Angles and Bearings

Year 10

#MathsEveryoneCan

White Rose Maths

	Week 1	Week 2	Week 3	Week 4	Week 5	Week 6	Week 7	Week 8	Week 9	Week 10	Week 11	Week 12
Autumn	**Similarity**						**Developing Algebra**					
	Congruence, similarity and enlargement			Trigonometry			Representing solutions of equations and inequalities			Simultaneous equations		
Spring	**Geometry**						**Proportions and Proportional Change**					
	Angles & bearings		Working with circles		Vectors		Ratios & fractions		Percentages and Interest		Probability	
Summer	**Delving into data**						**Using number**					
	Collecting, representing and interpreting data						Non-calculator methods		Types of number and sequences		Indices and Roots	

Spring 1: Geometry

Weeks 1 and 2: Angles and bearings

As well as the formal introduction of bearings, this block provides a great opportunity to revisit other materials and make links across the mathematics curriculum. Accurate drawing and use of scales will be vital, as is the use of parallel line angles rules; all of these have been covered at Key Stage 3. Students will also reinforce their understanding of trigonometry and Pythagoras from earlier this year, applying their skills in another context as well as using mathematics to model real-life situations.

National curriculum content covered:

- interpret and use bearings
- compare lengths…using scale factors
- apply Pythagoras' Theorem and trigonometric ratios to find angles and lengths in right-angled triangles {and, where possible, general triangles} in two dimensional figures
- {know and apply the sine rule and cosine rule to find unknown lengths and angles}
- use mathematical language and properties precisely
- reason deductively in geometry, number and algebra, including using geometrical constructions
- make and use connections between different parts of mathematics to solve problems

Weeks 4 and 5: Working with circles

This block also introduces new content whilst making use of and extending prior learning. The formulae for arc length and sector area are built up from students' understanding of fractions They are also introduced to the formulae for surface area and volume of spheres and cones; here higher students can enhance their knowledge and skills of working with area and volume ratios.

Higher tier students are also introduced to four of the circle theorems; the remaining theorems will be introduced in Year 11 when these four will be revisited.

National curriculum content covered:

- identify and apply circle definitions and properties, including: centre, radius, chord, diameter, circumference, tangent, arc, sector and segment
- calculate arc lengths, angles and areas of sectors of circles
- calculate surface areas and volumes of spheres, pyramids, cones and composite solids
- apply and prove the standard circle theorems concerning angles, radii, tangents and chords, and use them to prove related results

Weeks 5 and 6: Vectors

Students will have met vectors to describe translations during Key Stage 3 This will be revisited and used as the basis for looking more formally at vectors, discovering the meaning of $-\boldsymbol{a}$ compared to $\boldsymbol{a}$ to make sense of operations such as addition, subtraction and multiplication of vectors. This will connect to exploring 'journeys' within shapes linking the notation $\overrightarrow{AB}$ with $\boldsymbol{b}-\boldsymbol{a}$ etc. Higher tier students will then use this understanding as the basis for developing geometric proof, making links to their knowledge of properties of shape and parallel lines.

National curriculum content covered:

- describe translations as 2D vectors
- apply addition and subtraction of vectors, multiplication of vectors by a scalar, and diagrammatic and column representations of vectors; {use vectors to construct geometric arguments and proofs}.

