

Multiplying & Dividing Fractions

Year 8

#MathsEveryoneCan

White
Rose
Maths

	Week 1	Week 2	Week 3	Week 4	Week 5	Week 6	Week 7	Week 8	Week 9	Week 10	Week 11	Week 12
Autumn	Proportional Reasoning						Representations					
	Ratio and scale	Multiplicative change		Multiplying and dividing fractions		Working in the Cartesian plane			Representing data		Tables & Probability	
Spring	Algebraic Techniques						Developing Number					
	Brackets, equations and inequalities				Sequences	Indices	Fractions and percentages		Standard index form		Number sense	
Summer	Developing Geometry						Reasoning with Data					
	Angles in parallel lines and polygons	Area of trapezia and circles		Line symmetry and reflection		The data handling cycle				Measures of location		

Autumn 1: Proportional Reasoning

Weeks 1 and 2: Ratio and Scale

This unit focuses initially on the meaning of ratio and the various models that can be used to represent ratios. Based on this understanding, it moves on to sharing in a ratio given the whole or one of the parts, and how to use e.g. bar models to ensure the correct approach to solving a problem. After this we look at simplifying ratios, using previous answers to deepen the understanding of equivalent ratio rather than ‘cancelling’ purely as a procedure. We also explore the links between ratio and fractions and understand and use π as the ratio of the circumference of a circle to its diameter. Students following the higher strand also look at gradient in preparation for next half term.

National Curriculum content covered includes:

- make connections between number relationships, and their algebraic and graphical representations
- use scale factors, scale diagrams and maps
- understand that a multiplicative relationship between two quantities can be expressed as a ratio or a fraction
- divide a given quantity into two parts in a given part : part or part : whole ratio; express the division of a quantity into two parts as a ratio
- solve problems involving direct and inverse proportion

Weeks 3 and 4: Multiplicative Change

Students now work with the link between ratio and scaling, including the idea of direct proportion, linking various form including graphs and using context such as conversion of currencies which provides rich opportunities for problem solving. Conversion graphs will be looked at in this block and could be revisited in the more formal graphical work later in the term. Links are also made with maps and scales, and with the use of scale factors to find missing lengths in pairs of similar shapes.

National Curriculum content covered includes:

- extend and formalise their knowledge of ratio and proportion in working with measures and in formulating proportional relations algebraically
- interpret when the structure of a numerical problem requires additive, multiplicative or proportional reasoning
- use scale factors, scale diagrams and maps
- solve problems involving direct and inverse proportion, including graphical and algebraic representations
- move freely between different numerical, algebraic, graphical and diagrammatic representations

Weeks 5 and 6: Multiplying and Dividing Fractions

Students will have had a little experience of multiplying and dividing fractions in Year 6; here we seek to deepen understanding by looking at multiple representations to see what underpins the (often confusing) algorithms. Multiplication and division by both integers and fractions are covered, with an emphasis on the understanding of the reciprocal and its uses. Links between fractions and decimals are also revisited. Students following the Higher strand will also cover multiplying and dividing with mixed numbers and improper fractions.

National Curriculum content covered includes:

- consolidate their numerical and mathematical capability from key stage 2 and extend their understanding of the number system and place value to include decimals and fractions
- select and use appropriate calculation strategies to solve increasingly complex problems
- use the four operations, including formal written methods, applied to integers, decimals, proper and improper fractions, and mixed numbers, all both positive and negative

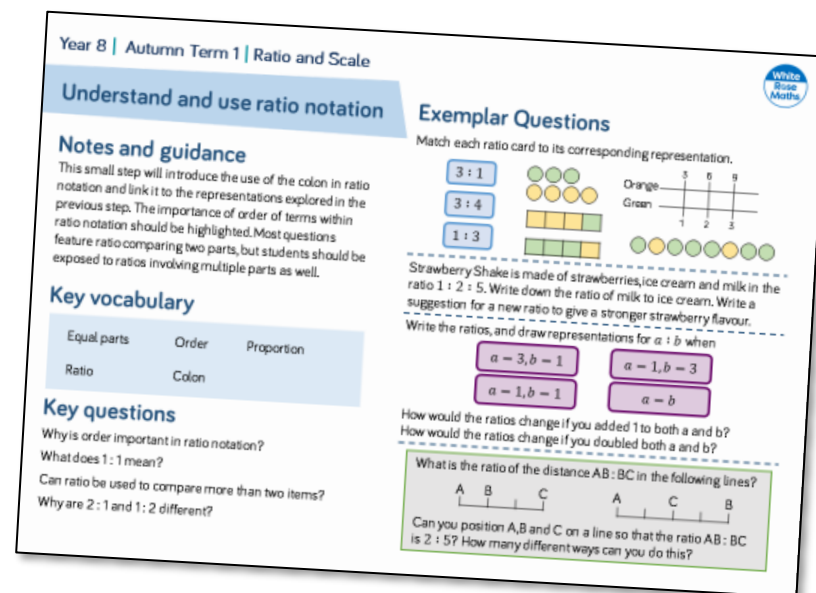
Why Small Steps?

