Gradients & lines

Year 11

#MathsEveryoneCan

White Rose Maths

	Week 1	Week 2	Week 3	Week 4	Week 5	Week 6	Week 7	Week 8	Week 9	Week 10	Week 11	Week 12
Autumn	**Graphs**						**Algebra**					
	Gradients & lines		Non-linear graphs		Using graphs		Expanding & factorising		Changing the subject		Functions	
Spring	**Reasoning**						**Revision and Communication**					
	Multiplicative		Geometric		Algebraic		Transforming & constructing		Listing & describing		Show that...	
Summer	**Revision**						**Examinations**					

Autumn 1: Graphs

Weeks 1 and 2: Gradients and lines

This block builds on earlier study of straight line graphs in years 9 and 10. Students plot straight lines from a given equation, and find and interpret the equation of a straight line from a variety of situations and given information. There is the opportunity to revisit graphical solutions of simultaneous equations. Higher tier students also study the equations of perpendicular lines.
National Curriculum content covered includes:

- move freely between different numerical, algebraic, graphical and diagrammatic representations
- plot and interpret graphs
- interpret the gradient of a straight line graph as a rate of change
- use the form $y = mc + c$ to identify parallel **{and perpendicular}** lines; find the equation of the line through two given points, or through one point with a given gradient
- find approximate solutions to two simultaneous equations in two variables (linear/linear **{or linear/quadratic})** using a graph

Weeks 3 and 4: Non-linear graphs

Students develop their knowledge of non-linear graphs in this block, looking at quadratic, cubic and reciprocal graphs, so they recognise the different shapes. They find the roots of quadratics graphically, and will revisit this when they look at algebraic methods in the Functions block during Autumn 2, where they will also look at turning points. Higher tier students also look at simple exponential graphs and the equation of a circle. Note that the equation of the tangent to a circle is covered later when the circle theorem of tangent/radius is met. Higher students also extend their understanding of gradient to include instantaneous rates of change considering the gradient of a curve at a point.
National Curriculum content covered includes:

- move freely between different numerical, algebraic, graphical and diagrammatic representations
- recognise, sketch and interpret graphs of linear functions, quadratic functions, simple cubic functions, the reciprocal function 1 **{the exponential function $y = k^x$ for positive values of k}**
- plot and interpret graphs (including reciprocal graphs **{and exponential graphs}**)
- find approximate solutions using a graph
- identify and interpret roots, intercepts of quadratic functions graphically
- **{recognise and use the equation of a circle with centre at the origin;}**

Weeks 5 and 6: Using graphs

This block revises conversion graphs and reflection in straight lines. Students also study other real-life graphs, including speed/distance/time, constructing and interpreting these. Higher tier students also investigate the area under a curve.
National Curriculum content covered includes:

- plot and interpret graphs of non-standard functions in real contexts, to find approximate solutions to problems such as simple kinematic problems involving distance, speed and acceleration
- **{interpret the gradient at a point on a curve as the instantaneous rate of change; apply the concepts of instantaneous and average rate of change (gradients of tangents and chords) in numerical, algebraic and graphical contexts}**
- **{calculate or estimate gradients of graphs and areas under graphs (including quadratic and other non-linear graphs), and interpret results in cases such as distance-time graphs, velocity-time graphs and graphs in financial contexts}**

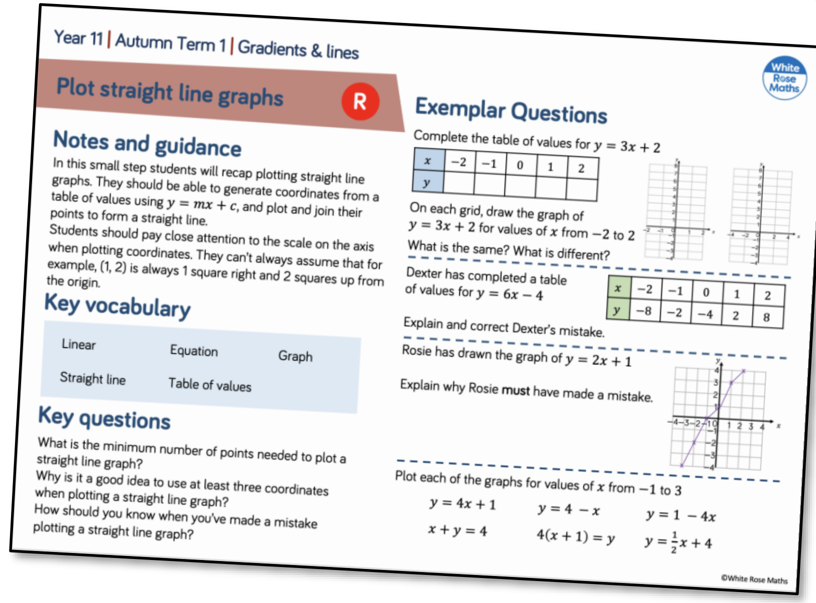
Why Small Steps?

