Directed Number

Year 7

#MathsEveryoneCan

2019-20

White Rose Maths

	Week 1	Week 2	Week 3	Week 4	Week 5	Week 6	Week 7	Week 8	Week 9	Week 10	Week 11	Week 12
Autumn	**Algebraic Thinking**						**Place Value and Proportion**					
	Sequences		Understand and use algebraic notation		Equality and equivalence		Place value and ordering integers and decimals			Fraction, decimal and percentage equivalence		
Spring	**Applications of Number**						**Directed Number**			**Fractional Thinking**		
	Solving problems with addition & subtraction		Solving problems with multiplication and division			Fractions & percentages of amounts	Operations and equations with directed number			Addition and subtraction of fractions		
Summer	**Lines and Angles**						**Reasoning with Number**					
	Constructing, measuring and using geometric notation			Developing geometric reasoning			Developing number sense		Sets and probability		Prime numbers and proof	

Spring 2: Directed Number and Fractional Thinking

Weeks 1 to 3: Directed number

Students will only have had limited experience of directed number at primary school, so this block is designed to extend and deepen their understanding of this. Multiple representations and contexts will be used to enable students to appreciate the meaning behind operations with negative integers rather than relying on a series of potentially confusing "rules". As well as exploring directed number in its own right, this block provides valuable opportunities for revising and extending earlier topics, notably algebraic areas such as substitution and the solution of equations; in particular students will be introduced to two-step equations for the first time in this block.

National curriculum content covered:

- select and use appropriate calculation strategies to solve increasingly complex problems
- use the four operations, including formal written methods, applied to integers, both positive and negative
- recognise and use relationships between operations including inverse operations
- use square and square roots
- use a calculator and other technologies to calculate results accurately and then interpret them appropriately
- substitute numerical values into formulae and expressions, including scientific formulae
- understand and use the concepts and vocabulary of expressions, equations, inequalities, terms and factors
- simplify and manipulate algebraic expressions to maintain equivalence
- understand and use standard mathematical formulae

<u>Interleaving/Extension of previous work</u>

- use conventional notation for the priority of operations
- forming and solving linear equations, including two-step equations

Weeks 4 to 6: Fractional thinking

This block builds on the Autumn term study of "key" fractions, decimals and percentages. It will provide more experience of equivalence of fractions with any denominators, and to introduce the addition and subtraction of fractions. Bar models and concrete representations will be used extensively to support this. Adding fractions with the same denominators will lead to further exploration of fractions greater than one, and for the Core strand adding and subtracting with different denominators will be restricted to cases where one is a multiple of the other.

National curriculum content covered:

- move freely between different numerical, graphical and diagrammatic representations [for example, equivalent fractions, fractions and decimals]
- express one quantity as a fraction of another, where the fraction is less than 1 and greater than 1
- order positive and negative integers, decimals and fractions; use the number line as a model for ordering of the real numbers; use the symbols =, ≠, , ≤, ≥
- select and use appropriate calculation strategies to solve increasingly complex problems
- use the four operations, including formal written methods, applied to integers, decimals, proper and improper fractions, and mixed numbers, all both positive and negative
- work interchangeably with terminating decimals and their corresponding fractions

<u>Interleaving/Extension of previous work</u>

- finding the range and the median
- substitution into algebraic formulae
- forming and solving linear equations, including two-step equations

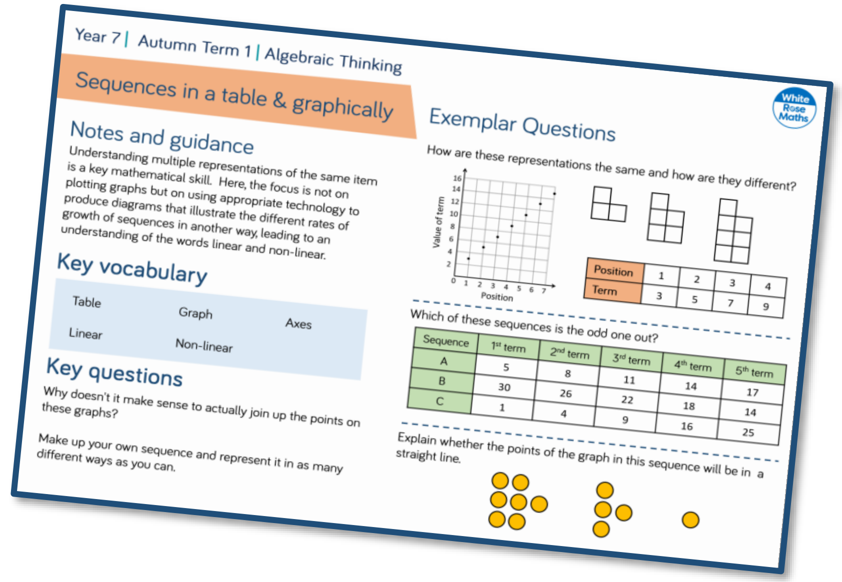
Why Small Steps?

