Listing and Describing

Year 11

#MathsEveryoneCan

White Rose Maths

	Week 1	Week 2	Week 3	Week 4	Week 5	Week 6	Week 7	Week 8	Week 9	Week 10	Week 11	Week 12
Autumn	**Graphs**						**Algebra**					
	Gradients & lines		Non-linear graphs		Using graphs		Expanding & Factorising		Changing the subject		Functions	
Spring	**Reasoning**						**Revision and Communication**					
	Multiplicative		Geometric		Algebraic		Transforming & Constructing		Listing & describing		Show that...	
Summer	**Revision**						**Examinations**					

Spring 2 : Revision & Communication

Weeks 1 and 2: Transforming & Constructing

Students revise and extend their learning from Key Stage 3, exploring all the transformations and constructions, relating these to symmetry and properties of shapes when appropriate. There is an emphasis on describing as well as performing transformations as using the language promotes deeper thinking and understanding. Higher tier students extend their learning to explore the idea of invariance and look at trigonometric graphs as a vehicle for exploring graph transformations.

National Curriculum content covered includes:

- describe translations as 2D vectors
- reason deductively in geometry, number and algebra, including using geometrical constructions
- interpret and use fractional **{and negative}** scale factors for enlargements
- **{describe the changes and invariance achieved by combinations of rotations, reflections and translations}**
- recognise, sketch and interpret graphs of **{the trigonometric functions (with arguments in degrees) for angles of any size}**
- **{sketch translations and reflections of the graph of a given function}**

Weeks 3 and 4: Listing & Describing

This block is another vehicle for revision as the examinations draw closer. Students look at organisation information, with Higher tier students extending this to include the product rule for counting. Links are made to probability and other aspects of Data Handling such as describing and comparing distributions and scatter diagrams. Plans and elevations are also revised. You can adapt the exact content to suit the needs of your class.

National Curriculum content covered includes:

- explore what can and cannot be inferred in statistical and probabilistic settings, and express their arguments formally
- calculate the probability of independent and dependent combined events, including using tree diagrams and other representations, and know the underlying assumptions
- **{calculate and interpret conditional probabilities through representation using expected frequencies with two-way tables, tree diagrams and Venn diagrams}**
- apply systematic listing strategies, **{including use of the product rule for counting}**
- construct and interpret plans and elevations of 3D shapes

Weeks 5 and 6: Show that

This is another block designed to be adapted to suit the needs of your class. Examples of communication in various areas of mathematics are provided in order to highlight gaps in knowledge that need addressing in the run up to the examinations. “Show that” is used to encourage students to communicate in a clear mathematical fashion, and this skill should be transferred to their writing of solutions to any type of question.

National Curriculum content covered includes:

- know the difference between an equation and an identity; argue mathematically to show algebraic expressions are equivalent, and use algebra to support and construct arguments **{and proofs}**
- apply the concepts of congruence and similarity
- make and use connections between different parts of mathematics to solve problems
- **{change recurring decimals into their corresponding fractions and vice versa}**
- apply addition and subtraction of vectors, multiplication of vectors by a scalar, and diagrammatic and column representations of vectors; **{use vectors to construct geometric arguments and proofs}**

