Using graphs

Year 11

#MathsEveryoneCan

White Rose Maths

	Week 1	Week 2	Week 3	Week 4	Week 5	Week 6	Week 7	Week 8	Week 9	Week 10	Week 11	Week 12
Autumn	**Graphs**						**Algebra**					
	Gradients & lines		Non-linear graphs		Using graphs		Expanding & factorising		Changing the subject		Functions	
Spring	**Reasoning**						**Revision and Communication**					
	Multiplicative		Geometric		Algebraic		Transforming & constructing		Listing & describing		Show that...	
Summer	**Revision**						**Examinations**					

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Autumn 1: Graphs

Weeks 1 and 2: Gradients and lines

This block builds on earlier study of straight line graphs in years 9 and 10. Students plot straight lines from a given equation, and find and interpret the equation of a straight line from a variety of situations and given information. There is the opportunity to revisit graphical solutions of simultaneous equations. Higher tier students also study the equations of perpendicular lines.
National Curriculum content covered includes:

- move freely between different numerical, algebraic, graphical and diagrammatic representations
- plot and interpret graphs
- interpret the gradient of a straight line graph as a rate of change
- use the form $y = mx + c$ to identify parallel **{and perpendicular}** lines; find the equation of the line through two given points, or through one point with a given gradient
- find approximate solutions to two simultaneous equations in two variables (linear/linear **{or linear/quadratic}**) using a graph

Weeks 3 and 4: Non-linear graphs

Students develop their knowledge of non-linear graphs in this block, looking at quadratic, cubic and reciprocal graphs so they recognise the different shapes. They find the roots of quadratics graphically, and will revisit this when they look at algebraic methods in the Functions block during Autumn 2, where they will also look at turning points. Higher tier students also look at simple exponential graphs and the equation of a circle. Note that the equation of the tangent to a circle is covered later when the circle theorem of tangent/radius is met. Higher students also extend their understanding of gradient to include instantaneous rates of change considering the gradient of a curve at a point.
National Curriculum content covered includes:

- move freely between different numerical, algebraic, graphical and diagrammatic representations
- recognise, sketch and interpret graphs of linear functions, quadratic functions, simple cubic functions, the reciprocal function 1 **{the exponential function $y = k^x$ for positive values of k}**
- plot and interpret graphs (including reciprocal graphs **{and exponential graphs}**)
- find approximate solutions using a graph
- identify and interpret roots, intercepts of quadratic functions graphically
- **{recognise and use the equation of a circle with centre at the origin;}**

Weeks 5 and 6: Using graphs

This block revises conversion graphs and reflection in straight lines. Students also study other real-life graphs, including speed/distance/time, constructing and interpreting these. Higher tier students also investigate the area under a curve.
National Curriculum content covered includes:

- plot and interpret graphs of non-standard functions in real contexts, to find approximate solutions to problems such as simple kinematic problems involving distance, speed and acceleration
- **{interpret the gradient at a point on a curve as the instantaneous rate of change; apply the concepts of instantaneous and average rate of change (gradients of tangents and chords) in numerical, algebraic and graphical contexts}**
- **{calculate or estimate gradients of graphs and areas under graphs (including quadratic and other non-linear graphs), and interpret results in cases such as distance-time graphs, velocity-time graphs and graphs in financial contexts}**

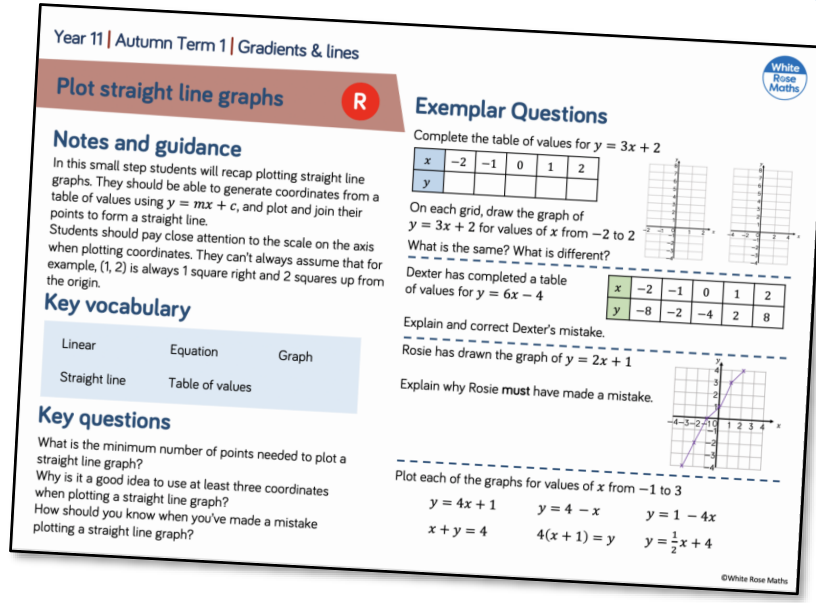
Why Small Steps?