We know that breaking the curriculum down into small manageable steps should help students to understand concepts better. Too often, we have noticed that teachers will try and cover too many concepts at once and this can lead to cognitive overload. We believe it is better to follow a "small steps" approach.

As a result, for each block of content in the scheme of learning we will provide a "small step" breakdown. ***It is not the intention that each small step should last a lesson – some will be a short step within a lesson, some will take longer than a lesson.*** We would encourage teachers to spend the appropriate amount of time on each step for their group, and to teach some of the steps alongside each other if necessary.

What We Provide

- Some ***brief guidance*** notes to help identify key teaching and learning points
- A list of ***key vocabulary*** that we would expect teachers to draw to students' attention when teaching the small step,
- A series of ***key questions*** to incorporate in lessons to aid mathematical thinking.
- A set of questions to help ***exemplify*** the small step concept that needs to be focussed on.


Year 11 | Autumn Term 1 | Gradients & lines

Plot straight line graphs R

Notes and guidance

In this small step students will recap plotting straight line graphs. They should be able to generate coordinates from a table of values using $y = mx + c$, and plot and join their points to form a straight line.
Students should pay close attention to the scale on the axis when plotting coordinates. They can't always assume that for example, (1, 2) is always 1 square right and 2 squares up from the origin.

Key vocabulary

Linear	Equation	Graph
Straight line	Table of values	

Key questions

What is the minimum number of points needed to plot a straight line graph?
Why is it a good idea to use at least three coordinates when plotting a straight line graph?
How should you know when you've made a mistake plotting a straight line graph?

Exemplar Questions

Complete the table of values for $y = 3x + 2$

x	-2	-1	0	1	2
y					

On each grid, draw the graph of $y = 3x + 2$ for values of x from -2 to 2
What is the same? What is different?

Dexter has completed a table of values for $y = 6x - 4$

x	-2	-1	0	1	2
y	-8	-2	-4	2	8

Explain and correct Dexter's mistake.

Rosie has drawn the graph of $y = 2x + 1$
Explain why Rosie **must** have made a mistake.

Plot each of the graphs for values of x from -1 to 3

$y = 4x + 1$ $y = 4 - x$ $y = 1 - 4x$
$x + y = 4$ $4(x + 1) = y$ $y = \frac{1}{2}x + 4$

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- These include reasoning and problem-solving questions that are fully integrated into the scheme of learning. Depending on the attainment of your students, you may wish to use some or all of these exemplars, which are in approximate order of difficulty. Particularly challenging questions are indicated with the symbol .
- For each block, we also provide ideas for key representations that will be useful for all students.

Some of the small steps are in **bold** and labelled with H to indicate this is Higher tier GCSE content. We would encourage teachers to use these with as many students as possible – if you feel your class can access any particular small step, then please include it in your planning. Steps that review content covered at Key Stage 3 are labelled R .

Gradients & lines

Small Steps

- Equations of lines parallel to the axis (R)
- Plot straight line graphs (R)
- Interpret $y = mx + c$ (R)
- Find the equation of a straight line from a graph (1) (R)
- Find the equation of a straight line from a graph (2)
- Equation of a straight-line graph given one point and gradient
- Equation of a straight-line graph given two points
- Determine whether a point is on a line

(H) denotes Higher Tier GCSE content

(R) denotes 'review step' – content should have been covered at KS3

Gradients & lines

Small Steps

- Solve linear simultaneous equations graphically R
- **Recognise when straight lines are perpendicular** H
- **Find the equations of perpendicular lines** H

H denotes Higher Tier GCSE content
R denotes 'review step' – content should have been covered at KS3

Lines parallel to the axis

R

Notes and guidance

In this small step students will revise and extend their learning from previous years. They should be able to recognise and use the equations of lines parallel to the axis. Students should understand that any point on a line satisfies the equation of that line. They should know that all lines of the form $y = a$ are parallel to the x-axis and each other, and all lines of the form $x = b$ are parallel to the y-axis and each other.