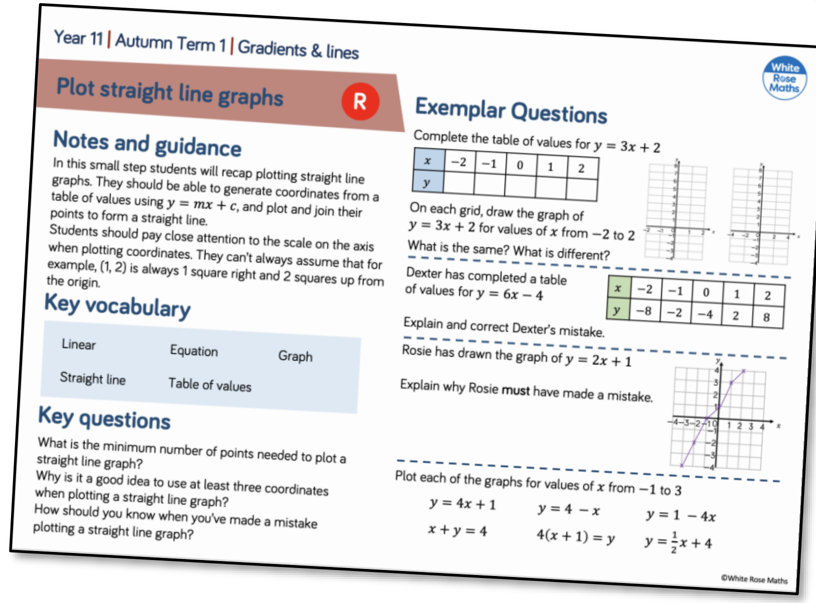
Why Small Steps?

We know that breaking the curriculum down into small manageable steps should help students to understand concepts better. Too often, we have noticed that teachers will try and cover too many concepts at once and this can lead to cognitive overload. We believe it is better to follow a "small steps" approach.

As a result, for each block of content in the scheme of learning we will provide a "small step" breakdown. ***It is not the intention that each small step should last a lesson – some will be a short step within a lesson, some will take longer than a lesson.*** We would encourage teachers to spend the appropriate amount of time on each step for their group, and to teach some of the steps alongside each other if necessary.

What We Provide

- Some ***brief guidance*** notes to help identify key teaching and learning points.
- A list of ***key vocabulary*** that we would expect teachers to draw to students' attention when teaching the small step.
- A series of ***key questions*** to incorporate in lessons to aid mathematical thinking.
- A set of questions to help ***exemplify*** the small step concept that needs to be focussed on.


Year 11 | Autumn Term 1 | Gradients & lines

Plot straight line graphs (R)

Notes and guidance

In this small step students will recap plotting straight line graphs. They should be able to generate coordinates from a table of values using $y = mx + c$, and plot and join their points to form a straight line.
Students should pay close attention to the scale on the axis when plotting coordinates. They can't always assume that for example, (1, 2) is always 1 square right and 2 squares up from the origin.

Key vocabulary

Linear, Equation, Graph, Straight line, Table of values

Key questions

What is the minimum number of points needed to plot a straight line graph?
Why is it a good idea to use at least three coordinates when plotting a straight line graph?
How should you know when you've made a mistake plotting a straight line graph?

Exemplar Questions

Complete the table of values for $y = 3x + 2$

x	-2	-1	0	1	2
y					

On each grid, draw the graph of $y = 3x + 2$ for values of x from -2 to 2
What is the same? What is different?

Dexter has completed a table of values for $y = 6x - 4$

x	-2	-1	0	1	2
y	-8	-2	-4	2	8

Explain and correct Dexter's mistake.

Rosie has drawn the graph of $y = 2x + 1$

Explain why Rosie **must** have made a mistake.

Plot each of the graphs for values of x from -1 to 3

$y = 4x + 1$ $\quad y = 4 - x$ $\quad y = 1 - 4x$

$x + y = 4$ $\quad 4(x + 1) = y$ $\quad y = \frac{1}{2}x + 4$

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- These include reasoning and problem-solving questions that are fully integrated into the scheme of learning. Depending on the attainment of your students, you may wish to use some or all of these exemplars, which are in approximate order of difficulty. Particularly challenging questions are indicated with the symbol 💡.
- For each block, we also provide ideas for key representations that will be useful for all students.

Some of the small steps are in **bold** and labelled with (H) to indicate this is Higher tier GCSE content. We would encourage teachers to use these with as many students as possible – if you feel your class can access any particular small step, then please include it in your planning. Steps that review content covered at Key Stage 3 are labelled (R).