We know that breaking the curriculum down into small manageable steps should help students to understand concepts better. Too often, we have noticed that teachers will try and cover too many concepts at once and this can lead to cognitive overload. We believe it is better to follow a "small steps" approach.

As a result, for each block of content in the scheme of learning we will provide a "small step" breakdown. ***It is not the intention that each small step should last a lesson – some will be a short step within a lesson, some will take longer than a lesson.*** We would encourage teachers to spend the appropriate amount of time on each step for their group, and to teach some of the steps alongside each other if necessary.

What We Provide

- Some ***brief guidance*** notes to help identify key teaching and learning points
- A list of ***key vocabulary*** that we would expect teachers to draw to students' attention when teaching the small step,
- A series of ***key questions*** to incorporate in lessons to aid mathematical thinking.
- A set of questions to help ***exemplify*** the small step concept that needs to be focussed on.



Year 11 | Autumn Term 1 | Gradients & lines

Plot straight line graphs R

Notes and guidance

In this small step students will recap plotting straight line graphs. They should be able to generate coordinates from a table of values using $y = mx + c$, and plot and join their points to form a straight line.

Students should pay close attention to the scale on the axis when plotting coordinates. They can't always assume that for example, (1, 2) is always 1 square right and 2 squares up from the origin.

Key vocabulary

Linear, Equation, Graph, Straight line, Table of values

Key questions

What is the minimum number of points needed to plot a straight line graph?

Why is it a good idea to use at least three coordinates when plotting a straight line graph?

How should you know when you've made a mistake plotting a straight line graph?

Exemplar Questions

Complete the table of values for $y = 3x + 2$

x	-2	-1	0	1	2
y					

On each grid, draw the graph of $y = 3x + 2$ for values of x from -2 to 2

What is the same? What is different?

Dexter has completed a table of values for $y = 6x - 4$

x	-2	-1	0	1	2
y	-8	-2	-4	2	8

Explain and correct Dexter's mistake.

Rosie has drawn the graph of $y = 2x + 1$

Explain why Rosie **must** have made a mistake.

Plot each of the graphs for values of x from -1 to 3

$y = 4x + 1$ $y = 4 - x$ $y = 1 - 4x$

$x + y = 4$ $4(x + 1) = y$ $y = \frac{1}{2}x + 4$

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- These include reasoning and problem-solving questions that are fully integrated into the scheme of learning. Depending on the attainment of your students, you may wish to use some or all of these exemplars, which are in approximate order of difficulty. Particularly challenging questions are indicated with the symbol .
- For each block, we also provide ideas for key representations that will be useful for all students.

Some of the small steps are in **bold** and labelled with H to indicate this is Higher tier GCSE content. We would encourage teachers to use these with as many students as possible – if you feel your class can access any particular small step, then please include it in your planning. Steps that review content covered at Key Stage 3 are labelled R .

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Using graphs

Small Steps

- Reflect shapes in given lines (R)
- Construct and interpret conversion graphs (R)
- Construct and interpret other real-life straight line graphs (R)
- Interpret distance/time graphs
- Construct distance/time graphs
- Construct and interpret speed/time graphs
- Construct and interpret piece-wise graphs
- Recognise and interpret graphs that illustrate direct and inverse proportion

(H) denotes Higher Tier GCSE content

(R) denotes 'review step' – content should have been covered at KS3

Using graphs

Small Steps

- Find approximate solutions to equations using graphs
- **Estimate the area under a curve** H

H denotes Higher Tier GCSE content

R denotes 'review step' – content should have been covered at KS3

Reflect shapes in given lines (R)

Notes and guidance

Students should be familiar with the equations of straight lines from the first block of the Autumn term. This step provides a reminder about lines of the form $x = a, y = a$ and $y = \pm x$ in the context of practising reflection. Students should be able to both perform and describe reflections in these lines using precise mathematical language; this key skill is revisited again in the Spring term.