Key vocabulary

Parallel	Horizontal	Vertical	Straight line
Axis	Equation	Graph	Intercept

Key questions

Which axis is $y = 4$ parallel to? How do you know?

All of the points on the line $x = 7$ have something in common. What is it?

What is the equation of the x-axis?

What is the equation of the y-axis?

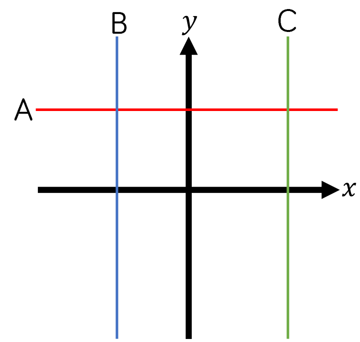
Exemplar Questions

Which of these points lie on the line $y = 9$?

$(9, 0)$ $(0, 0)$ $(0, 9)$ $(9, 9)$ $(9, 2.7)$

$(\frac{1}{2}, 9)$ $(\sqrt{81}, 3)$ $(8, 3^2)$ $(\frac{18}{2}, \frac{27}{3})$ $(1, -9)$

Lines A, B and C are all parallel to one of the axes.



Line A passes through the point $(2, 7)$

Line B passes through the point $(-3, -5)$

Lines A and C intercept at $(32, a)$

Write down the equation of each line.

What is the value of a?

Here are the equations of 8 lines, some of which need simplifying.

$y = 7$ $x + 4 = 11$ $y - 5 = 0$ $x - 5 = 0$

$x = 7$ $y + 3 = 7 - 2$ $-y = -9$ $y = 0$

- Which of these lines are parallel to the y-axis?
- Which of these lines are parallel to the x-axis?
- Which of these lines are parallel to neither the x- nor the y-axis?

Plot straight line graphs R

Notes and guidance

This step revisits plotting straight line graphs. Students should be able to generate coordinates for a table of values using $y = mx + c$, and plot and join their points to form a straight line. Students should pay close attention to the scale on the axis when plotting coordinates. They cannot always assume that for example, (1, 2) is always 1 square right and 2 squares up from the origin, so this misconception should be challenged.

Key vocabulary

Linear	Equation	Graph
Straight line	Table of values	

Key questions

What is the minimum number of points needed to plot a straight line graph?

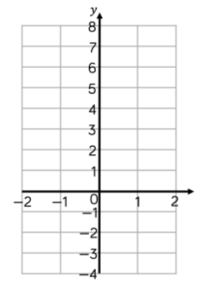
Why is it a good idea to use at least three coordinates when plotting a straight line graph?

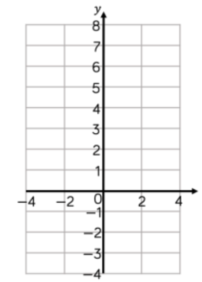
How can you tell when you've made a mistake plotting a straight line graph?

Exemplar Questions

Complete the table of values for $y = 3x + 2$

x	−2	−1	0	1	2
y					





On each grid, draw the graph of $y = 3x + 2$ for values of x from −2 to 2

What is the same? What is different?

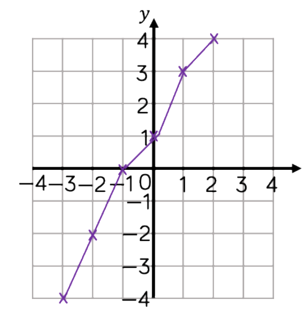
Dexter has completed a table of values for $y = 6x - 4$

x	−2	−1	0	1	2
y	−8	−2	−4	2	8

Explain and correct Dexter's mistake.

Rosie has drawn the graph of $y = 2x + 1$

Explain how you know Rosie **must** have made a mistake.