Listing and Describing

Small Steps

- Work with organised lists
- Sample spaces and probability — R
- **Use the product rule for counting** — H
- Complete and use Venn diagrams — R
- Construct and interpret plans and elevations — R
- Use data to compare distributions — R
- Interpreting scatter diagrams — R

H denotes Higher Tier GCSE content

R denotes 'review step' – content should have been covered at KS3

Work with organised lists

Notes and guidance

Students revise how to generate a list systematically. They should be encouraged to explain how they know they have all possibilities. Thinking about what they have changed and what they have kept the same each time can help students to talk about a systematic method. Teachers may want to interleave stem-and-leaf diagrams in this small step, reminding students why an organised list is useful and revisiting averages.

Key vocabulary

Systematic	Exhaustive	Arrangement
Stem-and-Leaf	Median	Range

Key questions

Explain what the word systematic means.

Describe a good systematic method for listing items. What did you keep the same and what did you change each time?

Why is it useful to put the data in order when designing a stem-and-leaf diagram?

Exemplar Questions

Ron is listing all possible 3-digit numbers he can make from the digits 1, 2 and 3

123, 321, 213, 312

- Explain why it is difficult to tell if Ron has listed all possibilities.

Alex also starts a list.

123, 132, 213, 231, …

- Describe Alex's strategy and complete her list.
- Find all 24 arrangements of the letters A, B, C and D

To get the meal deal, you have to choose one of each option.

Sandwiches		Crisps		Drink
Cheese Ham	+	Plain Beef	+	Orange Water

Use a systematic method to show that there are 8 different options. Explain why your method was systematic.

Here are the times taken by some runners in a 100 m race.

Key: 9 | 9 means 9.9 secs

9	9 8 2 1 5 8 7 5 9
10	5 6 1 2 4 3 3 8 6 5
11	1 7 3 1

The range is $11.1 - 9.9 = 1.2$ secs

The median 10.1 secs

Why are the range and the median both incorrect?
Work out the correct range and median of the times.

Sample spaces and probability R

Notes and guidance

When students don't use a sample space for multiple trials, they often give an incorrect answer based on flawed intuition (e.g. P(1 Head and 1 Tail) $= \frac{1}{3}$). Remind students that probabilities can only be written as fractions, decimals or percentages. For some students, it might be appropriate to revise constructing and interpreting tree diagrams, moving on to problems without replacement for Higher tier students.

Key vocabulary

Sample space	Two-way table	Event
Tree diagram	Replacement	Outcome

Key questions

How can we list all possible outcomes so that none are missed out?
How can we work out a probability from a sample space?
How do we know which cells to consider when finding a probability from a two-way table?
How can we tell if events are equally likely or not?

Exemplar Questions

- Ron flips a coin twice.
  He says, "the probability of getting a head and a tail is 50%".
  Is Ron correct? Justify your answer.
- Annie rolls a dice twice.
  She says, "the probability of getting two sixes is $\frac{1}{12}$"
  Is Annie correct? Justify your answer.

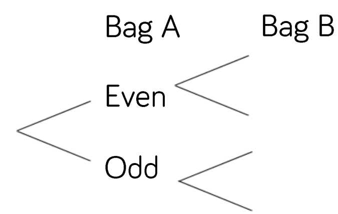
In a school, there are 24 teachers and 720 students.
20% of teachers eat a hot lunch and the rest eat a cold lunch.
The ratio of students who eat a hot lunch to a cold lunch is 3 : 5
Complete the two-way table.

	Hot lunch	Cold lunch	Total
Teachers			
Students			
Total			

A person is selected at random.
Work out the probability that they eat a hot lunch.

A student is selected at random.
Work out the probability that they eat a cold lunch.

Bag A contains cards numbered 1, 2, 3, 4 and 5
Bag B contains cards numbered 5, 6, 7, 8 and 9
A card is taken from Bag A and then from Bag B.
Copy and complete the tree diagram.