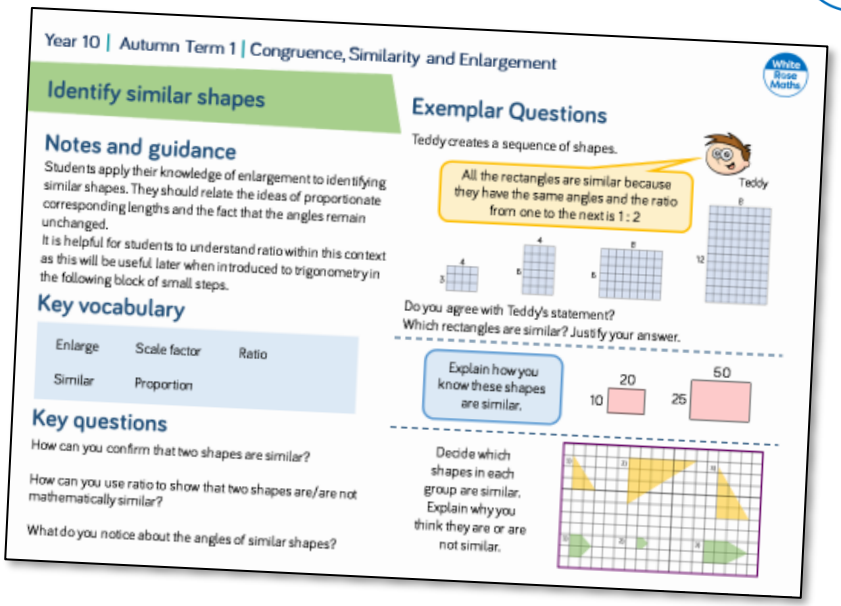
Why Small Steps?

We know that breaking the curriculum down into small manageable steps should help students to understand concepts better. Too often, we have noticed that teachers will try and cover too many concepts at once and this can lead to cognitive overload. We believe it is better to follow a "small steps" approach.

As a result, for each block of content in the scheme of learning we will provide a "small step" breakdown. ***It is not the intention that each small step should last a lesson – some will be a short step within a lesson, some will take longer than a lesson.*** We would encourage teachers to spend the appropriate amount of time on each step for their group, and to teach some of the steps alongside each other if necessary.

What We Provide

- Some ***brief guidance*** notes to help identify key teaching and learning points.
- A list of ***key vocabulary*** that we would expect teachers to draw to students' attention when teaching the small step.
- A series of ***key questions*** to incorporate in lessons to aid mathematical thinking.
- A set of questions to help ***exemplify*** the small step concept that needs to be focussed on.


Year 10 | Autumn Term 1 | Congruence, Similarity and Enlargement

Identify similar shapes

Notes and guidance

Students apply their knowledge of enlargement to identifying similar shapes. They should relate the ideas of proportionate corresponding lengths and the fact that the angles remain unchanged.
It is helpful for students to understand ratio within this context as this will be useful later when introduced to trigonometry in the following block of small steps.

Key vocabulary

Enlarge | Scale factor | Ratio | Similar | Proportion

Key questions

How can you confirm that two shapes are similar?

How can you use ratio to show that two shapes are/are not mathematically similar?

What do you notice about the angles of similar shapes?

Exemplar Questions

Teddy creates a sequence of shapes.

All the rectangles are similar because they have the same angles and the ratio from one to the next is 1 : 2

Teddy

Do you agree with Teddy's statement?
Which rectangles are similar? Justify your answer.

Explain how you know these shapes are similar.

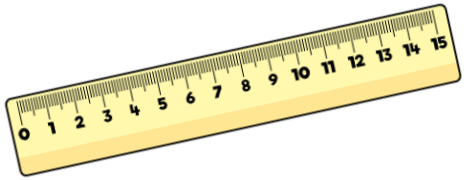
10 | 20 | 25 | 50

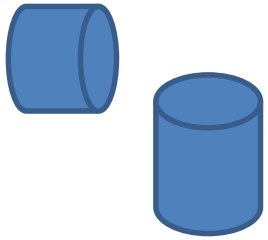
Decide which shapes in each group are similar. Explain why you think they are or are not similar.

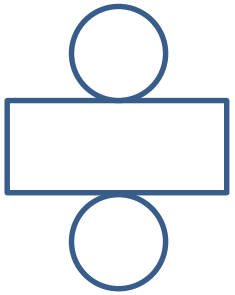
- These include reasoning and problem-solving questions that are fully integrated into the scheme of learning. Depending on the attainment of your students, you many wish to use some or all of these exemplars, which are in approximate order of difficulty. Particularly challenging questions are indicated with the symbol .
- For each block, we also provide ideas for key representations that will be useful for all students.