Plot each of the graphs for values of x from −1 to 3

$y = 4x + 1$ $\quad y = 4 - x$ $\quad y = 1 - 4x$

$x + y = 4$ $\quad 4(x + 1) = y$ $\quad y = \frac{1}{2}x + 4$

Interpret $y = mx + c$

R

Notes and guidance

Students may need reminding that when the equation of a line is given in the form $y = mx + c$, m represents the gradient, and the graph intercepts the y-axis at $(0, c)$.
Building on from the previous step, students could be encouraged to plot the straight lines $y = mx + a$ and $y = mx + b$ to see that they are parallel. Similarly, they could plot $y = mx + a$ and $y = nx + a$ to see that they intercept the y-axis at the same point.

Key vocabulary

Gradient	y-intercept	Equation
Parallel	Linear	Straight line

Key questions

In $y = mx + c$, what do m and c represent?

In $y = mx + c$, what do x and y represent?

What does it mean when two lines have the same gradient?

What does it mean when two lines have the same y-intercept?

Exemplar Questions

Draw each of the graphs on the same set of axis.

$y = 3x$ $\quad$ $y = 3x + 1$ $\quad$ $y = 3x + 2$ $\quad$ $y = 3x + 5$

What do you notice?
What do you think the graph of $y = -3x$ will look like?

Draw each of the graphs on the same set of axis.

$y = x + 1$ $\quad$ $y = 2x + 1$ $\quad$ $y = 3x + 1$ $\quad$ $y = 4x + 1$

What do you notice?

Write down the gradient and y-intercept of each line.

$y = 5x + 7$ $\quad$ $y = 5x - 7$ $\quad$ $y = 7 - 5x$ $\quad$ $y = -7 - 5x$

$y = \frac{1}{2}x$ $\quad$ $17 - 8x = y$ $\quad$ $y = 3(2x + 1)$ $\quad$ $2y = 10 + 6x$

$y = 9 + 2x$	$y = 8 - 2x$
$2y = 2x + 16$	$y = 4\left(\frac{1}{2}x + 9\right)$

Which lines are parallel?

Which lines have the same y-intercept?

How do you know?

Equation of a line from a graph R

Notes and guidance

Some students may need to revise finding the gradient of a line before they find its equation. This step reiterates that the gradient is m and the y-intercept is c, but sometimes students find it conceptually more difficult to 'work backwards' in this way. It is helpful to consider what information can be seen immediately from the graph (usually the y-intercept) before calculating the gradient. This step focuses on graphs will simple equal scales, with more complex scales to follow.

Key vocabulary

Gradient	y-intercept	Equation
Parallel	Linear	Straight line

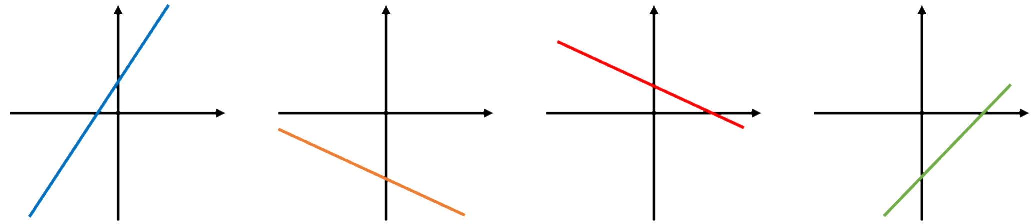
Key questions

How do you know if a straight line has a positive/negative gradient?

How do you know if a straight line has a positive/negative y-intercept?

How do you calculate the gradient of a line?