We know that breaking the curriculum down into small manageable steps should help students to understand concepts better. Too often, we have noticed that teachers will try and cover too many concepts at once and this can lead to cognitive overload. We believe it is better to follow a “small steps” approach.

As a result, for each block of content in the scheme of learning we will provide a “small step” breakdown. ***It is not the intention that each small step should last a lesson – some will be a short step within a lesson, some will take longer than a lesson.*** We would encourage teachers to spend the appropriate amount of time on each step for their group, and to teach some of the steps alongside each other if necessary.

What We Provide

- Some **brief guidance** notes to help identify key teaching and learning points
- A list of **key vocabulary** that we would expect teachers to draw to students' attention when teaching the small step,
- A series of **key questions** to incorporate in lessons to aid mathematical thinking.
- A set of questions to help **exemplify** the small step concept that needs to be focussed on.



Year 8 | Autumn Term 1 | Ratio and Scale

Understand and use ratio notation

Notes and guidance

This small step will introduce the use of the colon in ratio notation and link it to the representations explored in the previous step. The importance of order of terms within ratio notation should be highlighted. Most questions feature ratio comparing two parts, but students should be exposed to ratios involving multiple parts as well.

Key vocabulary

Equal parts	Order	Proportion
Ratio	Colon	

Key questions

- Why is order important in ratio notation?
- What does 1:1 mean?
- Can ratio be used to compare more than two items?
- Why are 2:1 and 1:2 different?

Exemplar Questions

Match each ratio card to its corresponding representation.

3:1
3:4
1:3

Orange
Green


Strawberry Shake is made of strawberries, ice cream and milk in the ratio 1:2:5. Write down the ratio of milk to ice cream. Write a suggestion for a new ratio to give a stronger strawberry flavour. Write the ratios, and draw representations for $a:b$ when

$a=3, b=1$
 $a=1, b=3$
 $a=1, b=1$
 $a=b$

How would the ratios change if you added 1 to both a and b ?
How would the ratios change if you doubled both a and b ?