Key vocabulary

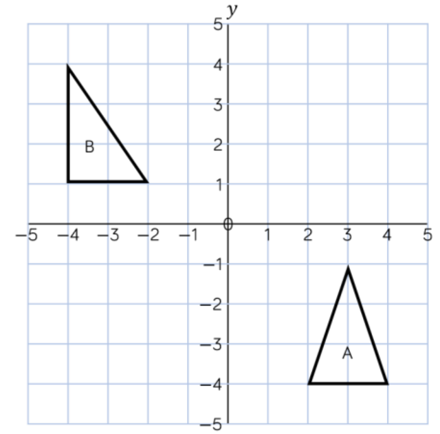
Parallel	Horizontal	Vertical	Straight line
Axis	Reflection	Mirror	

Key questions

What's the same and what's different about the equations of horizontal/vertical lines compared to diagonal lines?
What's the same and what's different about reflecting a shape in horizontal/vertical lines compared to diagonal lines?
Given two shapes that have been reflected, how would you find the equation of the mirror line?

Exemplar Questions

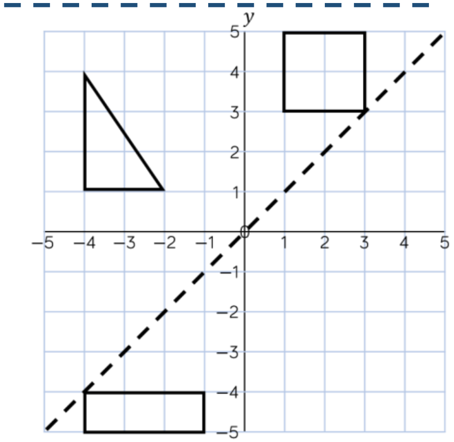
- Reflect triangle A in the line $y = -2$
Label the result X
- Reflect triangle A in the line $x = 1$
Label the result Y
- Reflect triangle B in the line $x = -2$
Label the result Z
- What is another way of saying "reflect in the line $y = 0$"?



What is the equation of the dotted line?

Reflect the shapes in this line.

On another grid, redraw the shapes and reflect them in the line $y = -x$



Draw a pair of coordinate axes from -5 to 5 in both directions.
Draw the trapezium with vertices at (1, 2), (5, 4), (5, 2) and (3, 2).
Reflect the trapezium in the x-axis.
What do you notice about the coordinates of the reflection of the trapezium? State the coordinates of the point (p, q) after

- a reflection in the x-axis
- a reflection in the y-axis

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Conversion graphs

R

Notes and guidance

Students may need reminding to use a ruler to draw lines to/from axes to the line rather than reading off 'by eye'. Many conversion graphs are particular examples of direct proportion, so that the point (0,0) is a point on a conversion graph line and the second point for constructing a graph should be as far from the origin as practical. Converting e.g. Fahrenheit to Celsius on a graph, would not go through (0, 0).

Key vocabulary

Convert	Axis	Gradient
Direct Proportion		

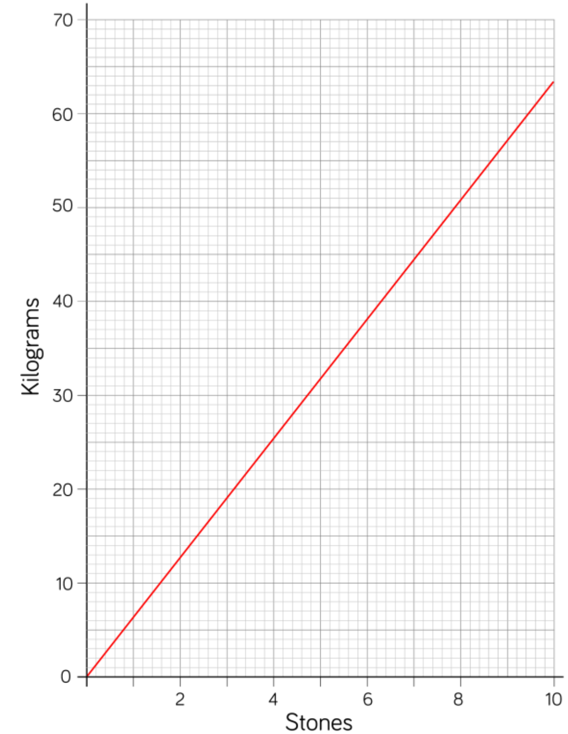
Key questions

Does it matter which axis represents which quantity when using a conversion graph?
What does the gradient of a conversion graph tell you?
Do all conversion graphs go through the origin?
How can you use a conversion graph to help work out conversions that are out of the range of the graph?

