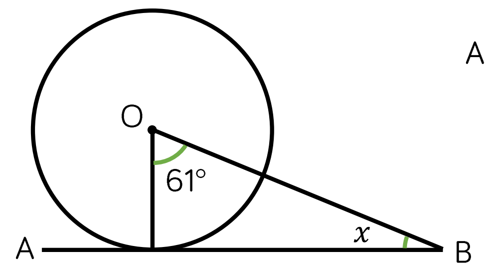
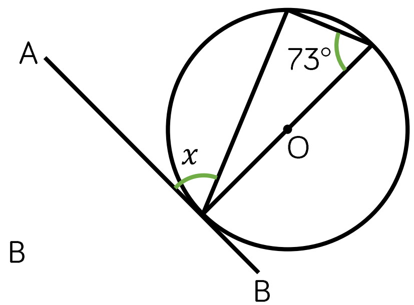
In each diagram, AB is a tangent to the circle centre O.
Work out the size of the angle marked $x$ in each diagram.

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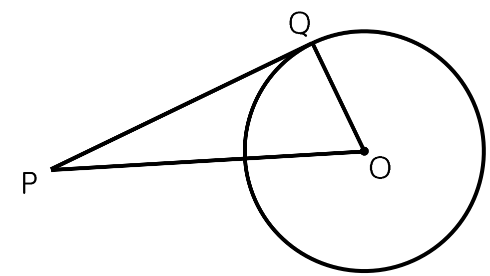
O is the centre of the circle
$\angle POQ = 79°$
$\angle OQP = 15°$
Is PQ a tangent to the circle?
Explain your answer.



# Two tangents from a point H

## Notes and guidance

Here students are introduced to the circle theorem that states that two tangents from the same point to the circumference of a circle are equal in length. They should relate this fact to other areas of mathematics, including isosceles triangles, pairs of congruent triangles or kites. Students can combine this theorem with their other knowledge of circle theorems to tackle more challenging problems. In particular, combining with the last step produces Pythagorean problems.

## Key vocabulary

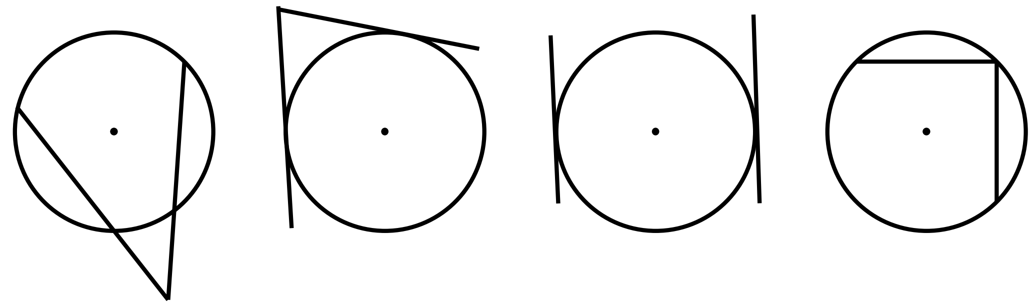
| | | |
|---|---|---|
| Circle | Circumference | Radius |
| Tangent | Right-angle | Length |

## Key questions

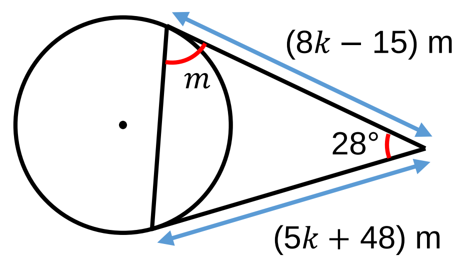
If two tangents drawn from point A meet the circumference of a circle at points B and C, what do you know about triangle ABC?
If the length of AB is 25 cm, what is the length of AC?
What types of triangle/quadrilateral can you see in the diagram?

## Exemplar Questions

Which diagram shows two tangents from a common point?



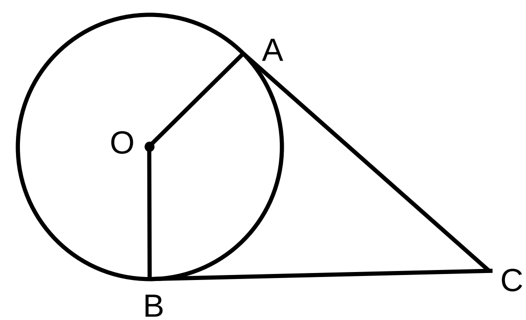
The diagram shows two tangents from a common point to a circle.




Work out the size of the angle marked $m$.

Work out the value of $k$.

AC and BC are tangents to a circle centre O.
Prove that triangles OAC and OBC are congruent.



# Alternate segment theorem H

## Notes and guidance

Here students are introduced to the alternate segment theorem. Students should know and understand that the angle between chord and tangent is equal to the angle in the alternate segment. Students could be introduced to this as a special case of the theorem about the angle between a radius and a tangent. It's important that students understand what is meant by the alternate segment so that they can correctly identify any pairs of equal angles.