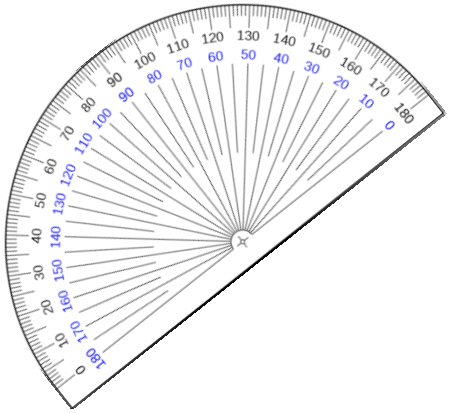
Some of the small steps are in **bold** and labelled with H to indicate this is Higher tier GCSE content. We would encourage teachers to use these with as many students as possible – if you feel your class can access any particular small step, then please include it in your planning. Steps that review content covered at Key Stage 3 are labelled R .

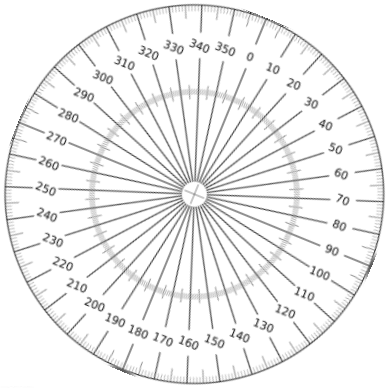
Key Representations

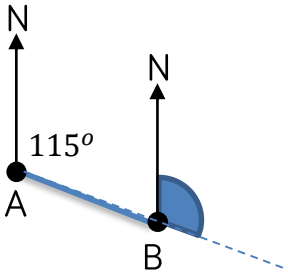


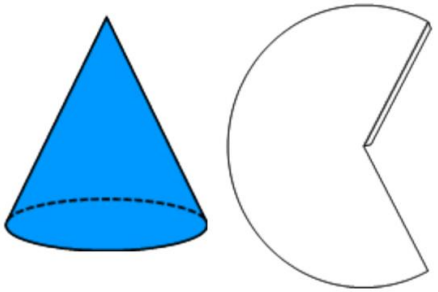












This is a very visual unit of work and students should be encouraged to make accurate diagrams and meaningful sketches throughout.

Using compasses in school or on a trip may be useful way in to bearings. Likewise, making and using actual scale drawings such as architect's drawings is helpful when looking at scale. Straws are always useful for reminding what happens with parallel and intersecting lines.

Creating nets of cylinders and cones is a good way of establishing how to find the formulae for their surface area. Likewise, sand can be used to demonstrate that the volume of a cone is one-third the volume of a cylinder of the same height.

Dynamic geometry is very useful to illustrate the circle theorems.

Angles and Bearings

Small Steps

- Use cardinal directions and related angles (R)
- Draw and interpret scale diagrams (R)
- Understand and represent bearings
- Measure and read bearings
- Make scale drawings using bearings
- Calculate bearings using angles rules
- Solve bearings problems using Pythagoras and trigonometry
- **Solve bearings problems using the sine and cosine rules** (H)

(H) Denotes Higher Tier GCSE content

(R) Denotes 'review step' – content should have been covered at KS3

Angles and compass points

R

Notes and guidance

In this small step, students will revisit their prior work on angles to prepare them for learning about bearings.

They should be comfortable with both measuring and drawing angles using a protractor and be able to identify angles using three letter notation. Familiarity with the major compass points and the angles between them will also be useful.