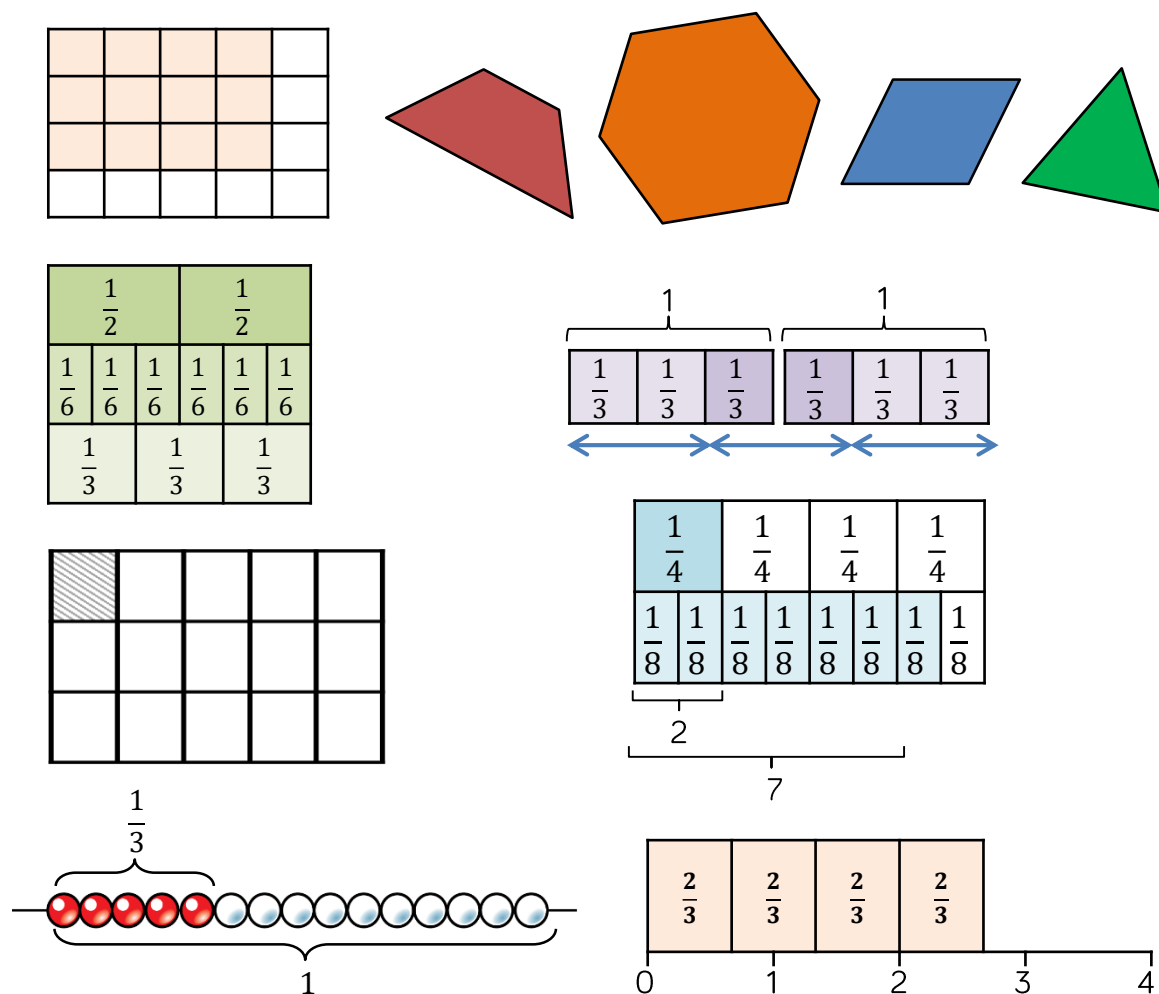
What is the ratio of the distance AB:BC in the following lines?

Can you position A, B and C on a line so that the ratio AB:BC is 2:5? How many different ways can you do this?

- These include reasoning and problem-solving questions that are fully integrated into the scheme of learning. Depending on the attainment of your students, you may wish to use some or all of these exemplars, which are in approximate order of difficulty. Particularly challenging questions are indicated with the symbol .
- For each block, we also provide ideas for key representations that will be useful for all students.

In many of the blocks of material, some of the small steps are in **bold**. These are content aimed at higher attaining students, but we would encourage teachers to use these with as many students as possible – if you feel your class can access any particular small step, then please include it in your planning.

Key Representations



Concrete, pictorial and abstract representations are an important part of developing students' conceptual understanding.

Grids are very useful for demonstrating finding a fraction of a fraction, and also introducing multiplication of fractions.

The algorithm for the division of two fractions is not intuitive and students benefit from seeing division as 'how many ... in ...' to understand the process and to get a sense of the size of the answer of a fraction division. As well as the use of bar models and number lines, pattern blocks are a useful representation; by varying which shapes is the whole we can explore a large number of different divisions.

Multiplying & Dividing Fractions

Small Steps

- ▶ Represent multiplication of fractions
- ▶ Multiply a fraction by an integer
- ▶ Find the product of a pair of unit fractions
- ▶ Find the product of a pair of any fractions
- ▶ Divide an integer by a fraction
- ▶ Divide a fraction by a unit fraction
- ▶ Understand and use the reciprocal
- ▶ Divide any pair of fractions

Multiplying & Dividing Fractions

Small Steps

- ▶ Multiply and divide improper and mixed fractions
- ▶ Multiply and divide algebraic fractions

H

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H denotes higher strand and not necessarily content for Higher Tier GCSE

Represent fraction multiplication

Notes and guidance

Repeated addition is used here to help understand the multiplication of fractions. Students will also explore familiar representations of fractions from previous years.