Exemplar Questions

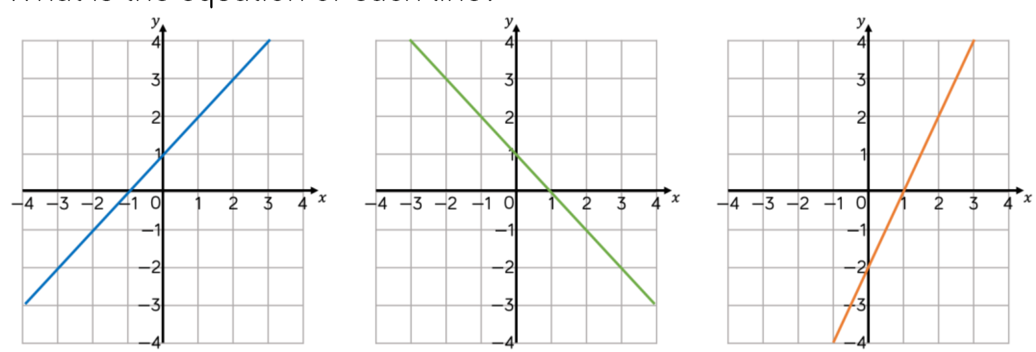


Is the gradient of each line positive or negative? How do you know?

Is the y-intercept of each line positive or negative? How do you know?

What is the gradient of each line?

What is the equation of each line?



Equation of a line from a graph (2)

Notes and guidance

Building on from the previous small step, students will now look at finding the equation of a line from a graph where the axes are more complex. Rather than thinking of the gradient as ‘for every 1 square across, how many squares up/down’, students now need to shift their thinking to consider the gradient as being ‘for every 1 unit across, how many units up/down’ and then extending further to ‘change in y divided by change in x’.

Key vocabulary

Gradient	y-intercept	Equation	Axis
Parallel	Linear	Straight line	Scale

Key questions

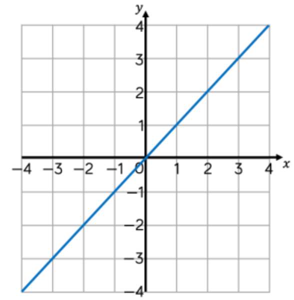
What is the scale on each axis?

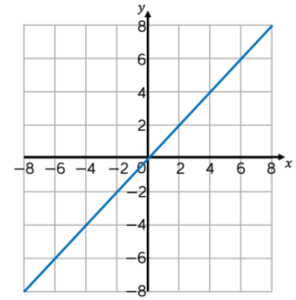
How does the scale affect the gradient?

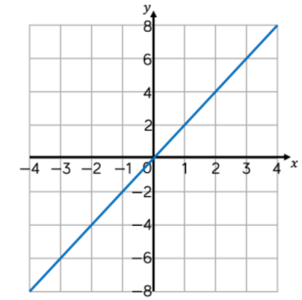
Does the scale on the axis affect how you find out the y-intercept?

Exemplar Questions

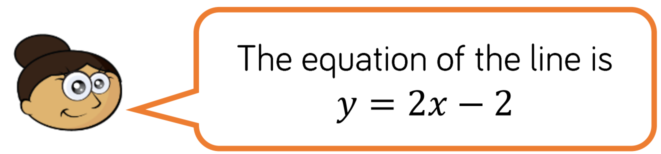
What is the equation of each line?

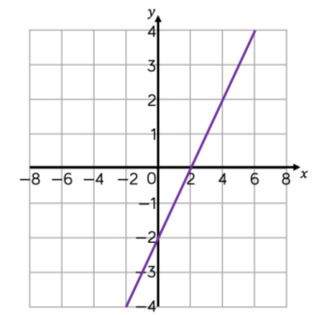






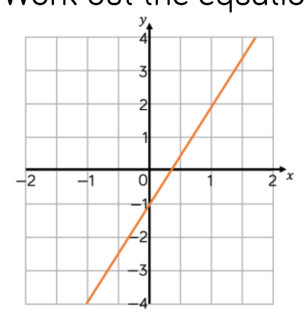
What is the same? What is different?

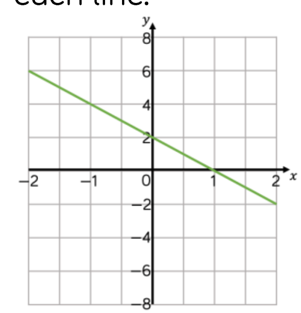


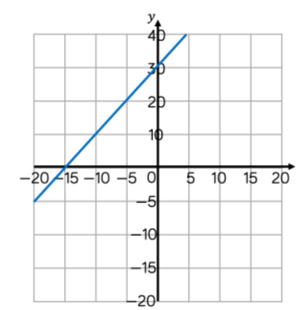


What mistake has Dora made?

Work out the equation of each line.