Exemplar Questions

You can use this graph to change between stones and kilograms.

- Change 5 stones to kilograms.
- Change 40 kilograms to stones.
- Explain how you could use your answers to change 40 kilograms to stones, and to change 35 stones to kg.



Use the fact that 1 inch is approximately 2.5 cm to draw a conversion graph for inches and cm. Use 0 to 12 inches on the horizontal axis and a suitable number of cm on the vertical axis.

- What two points should you plot to help draw the straight line?
- What scales should you use on the axes?
- Use your graph to convert 8.5 inches to cm and 45 cm to inches.
- Compare with answers found using a calculator.

Other real-life graphs

R

Notes and guidance

In this small step students look at linear relationships that do not go through the origin. Comparison could be made with direct proportion noting in this case that e.g. when one value doubles, the other does not. With these graphs, it is useful to consider the practical meaning of the gradient and intercept e.g. the unit increase and the fixed charge. Students could also be challenged to find the equation of the line.

Key vocabulary

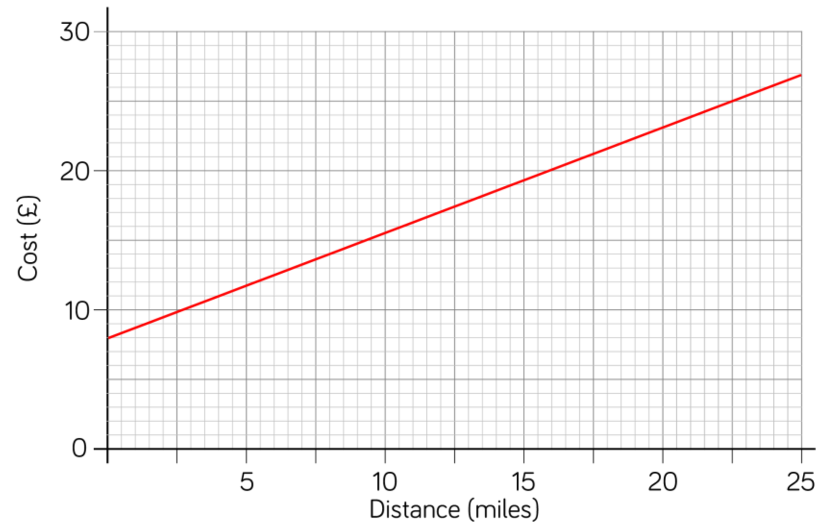
Gradient	Intercept	Interpret
Model		

Key questions

What is the value when $x = 0$? What does this mean in the context of the question?
What does the gradient of the line mean in the context of the question?
What is the same and what is different about these graphs and currency conversion graphs?

Exemplar Questions

This graph shows the cost of taxi journeys for different distances.



The taxi fare consists of a fixed charge plus a charge for each mile travelled.

- How much is the fixed charge?
- How much more does a 15 mile journey cost than a 5 mile journey?

A salesperson is paid £60 per day plus £30 for every sale they make. Draw a graph showing how much they are paid for up to 10 sales a day. Does this graph show direct proportion? Explain why or why not.

Another salesperson is paid £80 per day and £30 for every sale they make. Draw a graph for this salesperson on the same axes and compare the two wages.

Interpret distance/time graphs

Notes and guidance

In this step, students focus on the reading and interpretation of graphs, with construction covered in the next step. The key point is to understand that the gradient represents the speed of travel, e.g. a straight line is constant speed and a flat section implies the object is stationary. Various scales should be used, and students will need support to calculate speed in sections of less than one hour. Misconceptions about uphill and downhill direction of travel should be addressed.