Work out the probability of getting

- 2 odd numbers
- An even and an odd number
- At least one even number

Product rule for counting (H)

Notes and guidance

Systematic listing and structures like tree diagrams will support students to see the general rule that the total number of arrangements is the product of the number of possible choices for each 'part'. Students need to understand the difference between working out the total number of possible options with and without replacement and when "repeats" occur e.g. the total number of handshakes in a group if each pair shakes hands once.

Key vocabulary

Systematic	Generalise	Product
Product rule	Repeats	Replacement

Key questions

How can you represent the different options?

Can you generalise my findings?

What happens to the total number of options if I can't re-use an item? What happens if I can re-use an item?

Why do I divide by 2 when working out the number of options for pairs of students from a group?

Exemplar Questions

Starter	Main course
Samosa (S)	Meat Curry
Bhaji (B)	Vegetable Curry
	Paneer Cheese Curry

A meal is a starter and main course.

Show that there are 6 possible meal options.

> Whitney notices that there are 2 starters and 3 main courses. $2 \times 3 = 6$, and 6 is the number of meal options.

Explore Whitney's idea further by changing the number of starters and number of main courses. What can you conclude?

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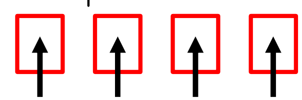
0 1 2 3 4 5 6 7 8 9

Ron is working out how many 4-digit numbers he can make if he uses each card once. Complete his workings.



No. of options: 10 9 ___ ___

Total no. of possible options $= 10 \times 9 \times$ ___ $\times$ ___ $=$ ___

How many 4-digit numbers can be made if each number can be reused?

There are 4 boys in a group. A teacher selects one pair of boys. Eva thinks that the teacher has 12 options ($4 \times 3 = 12$).

Joe thinks that the teacher has 6 options ($\frac{4 \times 3}{2} = 6$)

Joe is correct. Explain why.

How many options does the teacher have if there are 5 boys in the group? Can you make a generalisation?

Venn diagrams

R

Notes and guidance

Teachers may want to use a 'shading activity' to review what the regions on a Venn diagram represent before using these to work out probabilities. A common error is to read a sentence such as '15 people had mustard' in the second exemplar question as 'having mustard only', when in fact, they may also have had tomato sauce. Encourage students to start with the intersection of two events when completing a Venn diagram to overcome this misconception.

Key vocabulary

Venn diagram	Probability	Union
Intersect	Complement	

Key questions

Do the circles in a Venn diagram always overlap? Why or why not?
Can one set lie completely within another set?
Where do we start when completing a Venn diagram? Why?
What do we mean by the words union, intersect and complement?

Exemplar Questions

Draw a Venn diagram to represent the following information.

$\xi = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15\}$
$A = \{\text{multiples of } 3\}$ $B = \{\text{factors of } 48\}$

One of the numbers is selected at random.
Work out:

- $P(A \cap B)$
- $P(A \cup B)$
- $P(A)$
- $P(A')$
- $P(A' \cap B)$
- $P(A' \cup B)$
- $P(A \cap B)'$
- $P(A \cup B)'$

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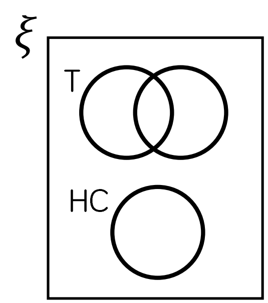
50 people bought hot dogs.
33 people had tomato sauce on their hot dog.
15 people had mustard on their hot dog.
12 people had both tomato sauce and mustard on their hot dog.
Draw a Venn diagram to represent this information.
A person is selected at random. Work out the probability that they had

- Only tomato sauce on their hot dog.
- Tomato sauce given that they had mustard.



33 teenagers were asked about drinks.
20 drink tea (T) and 10 drink coffee (C).
8 people drink hot chocolate.
All 33 teenagers drink tea, coffee or hot chocolate.
How can you tell from the diagram that no-one who drinks tea or coffee drinks hot chocolate (HC)?
Complete the Venn diagram.