Key vocabulary

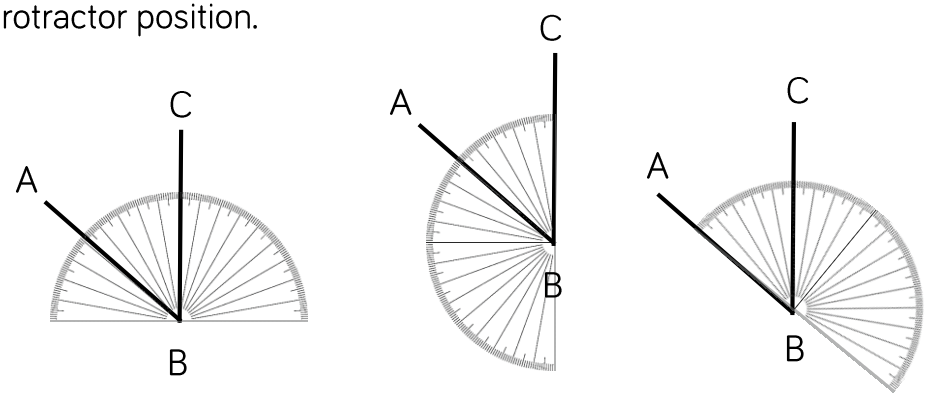
Compass	Point	Angle
Turn	Three letter notation	

Key questions

How can you draw a 200° angle using a 180° protractor?
What does due East mean?
What does the hat on $A\hat{C}D$ mean?
What else does the notation $A\hat{C}D$ tell you?
Are the angles $A\hat{C}D$ and $\angle ACD$ the same or different?

Exemplar Questions

Dora is measuring the angle $A\hat{B}C$ but isn't sure how to use a protractor. Explain how she could work out the angle using each protractor position.



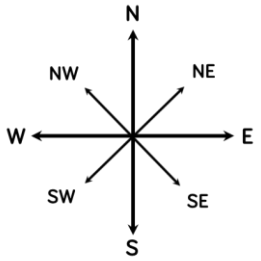
$C\hat{B}A$ is the angle extended around B from C to A. If $A\hat{B}C$ is θ, which of these expressions will find angle $C\hat{B}A$?

$180 - \theta$ $180 + \theta$ $360 - \theta$ $90 + \theta$

Turning clockwise, the angle between North and South-East is $135°$

Find three pairs of compass points so that the clockwise turn between them is

- $180°$
- $45°$
- $315°$
- $225°$



Draw/interpret scale diagrams R

Notes and guidance

This review step reminds students of Key Stage 3 work on scale, constructions and ratio. Students should be able to interpret scales as well as make scale drawings. It is useful to measure and draw angles from a variety of starting points and different inclinations rather than just from the horizontal. Likewise, a variety of scales should be explored using both 1 cm = 500 m and the 1 : 50 000 formats.

Key vocabulary

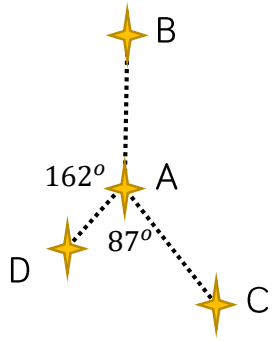
Enlarge	Scale factor	Ratio
Protractor	Convert	Similar

Key questions

Why is a scale drawing useful?
How accurate is a scale drawing?
On a scale drawing, what measurements always remain the same and which ones change? Think about lengths, areas, angles…
Which is more detailed, a 1 : 25 000 map or a 1 : 50 000 map?

Exemplar Questions

There are four stars in a constellation.
B and C are both 160 light years from A.
D is 60 light years from A.
Using a scale of 1 cm to represent 20 light years, draw a scale diagram of the constellation.
Use your diagram to find the distance between B and C in light years.



A tree is drawn to a scale of $1:30$. Explain what $1:30$ means.
Which part of the ratio represents measurements on the actual tree?
Complete the table.

	Actual Tree	Scale Drawing
Radius of trunk	69 cm	
Height of tree	2.8 m	
Width of leaf		6 mm
Acute angle between branch and trunk	7°	

Which measurement has not changed? Why?

What would be a sensible scale to use to make a scale drawing of the floorplan of your school?
Give your answer in the form $1:n$

Understand & represent bearings

Notes and guidance

Students will learn that bearings are always measured clockwise from North and always given as 3 figures.
The wording 'of A from B' can often confuse students and is worth addressing as a class, identifying a wide variety of start and end points. It is also useful to discuss the convention that the North line is usually drawn vertically up the page, regardless of the actual direction of North.

