Fractions & Percentages of Amounts

Year 7

#MathsEveryoneCan

2019-20

White Rose Maths

	Week 1	Week 2	Week 3	Week 4	Week 5	Week 6	Week 7	Week 8	Week 9	Week 10	Week 11	Week 12
Autumn	**Algebraic Thinking**						**Place Value and Proportion**					
	Sequences		Understand and use algebraic notation		Equality and equivalence		Place value and ordering integers and decimals			Fraction, decimal and percentage equivalence		
Spring	**Applications of Number**						**Directed Number**			**Fractional Thinking**		
	Solving problems with addition & subtraction		Solving problems with multiplication and division			Fractions & percentages of amounts	Operations and equations with directed number			Addition and subtraction of fractions		
Summer	**Lines and Angles**						**Reasoning with Number**					
	Constructing, measuring and using geometric notation			Developing geometric reasoning			Developing number sense		Sets and probability		Prime numbers and proof	

Spring 1: Application of Number

Weeks 1 & 2: Solving problems with addition & subtraction

The focus for these two weeks is building on the formal methods of addition and subtraction students have developed at Key Stage 2. All students will look at this in the context of interpreting and solving problems, for those for whom these skills are secure, there will be even more emphasis on this. Problems will be drawn from the contexts of perimeter, money, interpreting bar charts and tables and looking at frequency trees; we believe all these are better studied alongside addition and subtraction rather than separately. Calculators should be used to check and/or support calculations, with significant figures and equations explicitly revisited.

National curriculum content covered:

- use formal written methods, applied to positive integers and decimals
- recognise and use relationships between operations including inverse operations
- derive and apply formulae to calculate and solve problems involving: perimeter
- construct and interpret appropriate tables, charts, and diagrams, including frequency tables, bar charts and pictograms for categorical data, and vertical line (or bar) charts for ungrouped numerical data

Weeks 3 to 5: Solving problems with multiplication & division

The rest of the term is dedicated to the study of multiplication and division, so allowing for the study of forming and solving of two-step equations both with and without a calculator. Unit conversions will be the main context as multiplication by 10, 100 and 1000 are explored. As well as distinguishing between multiples and factors, substitution and simplification can also be revised and extended. Again, the emphasis will be on solving problems, particularly involving area of common shapes and the mean. Choosing the correct operation to solve a problem will also be a focus. There will also be some exploration of the order of operations, which will be reinforced alongside much of this content next term when studying directed number.

National curriculum content covered:

- use formal written methods, applied to positive integers and decimals
- select and use appropriate calculation strategies to solve increasingly complex problems
- recognise and use relationships between operations including inverse operations
- use the concepts and vocabulary factors (or divisors), multiples, common factors, common multiples, highest common factor, lowest common multiple
- change freely between related standard units [time, length, area, volume/capacity, mass]
- derive and apply formulae to calculate and solve problems involving: perimeter and area of triangles, parallelograms, and trapezia **(H)**
- substitute numerical values into formulae and expressions, including scientific formulae
- use algebraic methods to solve linear equations in one variable (including all forms that require rearrangement)
- describe, interpret and compare observed distributions of a single variable through: the mean

Week 6: Fractions and percentages of amounts

This short block focuses on the key concept of working out fractions and percentages of quantities and the links between the two. This is studied in depth in Year 8

National curriculum content covered:

- use the four operations, including formal written methods, applied to integers, decimals, proper and improper fractions
- interpret fractions and percentages as operators

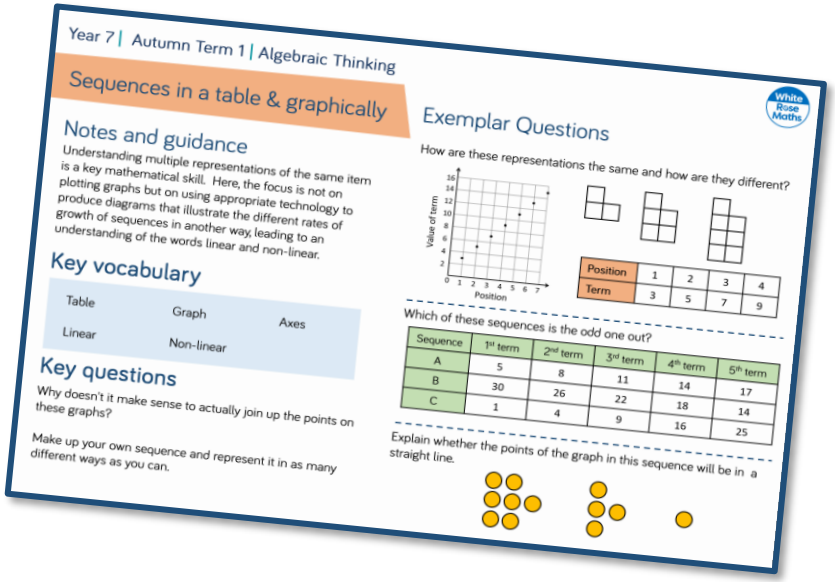
Why Small Steps?