We know that breaking the curriculum down into small manageable steps should help students to understand concepts better. Too often, we have noticed that teachers will try and cover too many concepts at once and this can lead to cognitive overload. We believe it is better to follow a "small steps" approach.

As a result, for each block of content in the scheme of learning we will provide a "small step" breakdown. ***It is not the intention that each small step should last a lesson – some will be a short step within a lesson, some will take longer than a lesson.*** We would encourage teachers to spend the appropriate amount of time on each step for their group, and to teach some of the steps alongside each other if necessary.

What We Provide

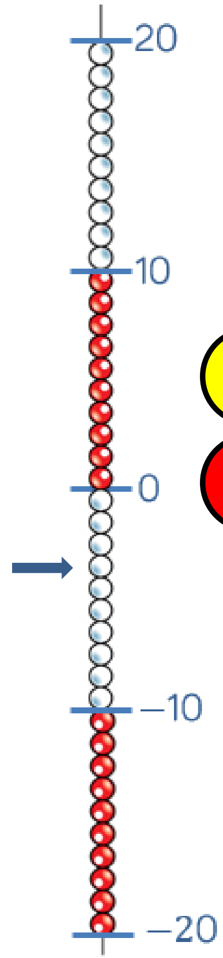
- Some ***brief guidance*** notes to help identify key teaching and learning points
- A list of ***key vocabulary*** that we would expect teachers to draw to students' attention when teaching the small step,
- A series of ***key questions*** to incorporate in lessons to aid mathematical thinking.
- A set of questions to help ***exemplify*** the small step concept that needs to be focussed on.

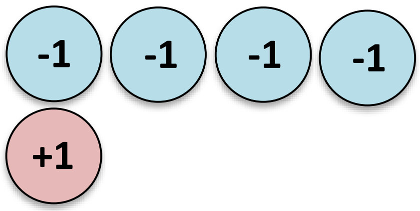


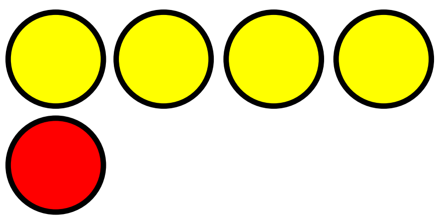
- These include reasoning and problem-solving questions that are fully integrated into the scheme of learning. Depending on the attainment of your students, you many wish to use some or all of these exemplars, which are in approximate order of difficulty. Particularly challenging questions are indicated with the symbol  .
- For each block, we also provide ideas for key representations that will be useful for all students.

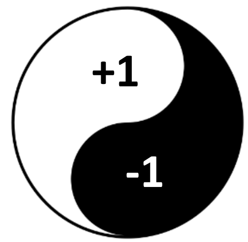
In many of the blocks of material, some of the small steps are in **bold**. These are content aimed at higher attaining students, but we would encourage teachers to use these with as many students as possible – if you feel your class can access any particular small step, then please include it in your planning.

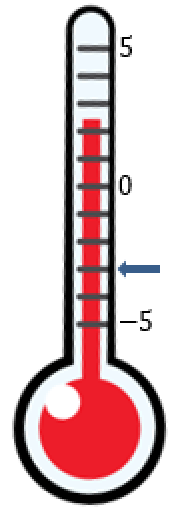
Key Representations

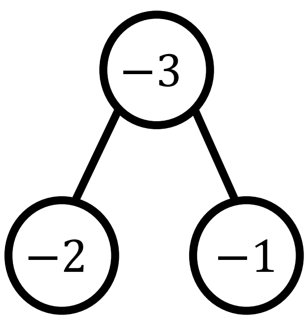


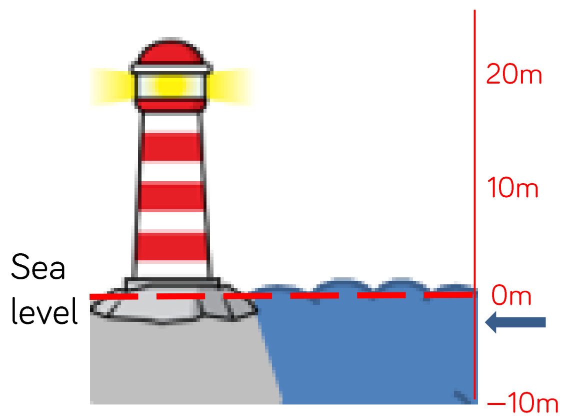


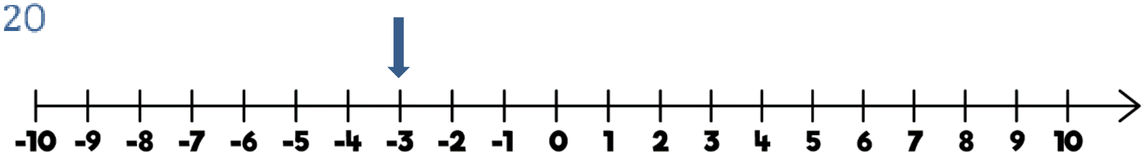












Concrete, pictorial and abstract representations are an important part of developing students' conceptual understanding.