Equation of a line – point & gradient

Notes and guidance

Students need to be able to find the equation of a line given the gradient and a point that lies on the line. Using their knowledge of parallel lines having the same gradient, they can find the equation of a line parallel passing through a point. Students need to be exposed to examples where the point is the y-intercept and where they need to calculate the y-intercept themselves.

Key vocabulary

Equation	Line	Gradient	y-intercept
Parallel	Point	Coordinates	Substitute

Key questions

Is the point you've been given the y-intercept?

If not, how can you work out the y-intercept?

What does it mean when two lines are parallel?

Exemplar Questions

The gradient of line A is 4
Line A passes through the point (0, 5)
What is the equation of line A?

Line B is parallel to line A and passes through the point (0, −2)
What is the equation of line B?

A line has a gradient of −2 and passes through the point (1, −4)

What is the equation of the line?

A straight line has a gradient of $\frac{1}{2}$ and passes through the point (−2, 0)

The equation of the line is $y = \frac{1}{2}x - 2$

What mistake has Tommy made?

Work out the equation of a line parallel to $2y - 8 = 4x$ that passes through the point (−5, −7).

L_1 passes through the points (2, 7) and (12, 32)
L_2 is parallel to L_1 and passes through the point (4, 12)
Work out the equation of L_2

Equation of a line - two points

Notes and guidance

Students now need to be able to work out the equation of a line from two points. They should start by working out the equation of a line where one of the points is the y-intercept. They will then need to use their knowledge of substitution and solving equations to work out the y-intercept for themselves. It is essential they understand that to calculate the y-intercept of any line, they need to substitute $x = 0$

Key vocabulary

Gradient	y-intercept	Equation
Parallel	Linear	Straight line

Key questions

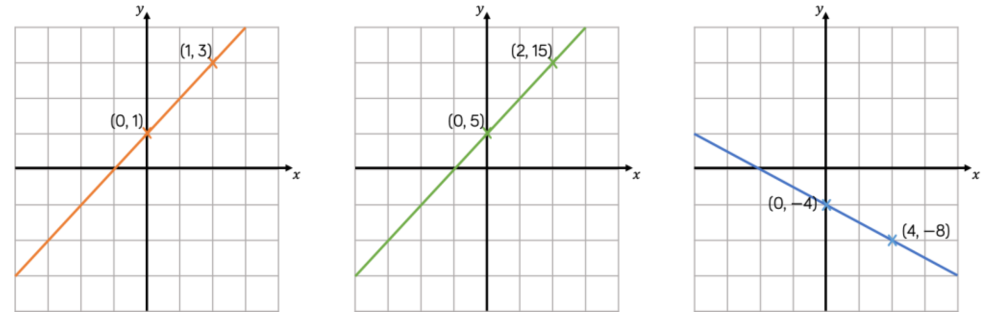
Is the gradient positive or negative? How do you know?

What is the gradient of the line? How do you know?

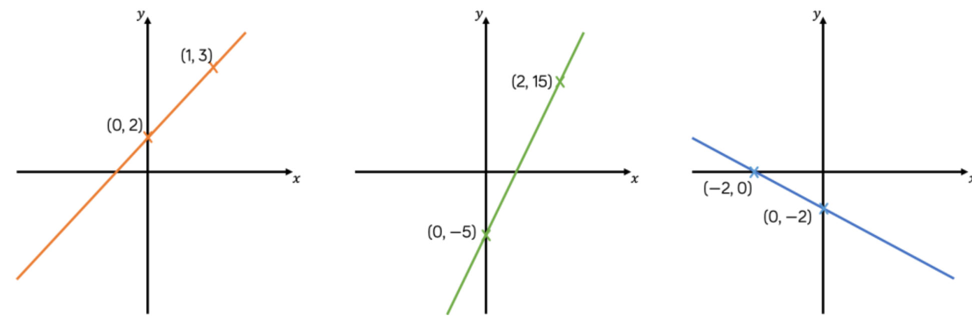
What is the x coordinate at the y-intercept? How do you know?

Exemplar Questions

Work out the equation of each line.



Work out the equation of each line.



Work out the equation of the line that passes through each pair of points.