Key vocabulary

Distance	Speed	Time
Gradient	Constant	Scale

Key questions

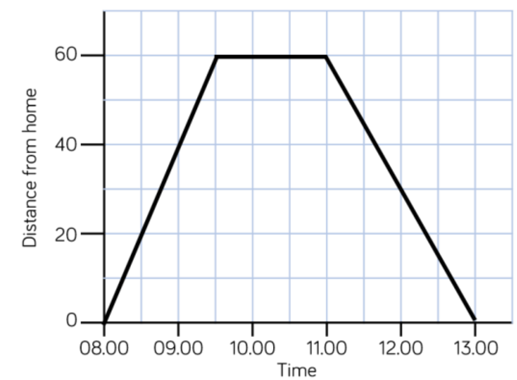
What is the connection between the gradient of a distance/time graph and the speed of travel?

Does a section of a distance/time graph with a negative gradient mean the journey is downhill? Why or why not?

What does a 'flat' section on a distance/time graph represent?

Exemplar Questions

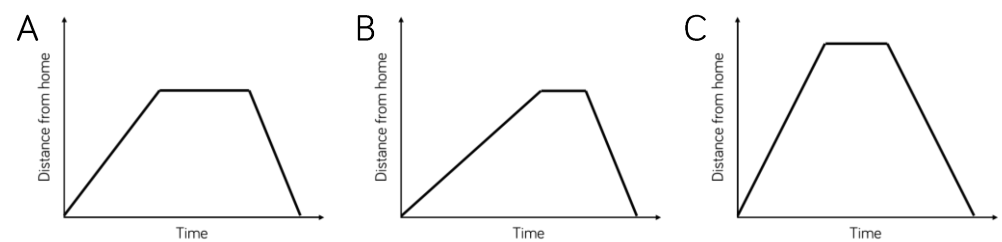
The distance-time graph shows Dora's journey to visit a friend and back.



- How long did it take to travel to see her friend?
- How long did she spend with her friend?
- How long was the journey back?

Use the formula Speed = Distance ÷ Time to work out Dora's speed for both the outward and the homeward journey.

How can you tell which part of the journey was faster just by looking at the graph?



Compare these distance/time graphs.

- Which graph shows the longest journey?
- Which graph has the fastest/slowest section?
- Which graph shows the longest break in a journey?
- Can you tell if any of the journeys were uphill/downhill?
- What else can you see?

Construct distance/time graphs

Notes and guidance

Students now move on to constructing graphs. This is relatively straightforward given times and distances, but can lead to difficulty if the speed is given, particularly if dealing with non-integer multiples of an hour. Students need to practice working out distances covered over periods of 10, 20, 30 and 45 minutes to inform their plotting of the graph. Discussion of how realistic the models are is also useful.

Key vocabulary

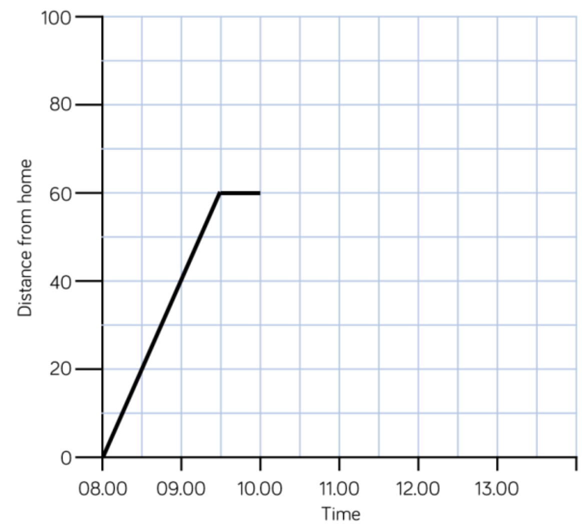
Distance	Speed	Time
Gradient	Constant	Scale

Key questions

What fraction of an hour is (e.g.) 20 minutes? If the car travels at (e.g.) 30 m.p.h., how far will it travel in 20 minutes?
What scale is the graph? How does this affect where we plot the parts of the journey?

Exemplar Questions

The graph shows part of Dani's journey to London from her home.



She takes a break, then drives the remaining 20 miles to London in half an hour. She then spends 90 minutes in London before returning directly home, arriving at 2 p.m.
Complete the graph to show this information.
Work out the speeds for each part of the journey.

Nijah goes on a cycle ride.
She sets off at 10:30 a.m., travelling at 24 k.p.h. for 45 minutes.
After a 15 minute break, she continues her journey for another 1.5 hours travelling at 16 k.p.h. She then rests for 45 minutes before returning home at a steady speed of 20 k.p.h.
Show this information on a distance/time graph.