When dealing with directed numbers, it is important to use both horizontal and vertical number lines. The vertical will be familiar from experience of temperature. It is preferable to refer to numbers below zero as e.g. "negative three" rather than "minus three" to try and avoid confusion between numbers and operators and the common misuse of language is a good discussion point.

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Directed Number

Small Steps

- Understand and use representations of directed numbers
- Order directed numbers using lines and appropriate symbols
- Perform calculations that cross zero
- Add directed numbers
- Subtract directed numbers
- Multiplication of directed numbers
- Multiplication and division of directed numbers
- Use a calculator for directed number calculations
- Evaluate algebraic expressions with directed number
- Introduction to two-step equations

Directed Number

Small Steps

- Solve two-step equations
- Use order of operations with directed numbers
- **Understand that positive numbers have more than one square root** H
- **Explore higher powers and roots** H

H denotes higher strand and not necessarily content for Higher Tier GCSE

Representations of directed number

Notes and guidance

Students should recognise and use negative numbers in a variety of different representations, including real-life contexts and more abstractly with concrete manipulatives and written notation. Students should be introduced to the reflective nature of positive and negative numbers on the number line e.g. knowing -4 and 4 are equidistant from 0. To avoid confusion "-4" should be read as "negative 4" etc.

Key vocabulary

Positive	Negative	Reflection
Symmetric	Sea level	

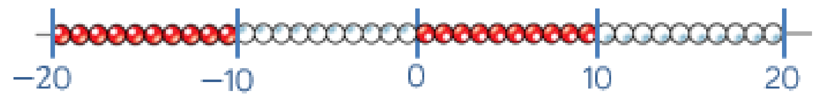
Key questions

How far is -3 from zero? How far is 3 from 0? How are they different?

What does this tell us about positive and negative numbers? (If using bead strings, they can be moved to emphasise the reflection about 0)

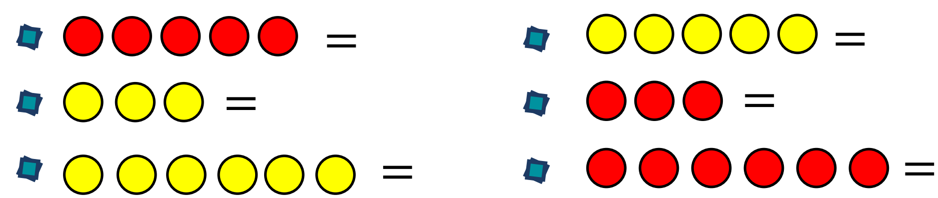
Exemplar Questions

Find the following pairs of numbers on the bead string. What do you notice about each pair?

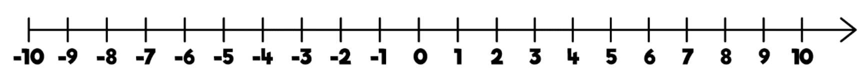


- −10 and 10
- −3 and 3
- −17 and 17
- Does 0 have a matching number?

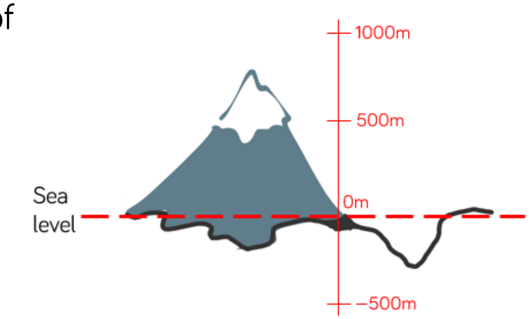
If = −1 and = 1, what is the total of each below?



Label them on the number line.



What is the approximate height of the mountain?
How deep is the valley?



Order directed numbers

Notes and guidance

In this small step, students practise ordering directed numbers. Order is established using both vertical and horizontal number lines,. The appropriate symbols are then used for comparison. Students should practise ordering negative fractions and decimals on a number line, as well as integers.

Key vocabulary

Ascending	Descending	Smaller/bigger than
Positive	Negative	Greater/less than

Key questions

Is ordering temperatures from hottest to coldest, putting them in ascending or descending order?

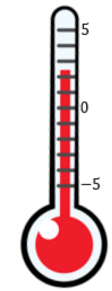
Where would $+\frac{1}{4}$ be on the number line? Is it closer to 0 or 1? How does this help us to put $-\frac{1}{4}$ on the number line?

Between which two consecutive integers does −1.5 lie?

Exemplar Questions

Put the following temperatures in order from coldest to hottest.