Key vocabulary

Three-figure	North line	Clockwise
Bearing	Bearing of … from …	

Key questions

Why are bearings always given as the clockwise angle?

Is it possible to have a bearing of $400°$? Why or why not?

Should bearings be written to one decimal place?

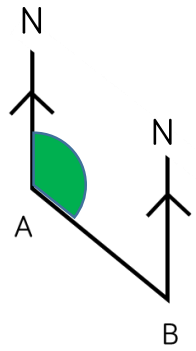
Exemplar Questions

Which of these are bearings? Explain why/why not in each case.

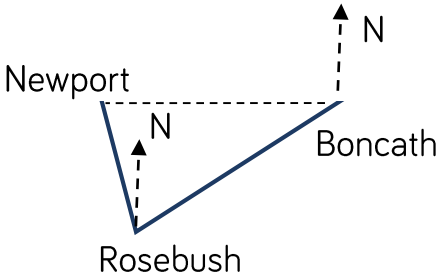
- 120°
- 24.5°
- 245°
- 12°
- 000°

Which of the statements are correct?
The highlighted angle shows,

- The bearing of A
- The bearing of A from B
- The bearing of B from A
- The bearing from A to B
- Between 090° and 180°



What other bearing could be shown on the diagram?



Newport is due West of Boncath. Amir says that the bearing of Newport from Boncath is 180° since they are on a straight line.
Is he correct?

Use the diagram to complete the sentences.
____ is due East from _____
The bearing of _____ from _____ is less than 090°
The bearing of _____ from_____ is close to 360°
The bearing of _____ from _____ is 270°

Measure and read bearings

Notes and guidance

Students need plenty of practice with the skill as confusion may arise with the wording of 'the bearing of A from B'.
Activities like the second exemplar question are very useful to let students explore and discover the relationships between angles and the relative positions of the points.
It is well worth working just with angles before proceeding to the next step and include scale as well.

Key vocabulary

Three figure bearing	Due East/West…. of
North line	Clockwise

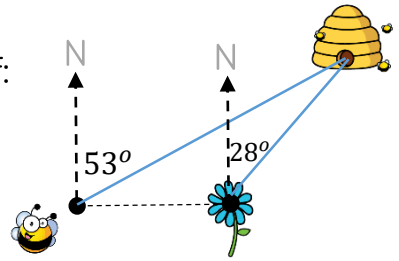
Key questions

From which compass point are bearings always measured?
Is the bearing of A from B the same as the bearing of B from A?
Why does it help to add auxiliary lines to diagrams when measuring bearings?

Exemplar Questions

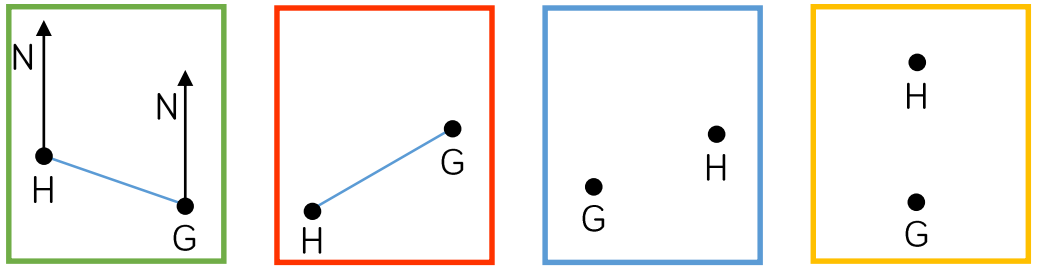
The bee is due West of the flower
From the list, choose the bearing of:

- the hive from the bee
- the flower from the bee
- the hive from the flower
- the bee from the flower



025°	028°	053°	090°	180°	270°

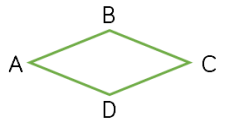
Draw the points G and H in each of the relative positions shown, including North lines for each point.



Measure the bearing of G from H and their bearing of H from G for each of your diagrams. Compare your answers with a partner's.