Speed/time graphs

Notes and guidance

Students need to know the difference between speed/time and distance/time graphs, appreciating that the gradient here represents the change in speed and that this is called acceleration. They should also understand that negative gradient now represents slowing down/deceleration. Higher tier students need to be aware that the area under a speed/time is the distance travelled, both in this step and in later non-linear examples.

Key vocabulary

Distance	Speed	Time
Gradient	Constant	Acceleration

Key questions

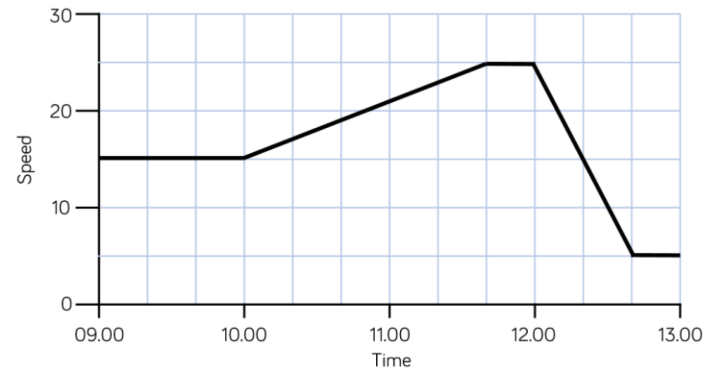
What is the difference between speed and acceleration?

What does a negative section mean on a speed/time graph? Why is it different from a distance/time graph?

What does the area under the graph represent?

Exemplar Questions

The graph shows the speed of a boat over a four hour period.



- What was the maximum speed of the boat?
- For how long altogether was the boat travelling at constant speeds?
- Between which times was the boat accelerating? Find the acceleration at this time
- Between which times was the boat decelerating?

The speed of boats at sea is measured in knots

- Given that 1 knot = 1.15 m.p.h., what was the fastest speed of the boat in knots over the four-hour period?

The area under a speed-time graph is the distance travelled.
Work out the total distance travelled by the boat.

A car accelerates from 0 m/s to 15 m/s in 12 seconds.
The car maintains this speed for 40 seconds before decelerating to rest in a further 20 seconds.
Represent this information in a speed/time graph.
Work out the acceleration and deceleration of the car.

Piece-wise graphs

Notes and guidance

Students now look at piece-wise graphs, which are discontinuous. These will be less familiar and may require careful explanation. Students can make links to the solutions of inequalities represented on number lines, as in this topic they again need to be careful when considering what values are included and not included. Piece-wise graphs could also be represented algebraically for each section, but if there are several sections this can become overwhelming.

Key vocabulary

Piece-wise

Key questions

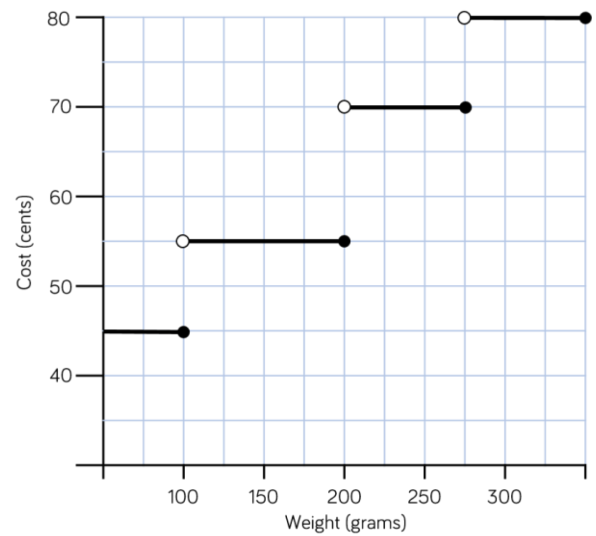
What is the gradient of each of the sections of the piece-wise graph?

When does the graph 'jump'? Is the boundary point included or excluded?

How does this compare to writing and solving inequalities?

Exemplar Questions

The graph shows the cost, in cents, of posting letters of different weights in a country.



Letters up to and including 100 g cost 40 c to send.

How much will a letter weighing 220 g cost to send?

Find the total cost of sending two letters weighing 150 g and a letter weighing 300 g.