2°C, −1°C, −7°C, 0°C, 12°C

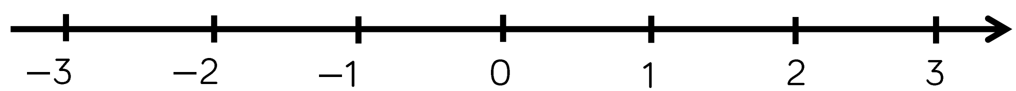


Complete the statements using $>$ or $<$

- −2°C ◯ −6°C
- −6°C ◯ 4°C
- −9°C ◯ 0°C
- −26°C ◯ −24°C
- 15°C ◯ −115°C
- −6°C ◯ −6.2°C

Use the number line to help you put these numbers in ascending order.

- $\frac{1}{4}$
- −1.5
- $-\frac{1}{4}$
- $-1\frac{3}{4}$
- −1
- 2.5



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Perform calculations that cross zero

Notes and guidance

Students can explore number pairs that add to 0 e.g. $-5 + 5$ to show that one negative and one positive of the same magnitude "cancel each other out". Students can use number lines to support adding and subtracting through partitioning: e.g. $-8 + 12 = -8 + 8 + 4 = 4$. A number line is also useful to illustrate the difference between two numbers e.g. -3 and $+4$.

Key vocabulary

Negative	Positive	Increase
Decrease	Difference	

Key questions

How could you use the number line to help perform this calculation?

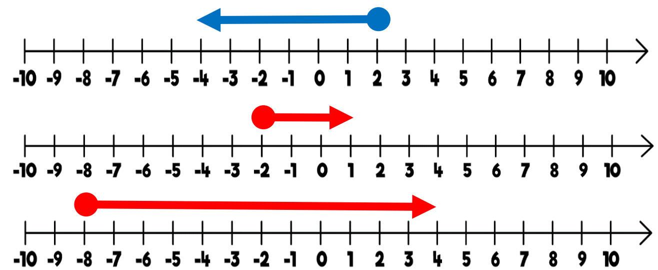
What is $4 - 4$? What is $-4 + 4$? What do you notice?

How is $-3m + 5m$ different from $-3 + 5$?

How are they the same?

Exemplar Questions

Complete the equation for each representation.



$2 - \square = -4$

$-2 + \square = \square$

$\square + \square = \square$

Work out the missing numbers.

$4 - 8 = \square$ $\qquad$ $2 - \square = -8$

$-3 + 23 + 30 = \square$ $\qquad$ $-2 + \square = 42$

- The temperature in Moscow is -3°C at 8am. By 2pm the temperature has gone up by 10°C. What is the temperature at 2pm?
- The temperature in Cardiff is 4°C at 4pm. At 8pm, the temperature has dropped by 5°C. What is the temperature at 8pm?
- The temperature in Paris is 2°C at 3pm. At 3am the temperature is -3°C. What is the temperature difference?

Simplify the expressions.

- $3m - 8m$
- $-3d + 8d$
- $-3w + 3w$

Add directed numbers

Notes and guidance

Students can use double sided counters to model adding negative and positive numbers. Introducing zero pairs will be helpful for both addition/subtraction of directed numbers and help with the use of partitioning e.g. $6 + -4$ as $2 + 4 + -4 = 2 + 0 = 2$. Students may then generalise that adding a negative number is equivalent to a subtraction, although the emphasis should be on understanding the calculation rather than memorising rules.

Key vocabulary

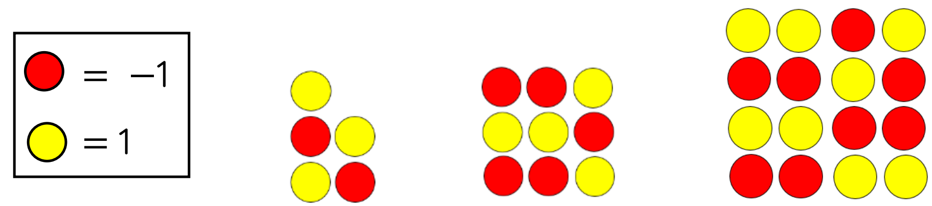
Add	Negative	Minus
Subtract	Partition	Zero pair

Key questions

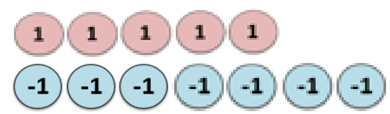
Why is adding a negative the same as subtracting?
Why is $100 + -102$ an easy calculation despite the large numbers? How does partitioning help?
Give an example to show the statement "Two negatives make a positive" is wrong.

Exemplar Questions

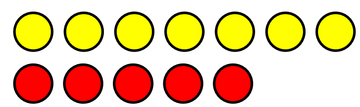
By making zero pairs, what is the total value of each set of counters?



Complete the calculations using the concrete manipulatives.
What does each counter represent?



$5 + -7 =$



$7 + -5 =$

How would you model $-7 + -5$?

Sort the number cards into pairs so that each pair has the same total.

−5	−9	−3	3	−11	1

Simplify the expressions by collecting like terms.