We know that breaking the curriculum down into small manageable steps should help students to understand concepts better. Too often, we have noticed that teachers will try and cover too many concepts at once and this can lead to cognitive overload. We believe it is better to follow a "small steps" approach.

As a result, for each block of content in the scheme of learning we will provide a "small step" breakdown. ***It is not the intention that each small step should last a lesson – some will be a short step within a lesson, some will take longer than a lesson.*** We would encourage teachers to spend the appropriate amount of time on each step for their group, and to teach some of the steps alongside each other if necessary.

What We Provide

- Some ***brief guidance*** notes to help identify key teaching and learning points
- A list of ***key vocabulary*** that we would expect teachers to draw to students' attention when teaching the small step,
- A series of ***key questions*** to incorporate in lessons to aid mathematical thinking.
- A set of questions to help ***exemplify*** the small step concept that needs to be focussed on.



Year 7 | Autumn Term 1 | Algebraic Thinking

Sequences in a table & graphically

Notes and guidance

Understanding multiple representations of the same item is a key mathematical skill. Here, the focus is not on plotting graphs but on using appropriate technology to produce diagrams that illustrate the different rates of growth of sequences in another way, leading to an understanding of the words linear and non-linear.

Key vocabulary

Table, Graph, Axes, Linear, Non-linear

Key questions

Why doesn't it make sense to actually join up the points on these graphs?

Make up your own sequence and represent it in as many different ways as you can.

Exemplar Questions

How are these representations the same and how are they different?

Position	1	2	3	4
Term	3	5	7	9

Which of these sequences is the odd one out?

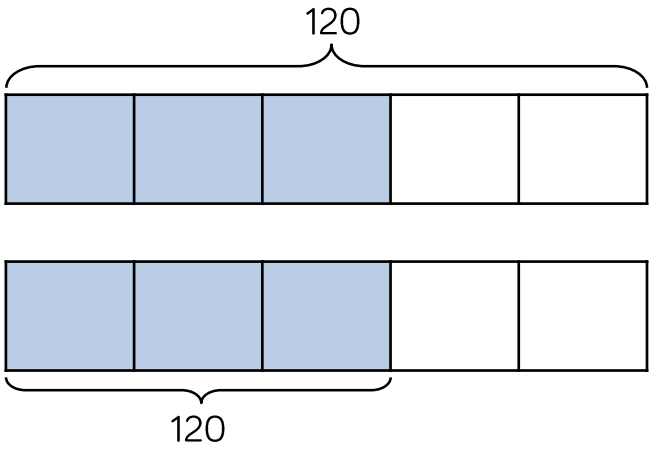
Sequence	1st term	2nd term	3rd term	4th term	5th term
A	5	8	11	14	17
B	30	26	22	18	14
C	1	4	9	16	25

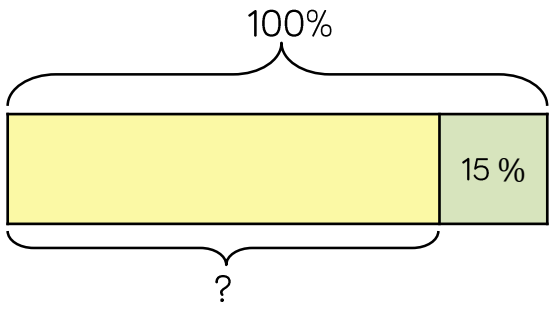
Explain whether the points of the graph in this sequence will be in a straight line.