Manipulatives such as paper plates and fraction pieces can be used to demonstrate the multiplications. Paper strips and Cuisenaire rods link well to pictorial representations as well as bar models.

Key vocabulary

Unit fraction	Numerator	Denominator
Product	Repeated addition	

Key questions

When making a representation of a fraction why is it important that each part is equal?

If 3 multilink blocks is 1 whole, what other facts do we know?

How is addition related to multiplication?

Exemplar Questions

3 paper strips are folded into quarters. One quarter of each is shaded. Which of the statements do you agree with? Why are they correct?



'This shows $\frac{3}{12}$ '

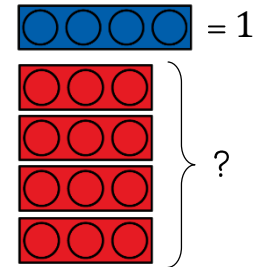
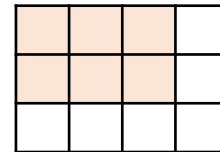
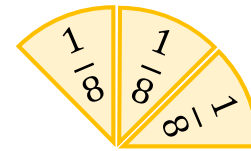
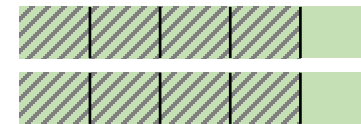
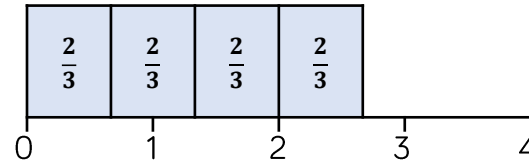
'It shows $\frac{1}{4} + \frac{1}{4} + \frac{1}{4}$ '

'The paper shows $3 \times \frac{1}{4}$ '

'I can see three quarters' 'It could be on 1 bar model'



What multiplication do each of the diagrams show?



Seth says that to work out $7 \times \frac{2}{3}$ on a number line, you just jump 2 forward then 3 back and repeat that 7 times. Is he correct?

Multiply a fraction by an integer

Notes and guidance

In this small step, students explore and formalise multiplication of a fraction by an integer. Calculations supported with pictorial representations are still encouraged at this stage. Multiple methods will allow students to pick the strategy that best suits the question. It could be useful to remind students of the word ‘product’ at this stage.

Key vocabulary

Unit fraction	Numerator	Denominator
Product	Repeated addition	

Key questions

How is finding a fraction of an amount the same as multiplying by a fraction?

Does multiplying by a fraction always give an answer less than 1?

Exemplar Questions

Mo is working out $3 \times \frac{2}{7}$. His friends are helping him, here is what they say. Which comment do you think is most helpful and why?

The answer will be less than one

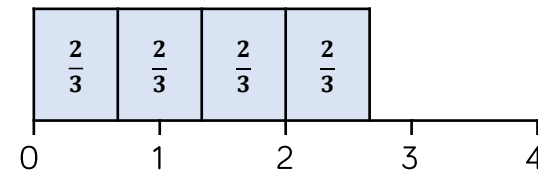
3 lots of 2 is 6
So 3 lots of 2 *sevenths* is 6 *seventh*.

It's just the same as
 $\frac{2}{7} + \frac{2}{7} + \frac{2}{7}$

It's the same as $\frac{3 \times 2}{7}$

Divide 3 by 7, then multiply it by 2

This bar model shows that $4 \times \frac{2}{3} = \frac{8}{3}$. It also shows $\frac{8}{3} = 2\frac{2}{3}$



Use the bar model to work out $3 \times \frac{2}{3}$ $8 \times \frac{2}{3}$ $\frac{2}{3} \times 5$ $1.5 \times \frac{4}{3}$

What is the same and what is different about each pair of calculations and their answers? Use bar models to help explain.

$$3 \times \frac{2}{5} \quad 2 \times \frac{3}{5}$$

$$3 \times \frac{2}{5} \quad 3 \times \frac{2}{7}$$

$$3 \times \frac{2}{5} \quad 6 \times \frac{1}{5}$$

Product of unit fractions

Notes and guidance

This step gives students the chance to understand the underlying mathematics of multiplying any fractions together. When folding paper, remind students that each side of the original shape has a unit length of 1. This links it to grid method multiplication and clearly shows the size of the product of unit fractions is always smaller than one.