- $5a + -12a$
- $-5a + -12a$
- $-5a + 12a$

Subtract directed numbers

Notes and guidance

Students can explore sequences of equations in order to generalise and gain a stronger understanding of this concept. Another useful approach is to have a collection of mixed double-sided counters and see what happens to the total when some/all of the negative counters are removed. Avoid phrases such as "two negatives make a positive" as this leads to misconceptions such as "$-1 - 2 = +3$".

Key vocabulary

Subtract	Negative	Minus

Key questions

Using the manipulatives, what happens to the total when I take away 2 negatives?

What happens when the lowest score is removed? Does the total increase or decrease?

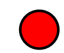
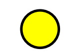
What happens when you subtract a negative number from a positive total? How can you represent this visually?

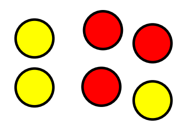
Exemplar Questions

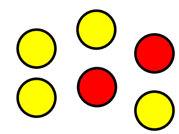
Complete the sequence of questions on the left, and then answer the questions on the right.

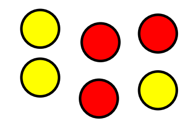
$3 - 3 =$
$3 - 2 =$
$3 - 1 =$
$3 - 0 =$
$3 - (-1) =$
$3 - (-2) =$
$3 - (-3) =$

$5 - 8 =$
$5 - (-8) =$
$-5 - (-8) =$
$-5 - 8 =$
$0 - 8 =$
$0 - (-8) =$

Find the totals of these sets of counters.  $= -1$  $= 1$

 =

 =

 =

How does the total change if you remove a red counter?
How does the total change if you remove a yellow counter?

In a singing competition, four judges give a competitor a score between -10 and 10 points. The scores given are as follows:

8	7	5	-3

- What is the total score?
- What is the mean score?
- If the lowest score is taken away, what is the new total score?

Multiplication with directed numbers

Notes and guidance

Students can use jumps on a number line and manipulatives to model multiplication with directed numbers. Drawing a carefully labelled bar model can also help (see example in the next step). The result of multiplication of two negatives can be justified with continuing patterns within a multiplication grid. It may be useful to teach this and the next step concurrently.

Key vocabulary

Product Multiply Commutative

Inverse

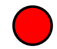
Key questions

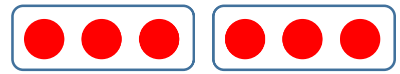
How could we use the number line to answer this question?

If $3 \times -2 = -6$, what is -3×-2? How do you know?

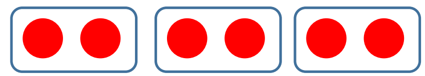
Why is $-3 \times 5a$ equal to $3 \times -5a$?
What other calculations give the same answer?

Exemplar Questions

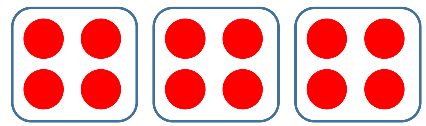
If  = -1, write calculations for the manipulatives below.



$-3 \times \square = \square$ and $2 \times \square = \square$



$-2 \times \square = \square$ and $3 \times \square = \square$



$-4 \times \square = \square$ and $3 \times \square = \square$

×	−2	−1	0	1	2
−2					
−1					
0					
1					
2					

Complete the multiplication grid and use it to answer the following questions.

- $2 \times -1 =$
- $-2 \times 1 =$
- $-1 \times -2 =$
- $-1 \times -2 =$

Calculate:

- $3 \times 2 =$
- $-3 \times 2 =$
- $3 \times -2 =$
- $-3 \times -2 =$
- $10 \times -12 =$
- $-10 \times 1.2 =$
- $100 \times -7.13 =$
- $-100 \times 0.713 =$

Multiplication and division

Notes and guidance

Students can use jumps on a number line and manipulatives to model multiplication with directed numbers. Drawing a carefully labelled bar model can also help (see example in the next step). The result of multiplication of two negatives can be justified with continuing patterns within a multiplication grid. It may be useful to teach this and the next step concurrently.

Key vocabulary

Product	Multiply	Commutative
Inverse		

Key questions

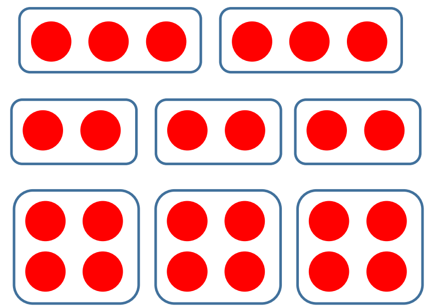
How could we use the number line to answer this question?

If $3 \times -2 = -6$, what is -3×-2? How do you know?

Why is $-3 \times 5a$ equal to $3 \times -5a$?
What other calculations give the same answer?

Exemplar Questions

If ● = −1, write calculations for the manipulatives below.