- $(0, 5)$ and $(3, 14)$
- $(0, 2)$ and $(2, -4)$
- $(-6, -2)$ and $(0, 0)$
- $(2, 5)$ and $(3, 14)$
- $(4, 2)$ and $(2, -4)$
- $(0, a)$ and $(4, a+12)$

Determine whether a point is on a line

Notes and guidance

Students need to understand that the equation of a line is a relationship between the x and y coordinates at any point on that line. For example, on the line $y = x + 3$, every y coordinate is 3 more than the x coordinate. Any point on a grid that does not satisfy this equation, therefore does not lie on the line. Students could be extended further to explore whether a point not on the line is either above or below the line.

Key vocabulary

Equation	Satisfies	Coordinate
Below	Above	Substitute

Key questions

What is the relationship between the x and y coordinates at any point on the line $y = 2x$?

How do you know if a line passes through a point?

How does drawing the graph help you decide if a point is above or below the line? Can you tell without a graph?

Exemplar Questions

Circle the points where the y-coordinate is 3 greater than the x-coordinate.

(5, 2) (4, 12) (7, 10) (1, 2) $(\frac{5}{2}, 5\frac{1}{2})$

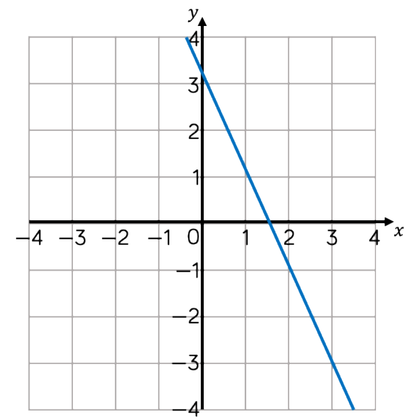
Hence, determine which points lie on the line $y = x + 3$

Does the point (7, 5) lie on the line $y = 2x - 9$?

How do you know?

Show that the point (8, − 8) does not lie on the line $y = -\frac{1}{2}x + 4$

The graph shows the line $y = 3 - 2x$



- Does the point (1, − 1) lie above or below the line?
- Does the point (3, 4) lie above or below the line?
- Does the point (17, 12) lie above or below the line?
- The point $(a, -15)$ lies on the line. Work out the value of a.

Simultaneous Equations

R

Notes and guidance

This small steps provides students with opportunity to revise and extend their knowledge of both solving linear simultaneous equations and plotting linear graphs. They should understand that two straight lines will only ever intercept at a single point, and the coordinates of this point provide the solutions to the pair of simultaneous equations. Students should be aware that where the point of intersection is difficult to interpret, their solutions are estimates.

Key vocabulary

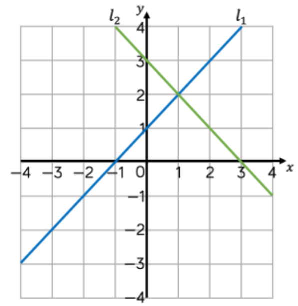
Simultaneous	Equations	Linear
Interception	Coordinates	Solutions

Key questions

How many solutions do a pair of linear simultaneous equations have?
How many points of intersection do a pair of linear graphs have? Is this always the case?
How does knowing the coordinates of a point of intersection help you solve a pair of simultaneous equations?

Exemplar Questions

Two lines, l_1 and l_2 are shown on the graph.



- What is the equation of l_1?
- What is the equation of l_2?
- What are the coordinates of the point of intersection?

Solve the pair of simultaneous equations.

$$3x + y = 8$$
$$5x + y = 14$$

Draw the graph of each line.
$$3x + y = 8$$
$$5x + y = 14$$

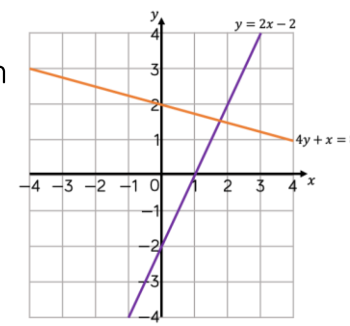
What are the coordinates of the point of intersection?

What do you notice? What is the same? What is different?

The graphs of the straight lines with equations $y = 2x - 2$ and $4y + x = 8$ have been drawn on the grid.