Key vocabulary

Denominator	Product	Square
Whole	Unit fraction	

Key questions

Does multiplying always make numbers larger?

Why will the product of two unit fractions always have 'one' as a numerator?

How many other fractions can we find by folding paper in different ways?

Exemplar Questions

A square napkin is folded in half vertically, then into thirds horizontally.

How many parts will there be when it is unfolded fully?

How does this help to work out $\frac{1}{3} \times \frac{1}{2}$?

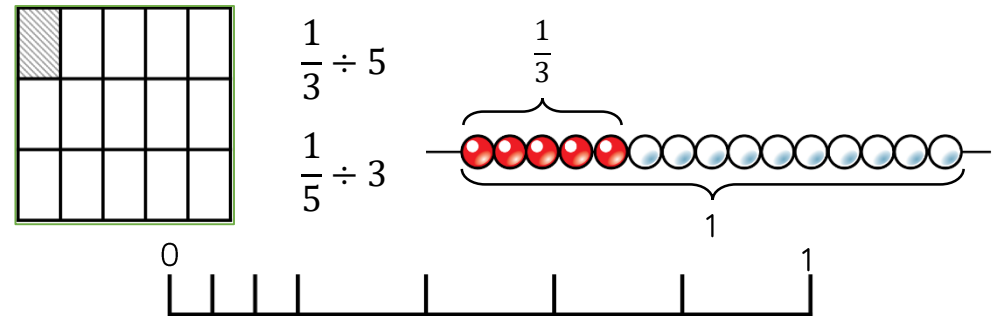
Does the napkin have to be square?

Use a piece of A4 paper as the napkin to investigate this.

Whitney has worked out $\frac{1}{3} \times \frac{1}{5} = \frac{1}{15}$

How do the following link to her calculation?

A third of one fifth is three times smaller than one fifth.



Find the missing numbers. Do any have more than one answer?

$$\frac{1}{5} \times \frac{1}{6} = \frac{1}{?}$$

$$\frac{1}{3} \times \frac{1}{?} = \frac{1}{30}$$

$$\frac{1}{?} \times \frac{1}{?} = \frac{1}{30}$$

Product of any fractions

Notes and guidance

This small step will look at the multiple ways in which the students might approach finding the product of any two fractions, allowing students to come up with their own conjectures for 'quick methods.'

Again, using familiar concrete and/or pictorial representations from previous steps will support abstract understanding.

Key vocabulary

Non-unit fraction

Commutative

Numerator

Denominator

Key questions

How can multiplying by a fraction be expressed as a multiplication and a division?

Is it always, sometimes or never appropriate to convert fractions to decimals before multiplying?

Exemplar Questions

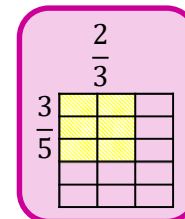
Which method is the most effective to work out $\frac{2}{3} \times \frac{3}{5}$?

$$\frac{2 \times 3}{3 \times 5} = \frac{6}{15} = \frac{2}{5}$$

$$\frac{2 \times 3}{3 \times 5} = \frac{2 \times 3}{5 \times 3} = \frac{2}{5}$$

Rewrite the question as $2 \times 3 \times \frac{1}{3} \times \frac{1}{5}$

Find a third of $\frac{3}{5}$, then double it.



Put the following in ascending order.

$$\frac{1}{5} \times \frac{3}{8}$$

$$\frac{2}{5} \times \frac{3}{8}$$

$$-\frac{1}{15} \times \frac{9}{16}$$

$$\frac{2}{15} \times \frac{15}{16}$$

$$\left(\frac{3}{5}\right)^2$$

Tommy says that to multiply fractions you just need to multiply the numerators and multiply the denominators.

Explain whether this method would be suitable for the following.

$$\frac{3}{8} \times \frac{8}{7}$$

$$0.2 \times \frac{40}{81}$$

$$\frac{5}{7} \times \frac{3}{11}$$

Three-quarters of 40%

$$\frac{2}{3} \times \frac{3}{4} \times \frac{32}{56} \text{ of } 2$$

$$\frac{3}{4} + \frac{4}{10}$$

Divide an integer by a fraction

Notes and guidance

In this small step, students understand the link between multiplying and dividing integers to multiplying and dividing fractions. A fact family with integer values will be intuitive but students may want to ask more questions when the fact family involves division of fractions. Demonstrations with bar models and fraction strips will help explain this.