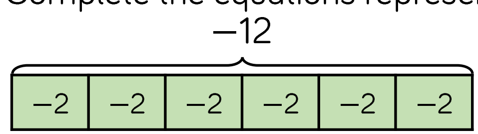


$-6 \div \square = \square$ and $-6 \div \square = \square$

$-6 \div \square = \square$ and $-6 \div \square = \square$

$-12 \div \square = \square$ and $-12 \div \square = \square$

Complete the equations represented by the diagrams.

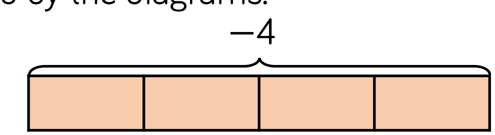


$-2 \times \square = \square$

$\square \times -2 = \square$

$-12 \div \square = \square$

$-12 \div \square = \square$



$\square \times \square = \square$

$\square \times \square = \square$

$\square \div \square = \square$

$\square \div \square = \square$

If we know $-3 \times -2 = 6$, we also know:
$6 \div -2 =$ ____
$6 \div -3 =$ ____

If we know $-5 \times -8 =$ ____, we also know:
____ $\div -5 =$ ____
____ $\div -8 =$ ____

White Rose Maths

Use a calculator for directed number

Notes and guidance

The main reason for this step is to develop is to develop students' calculator proficiency. Students should be introduced to the ± button through teacher modelling. Students could also be introduced to the fraction button as an alternative to the division button.

Key vocabulary

Calculator	Sign change	±
Fraction button		

Key questions

Explain how to use the ± on a calculator. How is it different from the – button?

What is the difference between -2.3^2 and $(-2.3)^2$

If there is no sign written in front of a number, is it positive or negative?

Exemplar Questions

Compare the calculations using <, > or =

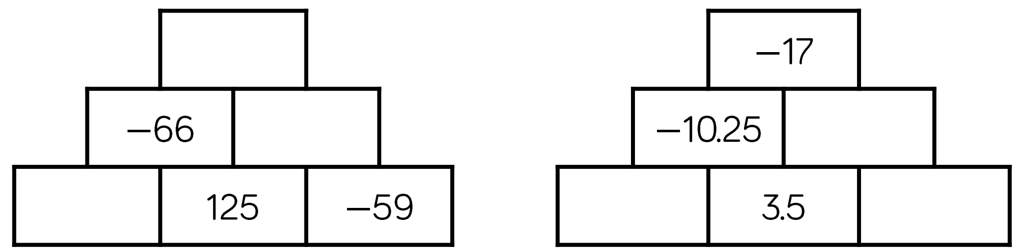
$17 - -0.5 + -2.7$ ◯ $17 - (2.7 - 0.5)$

$(-2.3)^2 \times -1.38$ ◯ $-2.3^2 \times -1.38$

$\frac{116.5+-8.9}{-2}$ ◯ $116.5 + -8.9 \div -2$

What's the same and what's different about the pairs of calculations?

Complete the addition pyramids.



–5 7 –2 10

Using **each** number card and any operations, can you make each of the target numbers? Can you find more than one way?

20 250 42 –40

Evaluate algebraic expressions

Notes and guidance

This small step continues to build on students' use of the order of operations, now through substitution. As in the previous small step, students should be encouraged to take care in organising their recording of work, ensuring they have substituted accurately and maintained the correct order of calculations throughout. Correct use of brackets around negative numbers should be modelled.

Key vocabulary

Substitute

Expression

Order of operations

Key questions

How do we substitute values into an expression?

What is the correct order of operations?

Why is it useful to put negative numbers in brackets when substituting?

Exemplar Questions

Evaluate the expressions by substituting the values $a = 5$, $b = -3$, $c = -1$ and $d = 0$

- $a - b$
- $3(a - b)$
- $-3(a - b)$
- bd
- $\frac{bd}{2}$
- c^2
- $2c$
- $2c - b^2$

Using the same values of a, b, c and d, write an algebraic expression that gives the values.

1   10   –10   55

What mistake has Tommy made?

Substitute $x = 3$ and $y = -5$ into the expression $x - y^2$

$= 3 - -5^2$

$= 3 + 5^2$

$= 28$

How could he make sure he doesn't make this mistake in future?

Substitute $m = -4$ and $n = -7$ into the expressions, then place in ascending order.

$3m$, $-3m$, $2n$, $-2n$, $5m + 3n$, $5m - 3n$

Introduction to two-step equations

Notes and guidance

Students have met one-step equations and these should be revised in order to move on to two-step equations. Practice of one-step equations can now of course include ones with negative solutions. Students could use concrete manipulatives, such as cups and counters (including 'zero pairs') and bar models, to represent the ideas pictorially. These should be used alongside written calculations.

Key vocabulary

Solve	Equation	Balance
Solution	Function machine	Zero pair

Key questions

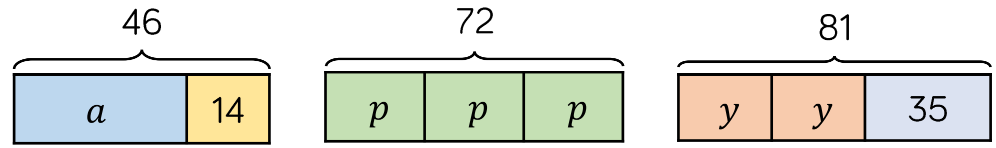
How do you know if an equation can be solved in one step or more than one step?