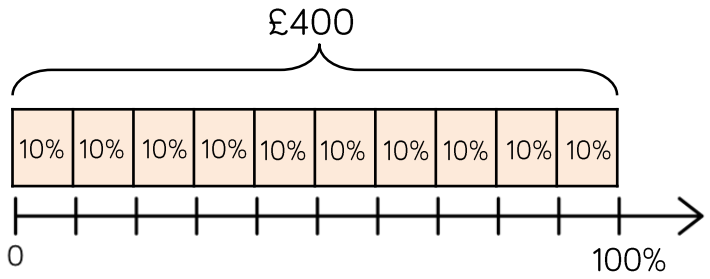
- These include reasoning and problem-solving questions that are fully integrated into the scheme of learning. Depending on the attainment of your students, you many wish to use some or all of these exemplars, which are in approximate order of difficulty. Particularly challenging questions are indicated with the symbol  .
- For each block, we also provide ideas for key representations that will be useful for all students.

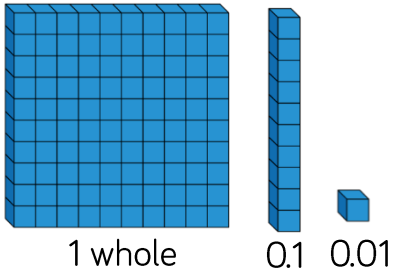
In many of the blocks of material, some of the small steps are in **bold**. These are content aimed at higher attaining students, but we would encourage teachers to use these with as many students as possible – if you feel your class can access any particular small step, then please include it in your planning.

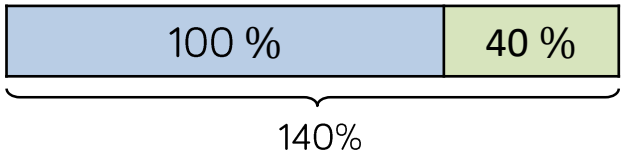
Key Representations

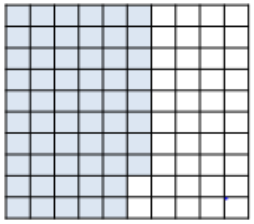












Making links between concrete, pictorial and abstract representations are an important part of developing students' conceptual understanding.

Bar models are particularly useful when working with fractions of amounts. The need for equal parts is emphasised and carefully labelling will help support distinguishing between problems where the whole or where a part is known.

A number line or a bar can also help to visualise percentages. This is especially helpful when working with e.g. multiples of 10%, 20% or 25%. They can also be useful to relate to the whole e.g. showing that to work out 90% of a quantity you can just subtract 10% form the whole rather than multiply 10% by 9

Fractions & Percentages of Amounts

Small Steps

- Find a fraction of a given amount
- Use a given fraction to find the whole and/or other fractions
- Find a percentage of a given amount using mental methods
- Find a percentage of a given amount using a calculator
- **Solve problems with fractions greater than 1 and percentages greater than 100%** (H)

(H) denotes higher strand and not necessarily content for Higher Tier GCSE

Fractions of amounts

Notes and guidance

Students should have met finding fractions of an amount throughout primary school. This step provides an opportunity for students to consolidate their understanding and attempt increasingly difficult problems. In order to aid understanding students should be able to represent and see the problem with a bar model They could use comparison bar models to look at e.g. one-third of 90 and two-thirds of 45

Key vocabulary

Fraction	Equivalent	Numerator
Denominator	Whole	

Key questions

How do you work out $\frac{3}{5}$ of a number?

Draw a diagram to explain why your method works.

What's the same and what's different about these two questions?

$\frac{2}{3}$ of 60 = ☐ $\frac{2}{3}$ of ☐ = 60

Why is one third of 90 equal to two-thirds of 45?

Exemplar Questions

Use the bar model to help you work out $\frac{2}{5}$ of £95

Work out: $\frac{1}{8}$ of 720 lbs $\frac{3}{8}$ of 720 lbs $\frac{5}{9}$ of 8.19 km $\frac{11}{10}$ of 120 kg

Ron bakes 280 cookies on Monday.
On Tuesday he bakes $\frac{1}{8}$ as many more cookies.
How many cookies did he bake altogether over the two days?

Tommy and Whitney each make a tower made up of red and blue bricks. They each use the same number of blue bricks.

- $\frac{3}{8}$ of Tommy's tower is made up of blue bricks.
- $\frac{1}{3}$ of Whitney's tower is made up of blue bricks.
- Tommy uses 48 red bricks.