Key vocabulary

Unit fraction

Whole

Quotient

Denominator

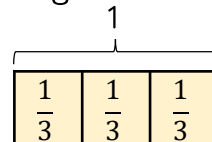
Key questions

How many unit fractions make a whole?

When we divide, is the answer always smaller than the dividend?

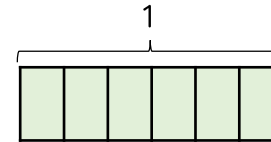
Exemplar Questions

Use the diagrams to help complete the sentences.



'There are ... thirds in 1 whole.'
'Therefore, there are ... thirds in 4 wholes.'

$1 \div \frac{1}{3} =$ $4 \div \frac{1}{3} =$ $40 \div \frac{1}{3} =$



'There are 6 ... in 1 whole.'
'Therefore, there are 30 ... in ... wholes.'

$1 \div \frac{1}{6} = 6$ $? \div \frac{1}{6} = 30$ $? \div \frac{1}{6} = 15$

Work out the following.

$$4 \div \frac{1}{2}$$

$$4 \div \frac{1}{3}$$

$$4 \div \frac{1}{4}$$

$$4 \div \frac{1}{5}$$

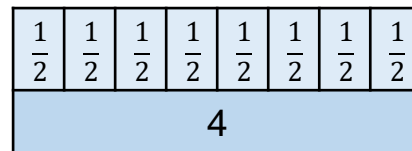
$$4 \div \frac{1}{13}$$

$$8 \div \frac{1}{13}$$

$$8 \div \frac{1}{n}$$

$$a \div \frac{1}{n}$$

Complete the facts shown in the representation below.



$$4 \div 8 =$$

$$1 \div \frac{1}{2} =$$

$$2 \div \frac{1}{2} =$$

$$4 \div \frac{1}{2} =$$

Are there any other facts you can see?

Divide a fraction by a unit fraction

Notes and guidance

As work with fractions becomes more abstract, it is useful to get students to reason their solutions. The questions asked in this small step will revolve around reasoning rather than procedure.

The language of dividing, 'How many ... in ...?', will help students to estimate answers before formally giving them.

Key vocabulary

Divide

Estimate

Numerator

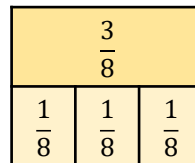
Denominator

Key questions

Shade in three quarters of a square. Count the number of quarters. How does this show $\frac{3}{4} \div \frac{1}{4}$?

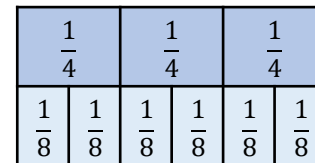
When we divide, the quotient is always smaller than the dividend. True or False?

Exemplar Questions



How many eighths are there in three-eighths?

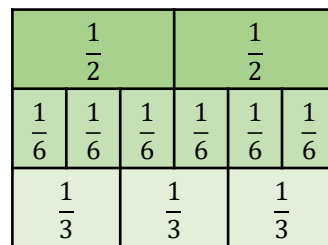
Complete $\frac{3}{8} \div \frac{1}{8} =$



How many eighths are there in one-quarter?

How many eighths are there in three-quarters?

Complete $\frac{3}{4} \div \frac{1}{8} =$



Use the equivalent fraction wall to calculate:

$$\frac{1}{2} \div \frac{1}{6}$$

$$\frac{1}{3} \div \frac{1}{6}$$



Using the wall, I can see that one-and-a-half thirds fit into one-half.

This means that $\frac{1}{2} \div \frac{1}{3} = \frac{3}{2}$

Is Eva right? Explain your answer.



Complete the statements using $<$, $>$ or $=$

$$\frac{3}{5} \div \frac{1}{5} \bigcirc \frac{3}{5} \div \frac{1}{6}$$

$$\frac{1}{2} \div \frac{1}{5} \bigcirc \frac{1}{4} \div \frac{1}{5}$$

$$\frac{1}{5} \div \frac{1}{3} \bigcirc \frac{3}{5} \div \frac{1}{9}$$

Understand and use the reciprocal

Notes and guidance

Here students will learn through investigation that the division of a number is equivalent to the multiplication by its reciprocal. They should be able to find the reciprocal of fractions and decimals and use these to answer questions on division.