Construct and label a rhombus ABCD.
How many examples of the following sentence can you find?

"The bearing of __ from __ is the same as the bearing of __ from __."



Scale drawings using bearings

Notes and guidance

When confident with the measuring and direction of bearings, students can move on to more complex problems requiring them to draw scale diagrams as well. It is a good idea to use plain paper rather than squared paper (as in examinations) to promote accurate use of a protractor.

The need for accuracy can be emphasised by comparing answers.

Key vocabulary

Scale	Ratio
Bearing	Construct

Key questions

Why is a scale represented as a ratio?

What units are used in a scale?

Which part of the scale represents the actual size?

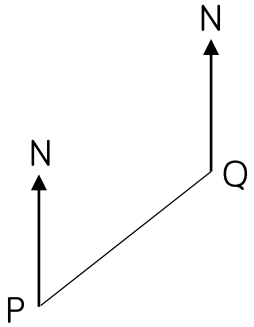
Exemplar Questions

The diagram shows the positions of two towns P and Q on a map.

- Measure the bearing of Q from P.

Another town R is on a bearing of 160° from Q, and on the map is 7 cm from Q.

- Mark and label the position of R.
- Measure the distance of R from P and the bearing of R from P.



A drone is 1.2 km from a school on a bearing of 075°
A football pitch is 500 m due South of the school.
Make a scale drawing of the drone, the school and the pitch, using 1 cm to represent 200 m.
Use your drawing to work out how far the drone needs to fly to get straight to the football pitch.

Here is a journey made by a boat.

- It sails 100 km East
- It sails 200 km on a bearing of 210°
- It sails 300 km West

Draw a rough sketch of the boat's journey.
Use your sketch to design a scale diagram of the boat's journey that fits on your page. What would be a sensible scale?
Use your scale diagram to find the distance and bearing the boat needs to sail back to get back to its starting point.

Bearings with angle rules

Notes and guidance

As well as providing a useful reminder of angle rules, this small step provides a good opportunity to allow students to compare multiple methods of solving a problem.
Adding auxiliary lines will help to emphasise where rules for angles in parallel lines might be used.
Encourage students to read the questions carefully, in particular noting where to measure the bearing from.

Key vocabulary

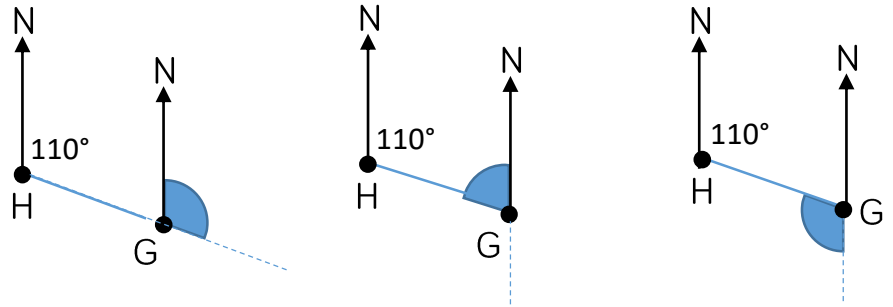
Parallel	Alternate	Corresponding
Co-interior	North line	Due South/West…

Key questions

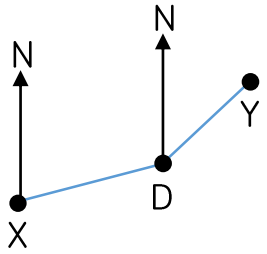
Why are rules for angles in parallel lines useful for solving bearings problems?
Why is the rule about co-interior angles different from the rules for alternate and corresponding angles?
When does adding the bearing of A from B and the bearing of B from A equal 360 degrees?

Exemplar Questions

The bearing of G from H is 110°
Explain how the diagrams help to find the bearing of H from G.