## Key vocabulary

| | | |
|---|---|---|
| Circle | Angle | Alternate |
| Segment | Equal | Tangent |

## Key questions

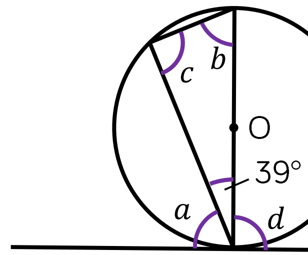
What is a segment?

What is meant by the alternate segment?

What do you know about the angle between a tangent and a radius? How can you use this here?

## Exemplar Questions

The diagram shows a tangent to a circle centre O.

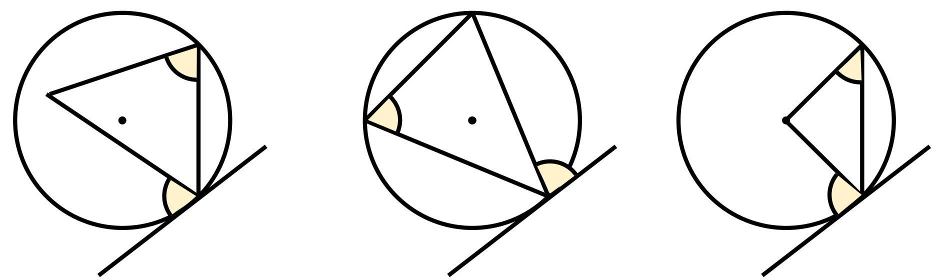



- Write down the size of angles $c$ and $d$.

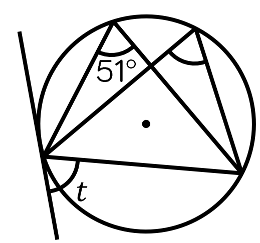
  What do you notice?
- Work out the size of angles $a$ and $b$.

  What do you notice?
- Why does this happen?

In which diagram are the pair of marked angles equal?



Work out the size of the angle marked $t$.
Give a reason for your answer.




Prove that the angle between a tangent and a chord is equal to the angle in the alternate segment.

# Review Pythagoras' & Trigonometry

## Notes and guidance

This small steps provides opportunity to revisit Pythagoras' Theorem and trigonometry. Students studying for foundation tier GCSE could go through key points from the Pythagoras block in year 10 during this block. Links can be made to different areas of the National Curriculum including coordinates and vectors. Students should be able to recognise when to use which rules to answer the questions.

## Key vocabulary

| | | |
|---|---|---|
| Triangle | Opposite | Adjacent |
| Hypotenuse | Ratio | Inverse |

## Key questions

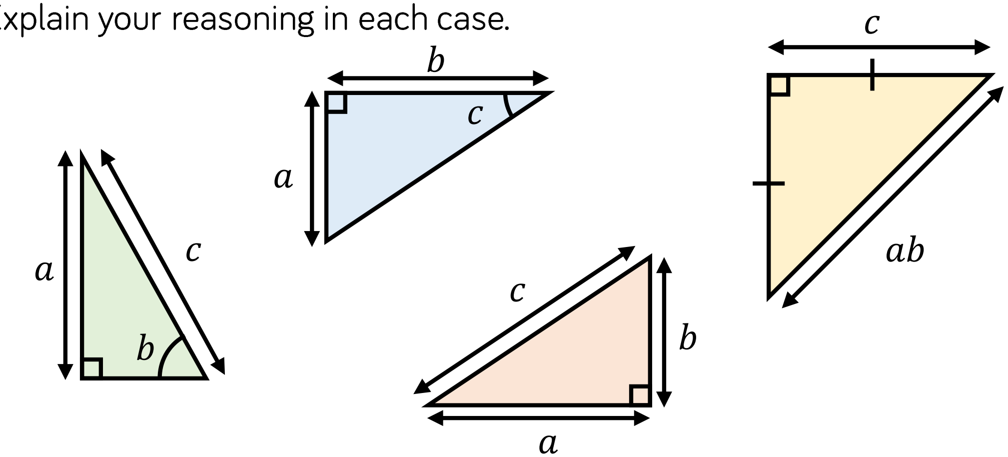
Does this question require Pythagoras' Theorem or trigonometry to solve? How do you know?

Where can you split the shape so that there is a right-angled triangle?

How can drawing a diagram support you?

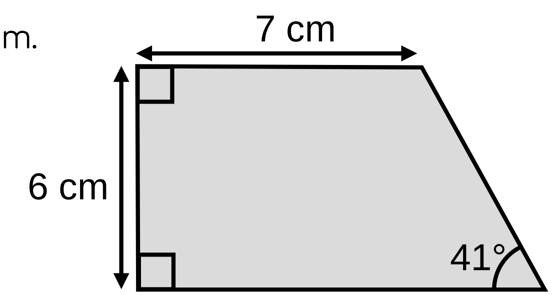
## Exemplar Questions

If you knew the value of $a$ and $b$ for each diagram, would you use Pythagoras' Theorem, Trigonometry or either to work out the value of $c$? Explain your reasoning in each case.




Given that $a = 41$ and $b = 37$, work out the value of $c$ in each diagram.

Work out the area of the trapezium.




Point A has coordinates $(-5, 3)$
Point B has coordinates $(2, -8)$
Work out the length of the line segment AB.