They should also understand that a number multiplied by its reciprocal is always 1

Key vocabulary

Reciprocal

Convert

Key questions

How would you find the reciprocal of a decimal?

Can we find the reciprocal of zero?

What do you notice about $\frac{1}{5} \times 5$? Try multiplying another number by its reciprocal. Is this true for all numbers?

Why is dividing by a fraction the same as multiplying by its reciprocal?

Exemplar Questions

Find the reciprocal of the following.

$$2 \quad \frac{1}{2} \quad \frac{2}{5} \quad 0.4 \quad a \quad \frac{2}{a}$$

Work out the following sets of calculations. What do you notice?

$$3 \div \frac{1}{4}$$

$$3 \times 4$$

$$3 \div 4$$

$$3 \times \frac{1}{4}$$

$$8 \div \frac{1}{5}$$

$$8 \times 5$$

$$8 \div 5$$

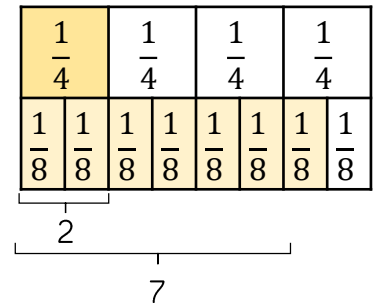
$$8 \times \frac{1}{5}$$

Dora reasons that $\frac{1}{4} \div \frac{7}{8}$ must give an answer of less than 1

Use the picture to explain why $\frac{1}{4} \div \frac{7}{8} = \frac{2}{7}$

How does this show that $\frac{1}{4} \div \frac{7}{8}$

is the same as $\frac{1}{4} \times \frac{8}{7}$?



Use the idea of division being the same as multiplying by the reciprocal to calculate.

$$2 \div \frac{1}{5}$$

$$2 \div \frac{2}{5}$$

$$\frac{3}{5} \div \frac{2}{5}$$

$$\frac{3}{10} \div \frac{2}{5}$$

$$\frac{1}{5} \div 2$$

$$0.4 \div \frac{1}{2}$$

Divide any pair of fractions

Notes and guidance

Students should now have developed their reasoning and so have many methods available for dividing fractions. This small step develops the concepts further so they can understand the division of any pair of fractions.

Encourage students to think about efficient methods depending on the question instead of relying solely on procedure.

Key vocabulary

Simplify

Factors

Denominators

Key questions

Why is a common denominator useful when dividing fractions?

Can there be a remainder when dividing by fractions?

Can a fraction be a factor of another fraction?

Exemplar Questions

Explain each of these methods for finding $\frac{2}{5} \div \frac{3}{4}$

$$\begin{aligned} \frac{2}{5} \div \frac{3}{4} \\ = \frac{8}{20} \div \frac{15}{20} \\ = \frac{8}{15} \end{aligned}$$

$$\begin{aligned} \frac{2}{5} \div \frac{3}{4} \\ = \frac{2}{5} \times \frac{4}{3} \\ = \frac{8}{15} \end{aligned}$$

$$\begin{aligned} \frac{2}{5} \div \frac{3}{4} \\ = \frac{0.4}{0.75} \\ = \frac{40}{75} \\ = \frac{8}{15} \end{aligned}$$

$$\begin{aligned} \frac{2}{5} \div \frac{3}{4} &= x \\ \frac{2}{5} &= \frac{3}{4}x \\ \frac{2}{5} \times 4 &= 3x \\ \frac{8}{5} \div 3 &= x \\ \frac{8}{15} &= x \end{aligned}$$

Calculate the answers to the following, thinking carefully about which method to use.

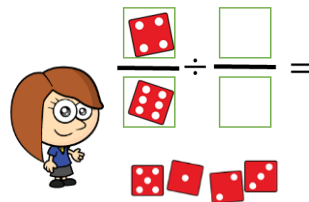
$$\frac{8}{9} \div 0.9$$

$$\frac{63}{35} \text{ divided by } \frac{70}{175}$$

How many times
does $\frac{4}{100}$ fit into 0.39

$$\frac{5}{6}y = \frac{35}{48}$$

Eva and Rosie are playing a game with 4 dice arranged in a calculation. If the answer is a whole number you win a point.