Can the solution to an equation be a negative number?

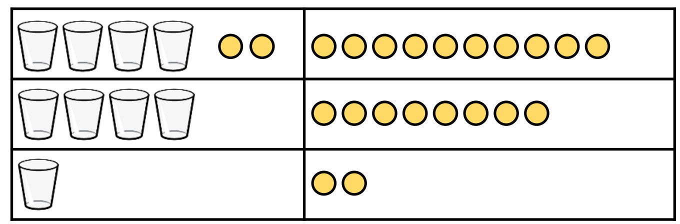
How does a bar model help you to decide what step to take first when solving a multi-step equation?

Exemplar Questions

Use the bar model to write an equation and solve it to find the unknown value.

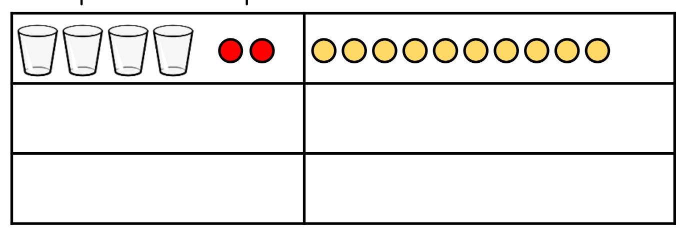


How does the diagram connect to the calculation?



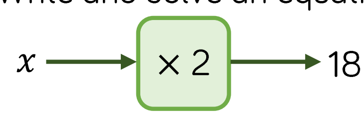
$$4x + 2 = 10$$
$$-2 \qquad -2$$
$$4x = 8$$
$$\div 4 \qquad \div 4$$
$$x = 2$$

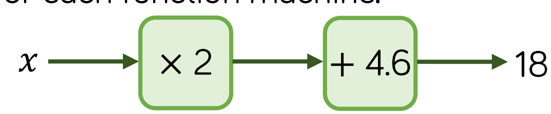
What's the same and what's different about these calculations?
Complete the representation and calculation.



$$4x - 2 = 10$$

Write and solve an equation for each function machine.





Solve the equations.

$4m = 96$ $\qquad 4m = -96$ $\qquad g + 46 = 91$ $\qquad g + 46 = 11$

Solve two-step equations

Notes and guidance

Students continue to develop their understanding of solving equations in this small step, which includes more negative number work and negative solutions. There are opportunities to consider how varying the signs, coefficients and operations in an equations affects its solution. Students should continue to use bar models and concrete representations as appropriate.

Key vocabulary

Solve, Equation, Balance, Positive/negative solution

Key questions

What is the same and what is different about these questions and answers?

When is it most useful to use a bar model for a two-step equation?

How do you know the order of steps to take to solve an equation?

Exemplar Questions

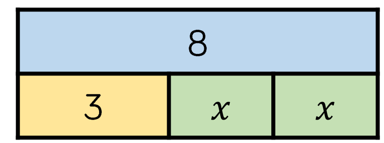
Explain why each equation has the same solution.

$2a + 5 = 1$ $\quad 4a + 10 = 2$ $\quad 2 = 10 + 4a$

$2a - 1 = -5$ $\quad 10 = 2 - 4a$ $\quad -2a - 5 = -1$

Find the solution and write three more equations with the same solution.

Use the bar model to solve $8 - 2x = 3$



Solve the equations.

$2x + 3 = 8$ $\quad 2x - 3 = 8$ $\quad \frac{x}{2} - 3 = 8$ $\quad 8 - \frac{x}{2} = 3$

Solve the equations.

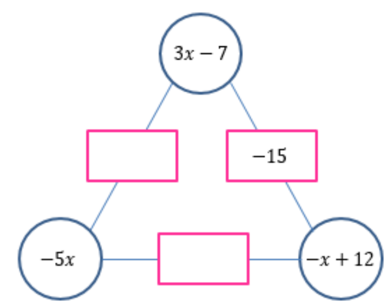
Which are most suited to be represented with a bar model?

$5x + 3 = 28$ $\quad 5x + 28 = 3$

$\frac{x}{5} - 3 = 28$ $\quad \frac{x - 3}{5} = 28$

The value in the rectangle is the total of the expressions in the circles on either side.

Complete the missing numbers in the rectangles.



Use order of operations

Notes and guidance

Students build on their understanding of the order of operations, now including negative numbers. Students should be encouraged to pay careful attention to their recording of solutions. Discussion of common misconceptions is useful here. A reminder about commutativity should help students to understand why e.g. multiplication and division are of equal priority.

Key vocabulary

Order of operations	Indices	Brackets
Commutative	Priority	

Key questions

What does it mean when there is a number directly in front of a bracket e.g. $3(6 + 4)$?