How many bricks are there in Tommy's tower?

Sort these cards into pairs with equal values. What do you notice?

$\frac{1}{2}$ of 30	$\frac{2}{3}$ of 60	$\frac{3}{8}$ of 160	$\frac{1}{4}$ of 60
$\frac{6}{7}$ of 210	$\frac{3}{4}$ of 80	$\frac{1}{3}$ of 120	$\frac{2}{7}$ of 630

Find the whole

Notes and guidance

Bar models are again a useful tool for 'working backwards' to find the whole given a particular fraction, either unit or non-unit. As with the previous step, they help make sense of the process involved rather than attempting rote memorisation of division/multiplication by the numerator and denominator. Once the whole is found, other fractions can be easily found. Students can be challenged by considering fractions of increasingly complex expressions.

Key vocabulary

Fraction	Equivalent	Numerator
Denominator	Whole	Original

Key questions

How can I work out a number if I know a fraction of the number?

What's different about these questions?

- What number is half of 12?
- 12 is half of what number?

Exemplar Questions

Half of a number is 24. What is the number?

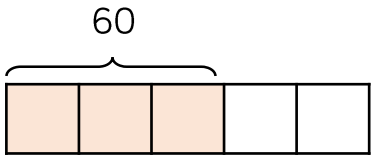
One-third of another number is 24. What is this number?

One-tenth of a third number is 24
What is the third number?

$\frac{3}{5}$ of a number is 60

What is $\frac{1}{5}$ of the number?

What is the number?



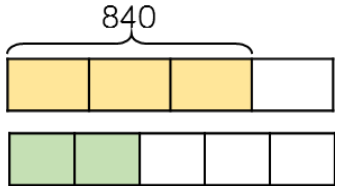
Work out the whole number if

- $\frac{2}{3}$ of the number is 60
- $\frac{3}{4}$ of the number is 60
- $\frac{5}{6}$ of the number is 60
- $\frac{5}{12}$ of the number is 60

Dexter spends two-fifths of his money on a book. He has £15 left.
How much money did he have to start with?

$\frac{3}{4}$ of a number is 840

What is $\frac{2}{5}$ of the number?



$\frac{2}{3}$ of an expression is $12x$. What is the expression?

Percentages of amounts: Mental

Notes and guidance

Students should have met finding percentages of an amount in KS2. They are likely to have focused on finding multiples of 5% and 10%, and many will be used to 'build-up' methods from key percentages. It is worth exploring alternative methods and discussing when which method would be appropriate e.g. 95% is best found by subtraction from the whole.

Key vocabulary

Place value	Percent	Percentage

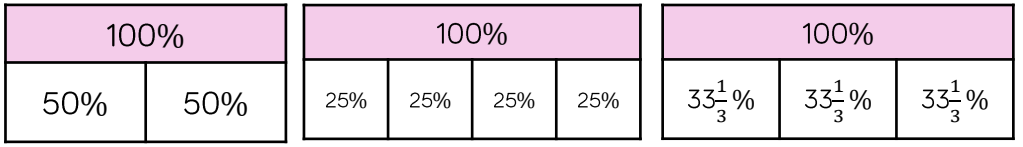
Key questions

Why is it that you divide by 10 to find 10% of a number, but you don't divide by 20 to find 20% of a number?

If you know 10% of a number, what other percentages can you easily work out?

Find as many ways as you can to work out 60% of 45

Exemplar Questions

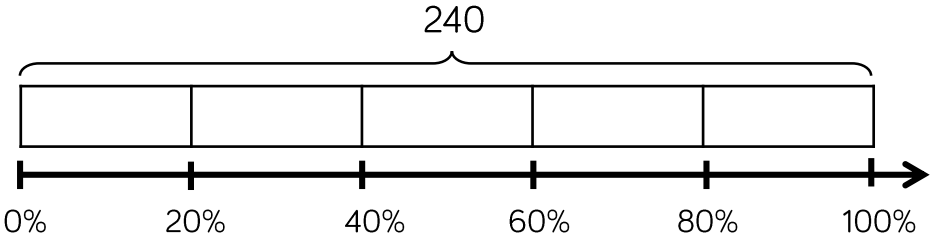


Use the bar models above to explain how you work out:

- 50% of 30
- 50% of 80
- $33\frac{1}{3}$% of 90
- 50% of 24
- 50% of 120
- $66\frac{2}{3}$% of 18

Draw a bar model that shows you how to work out 10% of a number.
What other percentages can you then work out?