Plans and elevations R

Notes and guidance

To support this step, students might use multi-link cubes so that they can move around the object to appreciate the different views. Dynamic geometry software can also be very helpful here. Include sketching elevations and plans as well as accurately drawing them and labelling dimensions. Students may need reminding on how to use isometric paper. When reviewing the language, check and compare the language that is used in other areas of the curriculum such as Technology.

Key vocabulary

Plan view	Side elevation	Front elevation
Isometric	Face	Edge

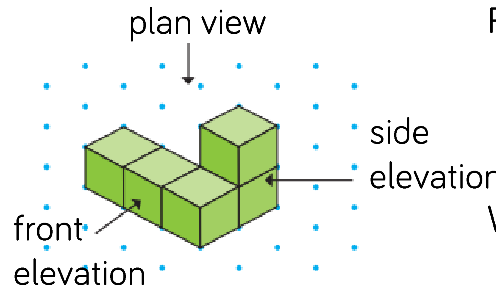
Key questions

What's the difference between the net of a 3-D shape and the plan view?

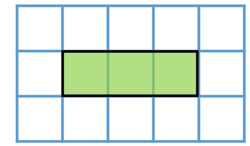
What can you see looking at the shape from the front/ side/ above? Are there any parts you cannot see?

How do you know the dimensions of the elevations and plan?

Exemplar Questions



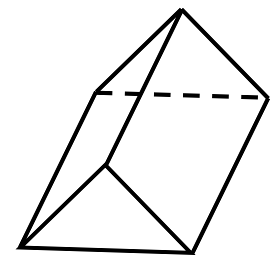
Ron has drawn the front elevation.

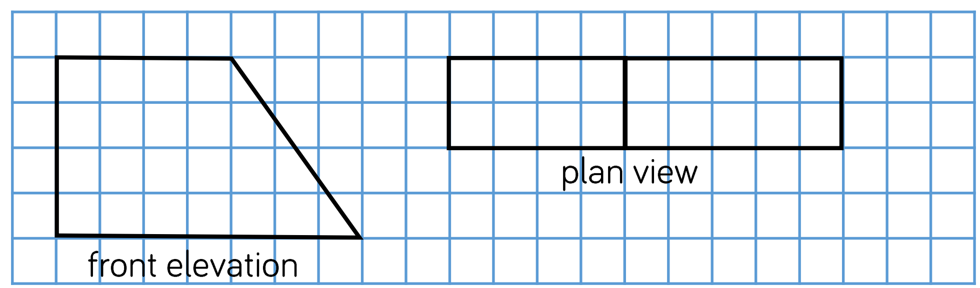


What mistake has he made?

- Build other 3D shapes using 5 multi-link cubes.
- Each time draw the front elevation, plan view and side elevation.

Sketch the front and side elevations and the plan view of the triangular prism.





The front elevation and the plan view of a trapezium-faced prism are shown. On squared paper, draw the 2 different side elevations.
Make a drawing of the 3-D shape on isometric paper.

Compare distributions

R

Notes and guidance

Students should appreciate that a good comparison involves both an average and a measure of spread. Students can find it difficult to construct these, so sentence stems may help. There may be other aspects that they can also comment on, such as outliers. Be aware students may need to compare their findings with a given hypothesis. Higher tier may need to revise cumulative frequency diagrams and box plots here.

Key vocabulary

Hypothesis	Average	Spread
Interquartile Range	Range	Outlier

Key questions

Are there any outliers in the data? How will an outlier affect the mean, median, mode, range, interquartile range?
Which are the most appropriate calculations to do?
What graphs can be drawn? How will they help to compare the data?
What can you conclude? Have you written in full sentences? Is your conclusion fully justified?

Exemplar Questions

Whitney says, "Year 7 students will be slower than Year 11 students at solving a puzzle."
Compare the data. Is Whitney correct? Justify your answer.