Use the graph to estimate the solution to the simultaneous equations.

$$y = 2x - 2$$
$$4y + x = 8$$



Use an algebraic method to find the exact solution.

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Recognise perpendicular lines H

Notes and guidance

Students should already be familiar with the fact that perpendicular lines intercept at right angles. They should look at lines $y = 2x$ and $y = -\frac{1}{2}x$ and recognising that the product of the gradients of a pair of perpendicular lines will always be −1. Students need to know that when two lines are perpendicular, one gradient is the negative reciprocal of the other.

Key vocabulary

Parallel	Perpendicular	Gradient
Product	Reciprocal	Negative reciprocal

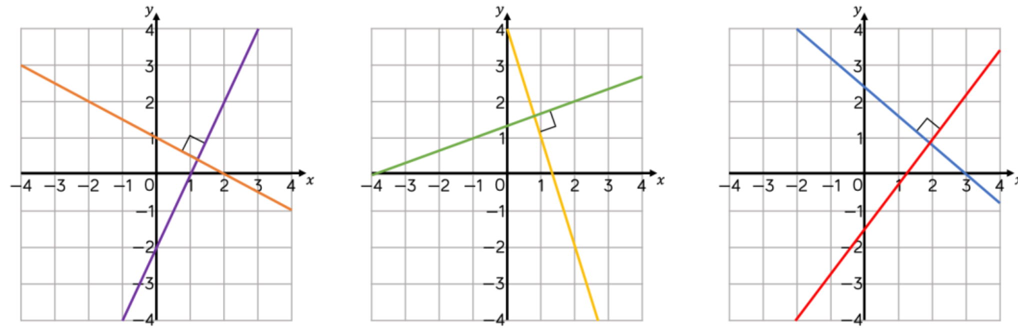
Key questions

When two lines are perpendicular, why must one gradient be positive and one be negative?

What is the product of the gradients of a pair of perpendicular lines?

Exemplar Questions

Each graph shows a pair of perpendicular lines.



Find the product of the gradients of each pair of lines.
What do you notice?

Fill in the missing numbers.

$5 \times \square = -1$ $\qquad -\frac{1}{7} \times \square = -1$ $\qquad \frac{2}{5} \times \square = -1$

Write down the negative reciprocal of each number.

$2 \qquad -9 \qquad \frac{3}{2} \qquad -\frac{4}{7} \qquad 2.5$

Line l_1 is given by the equation $y = 4x - 7$
Line l_2 is given by the equation $4y = 17 - 2x$

Show that l_1 and l_2 are not perpendicular.

Equations of perpendicular lines H

Notes and guidance

Students build on knowledge from the previous step and begin to find the equation of perpendicular lines. Using their understanding of the product of the gradients being −1, they first work out the gradient of a line that will be perpendicular. Once they are secure in this they can also start to calculate the y-intercept given a point on a line. Students could also find the equation of the perpendicular bisector of a given line segment.

Key vocabulary

Parallel	Perpendicular	Gradient
Product	Negative reciprocal	y-intercept

Key questions

How do you work out the gradient of a perpendicular line?
Once you know the gradient, how do you find the y-intercept?
How do you find the midpoint of a line segment? How does this help find the equation of the perpendicular bisector of the line segment?

Exemplar Questions

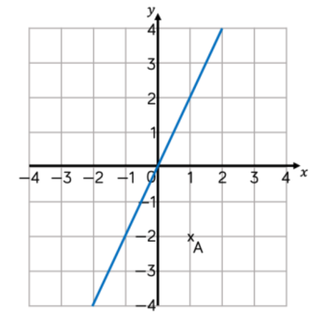
The line l_1 has the equation $y = 3x - 9$
The line l_2 is perpendicular to l_1 and passes through the origin.

What is the equation of l_2?

Point P has coordinates (3, 7).
Point Q has coordinates (9, 9).
Work out the equation of the line perpendicular to PQ that passes through the origin.

The graph of $y = 2x$ is shown on the grid.

Work out the equation of the line perpendicular to $y = 2x$ that passes through point A.



Two perpendicular lines, l_1 and l_2 are drawn on the grid.
The equation of l_1 is $y = 9 - 3x$
Work out the equation of l_2

Work out the equation of a line parallel to l_2 that passes through the point (8, 11).