What numbers could Eva roll to score a point?

Here is Rosie's roll. Can she score a point?

Improper and mixed fractions

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Notes and guidance

Teachers might introduce this by using a bar model to make links between repeated addition, multiplication and improper and mixed fractions. Being able to visualise this, helps to develop 'number sense' around multiplying and dividing fractions. It could be useful to highlight the common misconception that integer values cannot be multiplied and divided separately is worth noting.

Key vocabulary

Unit fraction	Repeated addition	Whole
Numerator	Denominator	

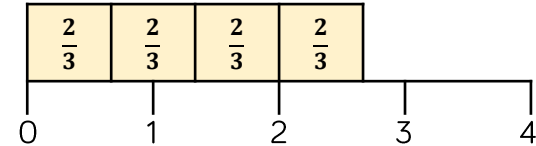
Key questions

Count up in thirds starting from 0
Did you count up in improper fractions or mixed fractions?

Is it easier to multiply and divide fractions as improper or mixed?

Exemplar Questions

How many different written representations can you find these for this bar model?



Alex is trying to work out $6\frac{4}{5} \div 2\frac{2}{5}$. What has Alex done wrong?



$$6 \div 2 = 3$$

$$4 \div 2 = 2$$

$$\frac{4}{5} \div \frac{2}{5} = 2$$

So the answer is $3 + 2 = 5$

Work out the following multiplications.

$$2\frac{1}{4} \times 3$$

$$2\frac{1}{4} \times 4$$

$$5 \times 2\frac{1}{4}$$

$$5\frac{5}{8} \times 1\frac{7}{9}$$

$$5\frac{5}{8} \times 1\frac{5}{9}$$

$$5\frac{5}{8} \times 1\frac{5}{7}$$

Work out the following divisions.

$$1 \div \frac{2}{5}$$

$$1 \div 2\frac{2}{5}$$

$$2 \div 2.4$$

$$4\frac{3}{8} \div 2\frac{1}{7}$$

$$2\frac{1}{7} \div 4\frac{3}{8}$$

$$\frac{22}{7} \div 4.375$$

Algebraic Fractions

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Notes and guidance

There have been opportunities throughout these small steps for students to generalise a method algebraically. Here we will look at calculations that involve algebraic fractions. Students should be encouraged to reason every step of their solution.

Pictorial representations, such as bar models, may still be appropriate when introducing this small step.

Key vocabulary

Generalise

Cancel

Term

Expression

Simplest Form

Key questions

How many different ways can you write a quarter of x ?

Can you have an improper algebraic fraction?

Can we use repeated addition for multiplying algebraic fractions?

Exemplar Questions

Imran is working out $\frac{2x}{5} \div \frac{x}{5}$

He says the answer is 2 without doing any working out.

How does he know this?

His next question is $\frac{2x}{5} \div \frac{5}{x}$

Will the same method work? Why or why not?

Work these out, giving your answers in their simplest form.

$$\frac{2}{5} \times \frac{w}{r}$$

$$\frac{3}{5w} \times \frac{w}{r}$$

$$\frac{4}{5w} \div \frac{r}{w}$$

$$\frac{2w}{5} \times \frac{w}{r}$$

$$\frac{3r}{5w} \times \frac{w}{3r}$$

$$5 \times \frac{4}{5w} \div \frac{r}{w}$$

$$\frac{2w}{5} \times \frac{w}{r} \div 2$$

$$\frac{3r}{5w} \times \frac{2w}{3r^2} \times \frac{1}{m}$$

$$5 \div \frac{4}{5w} \div \frac{r}{w}$$

What is the same and what is different about these pairs of calculations?

$$\frac{a}{2} \times \frac{4a}{3} \quad \frac{2}{a} \times \frac{4a}{3}$$

$$\frac{3}{b} \div \frac{b}{5}$$

$$\frac{b}{3} \div \frac{b}{5}$$

$$\frac{c^2}{4}$$

$$\left(\frac{c}{4}\right)^2$$

$$(d + 5) \div 2d$$

$$(d + 5) \div \frac{2}{d}$$