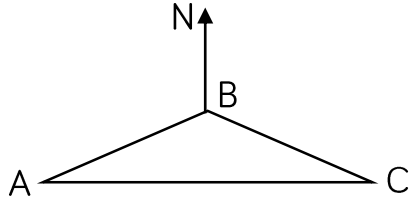


The bearing of Y from D is 055°
The angle XDY is 192°
Find the bearing of D from X.



The bearing of a ship from a lighthouse is $070°$
Showing your working clearly, calculate the bearing of the lighthouse from the ship.

Triangle ABC is isosceles.
Angle ABC is 134°, C is due East of A.
Find the bearings of each vertex of the triangle from each other vertex.



Bearings & right-angled geometry

Notes and guidance

This is good opportunity to revisit the use of trigonometry studied last term, and Pythagoras' theorem. Drawing the North line is especially revealing in these questions as it introduces right angles. Adding auxiliary lines and drawing separate triangles might help students to decide which geometric methods to use. They will need support initially to form diagrams from worded questions.

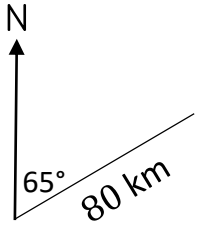
Key vocabulary

Trigonometry	$\sin\theta$, $\cos\theta$, $\tan\theta$
Due East	Perpendicular

Key questions

How do you know which trigonometric ratio to use?
Do you need to add an extra line to find a right-angled triangle in your diagram?
Do you need to use Pythagoras theorem, trigonometry or both?

Exemplar Questions



A ship sails for 80 km on a bearing of 065°
Calculate:

- How far East the ship has travelled.
- How far North the ship has travelled.

A plane flies due West for 200 km and then turns and flies due South for 90 km.

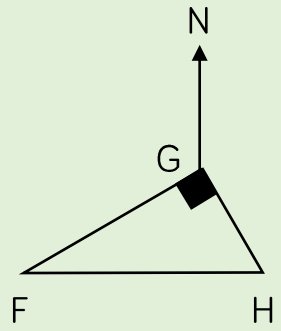
- Sketch a diagram of the plane's journey.
- Show that the plane is just over 219 km from its starting point.
- Work out the bearing the plane needs to fly on in order to return directly to its starting point.

H is due East of F.
The bearing of H from G is 158°

- Find the bearing of G from H.
- Find the bearing of G from F.

The distance from F to H is 14 km.

- What is the distance from F to G?



A hiker walks 3 km due East and then turns and walks 5 km due South. Work out the bearing of the hiker from their starting point.

Bearings: sine & cosine rule

H

Notes and guidance

This is another good opportunity to revisit and extend prior learning, using the sine and cosine rules. Teacher modelling will help students to construct their own sketch diagrams in time, but they will need support initially to correctly label information given and to identify lengths and angles needed. Scaffolding, by providing partly-drawn diagrams may be useful.

Key vocabulary

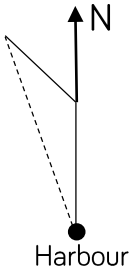
Sine Rule	Cosine Rule
Opposite	Included angle

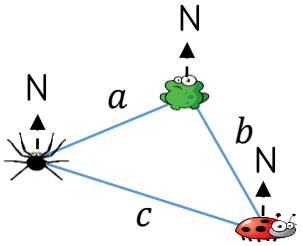
Key questions

What is the minimum amount of information required to use the sine/cosine rule?
How many angles inside the triangle can you find with the given information for your question?
How can you work out other angles?

Exemplar Questions

A boat starts from harbour and travels 30 km North, then turns to travel 12 km North West. Complete the diagram and work out how far does the boat need to travel to get back to the harbour?





The bearing of the frog from the spider is 062°
The bearing of the ladybird from the spider is 113°
The bearing of the spider from the lady bird is 295°
Draw a sketch of the diagram and find all the interior angles of the triangle in the diagram. Given that the distance from the spider to the frog is 98 cm, find lengths b and c.

A cyclist leaves a point on a bearing of 075° at a speed of 15 mph. A runner leaves the same point at a speed of 6 mph on a bearing of 318°. How far apart are they,

- after 10 minutes?
- after 1 hour?
- after x hours?