What's the difference between $(-6)^2$ and -6^2?

Does a negative number change the order of operations?

Exemplar Questions

Which is the correct answer? What have the others done wrong?

Student 1	Student 2	Student 3
$(5 - 3^2) \div 8$	$(5 - 3^2) \div 8$	$(5 - 3^2) \div 8$
$= 2^2 \div 8$	$= 5 - 9 \div 8$	$= (5 - 9) \div 8$
$= 4 \div 8$	$= 5 - 1.125$	$= -4 \div 8$
$= 0.5$	$= 3.875$	$= -0.5$

Calculate. Show each step of your working.

- $21 + 18 \div -3$
- $\frac{21 + 18}{-3}$
- $-3 \times 5 + 8 - 7$
- $3(5 + 8) - 7$
- $-6^2 + 14 \times 2$
- $(-6)^2 + 14 \times 2$
- $-3 + 4^2$
- $(-3 + 4)^2$

Substitute $n = 1$, $n = 2$, $n = 3$ and $n = 4$ into the expressions. Are either of the sequences linear?

$3(n - 5)$

n	1	2	3	4
Output				

$3(n - 5)^2$

n	1	2	3	4
Output				

Understand square roots

Notes and guidance

Students should be secure on what a square number is before this small step e.g. by using manipulatives to show why they are called square numbers. Students can logically come to the conclusion that positive numbers have more than one square root by exploring ideas from previous small steps, such as finding the square numbers in the multiplication grid shown.

Key vocabulary

Square	Square root	Inverse
Positive	Negative	Power

Key questions

What is a square number?

What is the inverse of multiplication/squaring a number?

What is the difference between $(-5)^2$ and -5^2?

Does 5 have a square root?

Exemplar Questions

×	−3	−2	−1	0	1	2	3
−3	9	6	3	0	−3	−6	−9
−2	6	4	2	0	−2	−4	−6
−1	3	2	1	0	−1	−2	−3
0	0	0	0	0	0	0	0
1	−3	−2	−1	0	1	2	3
2	−6	−4	−2	0	2	4	6
3	−9	−6	−3	0	3	6	9

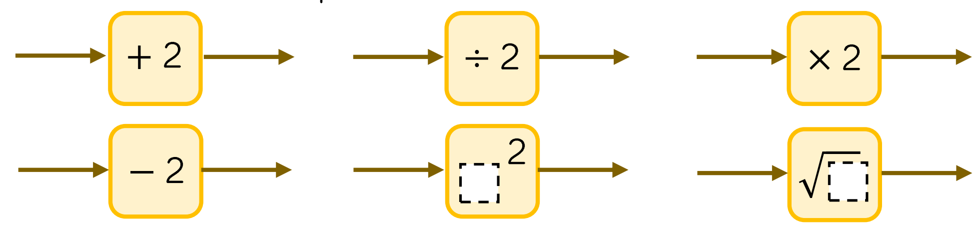
What do you notice about the numbers across the diagonal that have been shaded?

Group the calculations and their answers.

3^2 | $(-2)^2$ | 4 | $(-3)^2$ | 2^2

1^2 | 9 | $(-1)^2$ | 1

If 9 is the **output** for each function machine, what could the input be?
Is there more than one possible answer?



Calculate.

$\sqrt{4}$ $\sqrt{16}$ $\sqrt{100}$ $\sqrt{225}$

Between which integer values would $\sqrt{45}$ lie?
Complete the inequalities.

$\square < \sqrt{45} < \square$ or $\square < \sqrt{45} < \square$

The area of a square is 169 cm². What could the side length be?

Explore higher powers and roots H

Notes and guidance

Students continue to further their understanding of powers by extending their knowledge of square and cube numbers. If appropriate, extend to look at higher powers. Understanding roots as the inverse operation will help understanding of powers. Students need to be taught that a radical without a number ($\sqrt{}$) means square root.

Key vocabulary

Power	Indices	Inverse
Root	Exponent	

Key questions

What does cube mean?

How do you raise a number to the fourth power?

How do you find roots and powers on your calculator?

If a number has two square roots, does it have three cube roots?

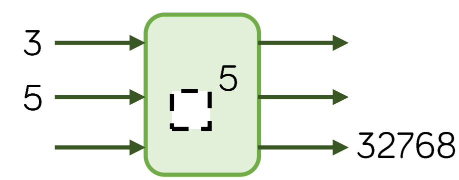
Exemplar Questions

Work out the calculations.
What do you notice about the digit in the ones column?

$3^2 = 3 \times 3 =$	$4^2 =$	$5^2 =$
$3^3 =$	$4^3 =$	$5^3 =$
$3^4 =$	$4^4 =$	$5^4 =$
$3^5 =$	$4^5 =$	$5^5 =$

Use a calculator to complete the function machines.

Use a calculator to complete the function machine.



What do you notice about the answers to the calculations?

256^1	16^2	4^4	2^8
729^1	27^2	9^3	3^6