	Time taken to complete the puzzle (to the nearest minute)									
Year 7	11	4	6	8	10	7	13	5	12	6
Year 11	4	5	16	20	3	5	5	7	8	9

The number of texts 30 students received on two different days of the week was recorded. Compare the data for the two days.

Number of texts on a Saturday (x)	Frequency
$0 < x \leq 20$	0
$20 < x \leq 40$	2
$40 < x \leq 60$	5
$60 < x \leq 80$	17
$80 < x \leq 100$	6

Number of texts on a Monday(x)	Frequency
$0 < x \leq 20$	5
$20 < x \leq 40$	8
$40 < x \leq 60$	8
$60 < x \leq 80$	9
$80 < x \leq 100$	0

The following data was obtained from a register at a national gym.
Eva says, "The youngest person on the register for the gym is male."
Is she right?
Explain your answer.
Compare the data.

	%	Mean age (years)	Interquartile range of ages (years)
Male	56	28.1	7.3
Female	44	34.0	11.7

Interpreting scatter diagrams R

Notes and guidance

Students may need reminding of the vocabulary around correlation. They should also know that a line of best fit has to be straight, but it does not have to go through the origin and nor does it have to join the first and last marked points. The issues with extrapolating from a line of best fit should be discussed. Students should also be aware the correlation does not imply causation.

Key vocabulary

Negative/Positive	Correlation	Causation
Estimate	Outlier	Extrapolate

Key questions

What steps do I take when drawing a line of best fit?

Why do I need a line of best fit when estimating values?

What are the problems caused by extending a line of best fit outside the range of given data?

Why does correlation not imply causation?

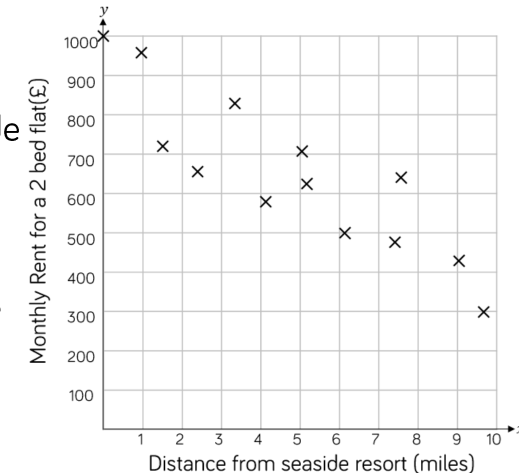
Can two data sets have a relationship which isn't linear?

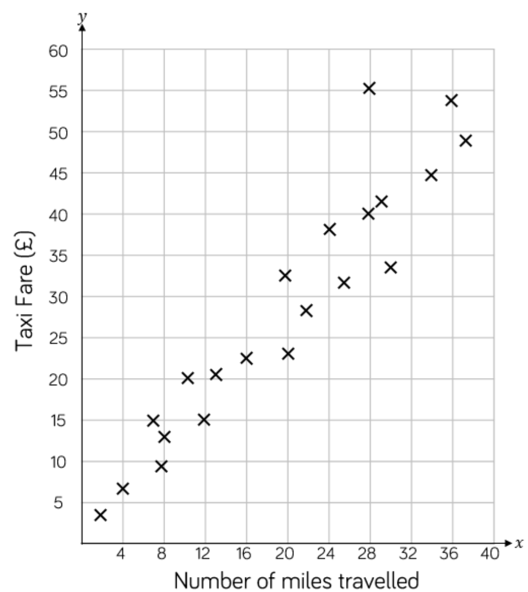
Exemplar Questions

What type of correlation is shown?

Mo says, "Being closer to the seaside resort causes monthly rent to rise". Why might Mo be incorrect?

Use a line of best fit to estimate the rent of a flat which is 7 miles away from the seaside resort.





What type of correlation is shown?

Describe the relationship shown.

Are there any outliers?

Use your graph to estimate the

- price of journey that is 32 miles long
- length of a journey that costs £50

Explain why it may not be appropriate to use your line of best fit to estimate the cost of a journey that is 100 miles long.