What percentages of 240 can you easily work out from this model?



Compare these (or other) methods to find 45% of a number e.g. 60

10% × 4 + 5%	20% + 25%	50% − 5%

Find the missing numbers in these calculations.
Could there be more than one answer?

20% of ☐ = ☐ of 200 20% of ☐ = 25% of ☐

Percentages of amounts: Calc

Notes and guidance

It is unlikely that students will have used a calculator to find percentages and this is a good opportunity to explore the variety of methods available, including the percentage button. In particular, students should consider when a calculator method is preferable to a mental method. This is a good step to discuss real life percentage problems such as interest rates, commission charges etc.

Key vocabulary

Place value	Percent	Percentage
Decimal	Convert	Equivalent

Key questions

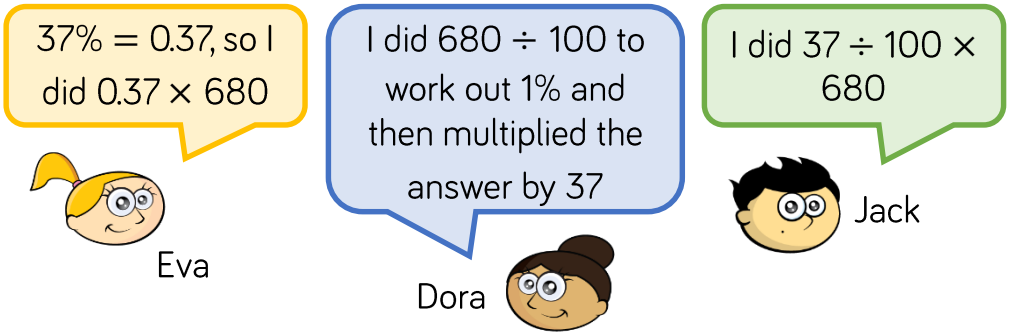
When is it easier to use a mental method rather than a calculator?

How do you know how to interpret the display on a calculator?

What does the % button on your calculator do?

Exemplar Questions

Jack, Eva and Dora are working out 37% of £680



Which method do you prefer? Can you find another way?
All three calculators gave the answer 251.6, what is the 6 worth in this number?

Use a calculator to work out these calculations.
What do you notice about your answers? Can you generalise?

34% of 47

47% of 34

Amir puts £300 into a bank account.
At the end of the year, he received 4% interest. How much is this?
How much would he receive if the interest rate was 4.3%?

Mo works out 17% of £84.10 on his calculator.

"It says 14.297.
Does this mean £14, £14.29 or £14.30?"

Which answer would you choose?

Percentages over 100%

H

Notes and guidance

As students understand percentages as 'out of a hundred' there is often confusion about going over 100%. It is therefore a worthwhile discussion as to when it is and isn't appropriate to have percentages over 100%. Bar models can then support finding the total percentage and the decimal conversion. This step is covered again in the core strand in Year 8.

Key vocabulary

Fraction	Equivalent	Numerator
Denominator	Whole	Original

Key questions

Can 110% of the class be absent on one day?

If the price of an item increases by 60%, what percentage is the new price of the old price?

Can a price increase/decrease by 180% or 200%?

Exemplar Questions

Which of these statements make sense, and which are impossible?

I'm going to give 150% effort

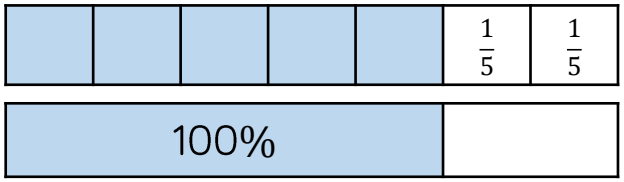
My pocket money has gone up by 150% Woo hoo!

My pocket money has gone down by 150%

The population of Mathtown today is 150% of its population last year.

A company's profits grew by $\frac{2}{5}$ between 2018 and 2019

What percentage of the 2018 profit is the 2019 profit?



Match the equivalent cards.

160% $\frac{8}{5}$ $\frac{5}{2}$ $\frac{7}{4}$ 1.75 2.5 1.6 175% 250%