The table shows the costs of using a car park in a town centre.

Represent this information on a piece-wise graph.

Duration	Up to 1 hour	Up to 2 hours	Up to 3 hours	Up to 5 hours	Up to 8 hours
Cost	50p	£1.00	£2.00	£3.50	£5

Rewrite the table using inequality notation e.g. $2 \leq t < 3$

Direct and inverse proportion

Notes and guidance

Direct and inverse proportion calculations using formulae are studied in depth next term under Multiplicative Reasoning, although the idea of constant multiplier and constant product could be explored here. Students explore the graphs of both types of proportion, with direct being more familiar. Teachers may wish to compare the graphs of inverse proportion relationships with that of the reciprocal function covered in the previous block.

Key vocabulary

Direct	Inverse	Proportion
Speed	Pressure	

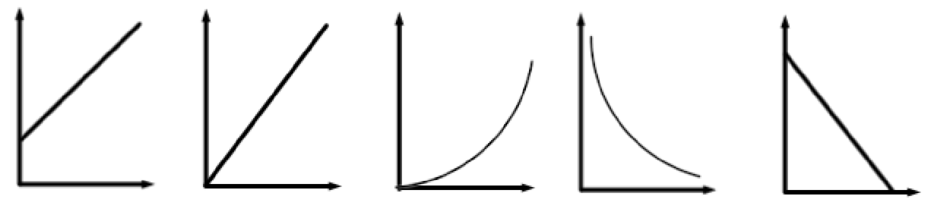
Key questions

How is the graph of an inverse proportion relationship different from the graph of a direct proportion relationship?

If you double the value of the x-axis quantity, what happens to the y-axis quantity? Is this the same for both direct and inverse proportion?

Exemplar Questions

Do the graphs show direct proportion, inverse proportion or neither? Explain your answers.



The table shows how long a journey takes at different speeds.

Speed (m.p.h.)	5	10	15	20	30	40	60	80	90	100
Time (hours)	24	12	8	6	4	3	2	1.5	1.333	1.2

Work out the length of the journey, and using this information or otherwise, find the time taken for the journey at each of 4, 3, 2, 1 m.p.h.
Draw a graph of time taken against speed, joining the points with a smooth curve.
Explain why the graph shows an inverse proportion relationship.

Sketch the shapes of the graphs for each of the situations.

- The time taken to complete a project against the number of people working on the project
- The distance travelled by a cyclist travelling at constant speed against the time spent cycling
- The number of biscuits each person has from a packet against the number of people sharing the packet

Approximate solutions

Notes and guidance

Students are familiar with finding the roots of a quadratic graphically from the previous block; their learning is reinforced both here and in later blocks when algebraic methods are considered. Students now also explore finding the approximate solutions of other equations by looking at the points where a graph and line intersect and can check answers by substitution. Higher tier students can also check their answers for quadratics that factorise, and will study other methods next term.

Key vocabulary

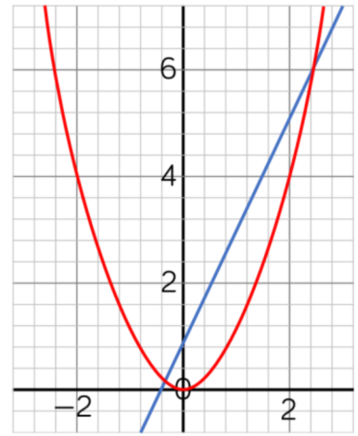
Quadratic	Solutions	Root
Estimate	Approximate	

Key questions

How can we check if the approximate solutions are close to the actual solutions? What values should we get when we substitute into the original equation?

What do we mean by the roots of a quadratic? Is this the same as, or different to, solutions?

Exemplar Questions



The diagram shows the graphs of $y = x^2$ and $y = 2x + 1$

By looking at the where the graphs intersect, find the values of x for which $x^2 = 2x + 1$

Draw suitable straight lines on the graph to find approximate solutions to the equations

- $x^2 = x + 3$
- $x^2 = 3 - x$

Complete the table of values for $y = x^2 - x - 6$

x	−3	−2	−1	0	1	2	3
y			−4				

On a grid with x from −3 to 3 and y from −7 to 7, draw the graph of $y = x^2 - x - 6$. (Use a scale of 2 cm to 1 unit on the x-axis and 1 cm to 1 unit on the y-axis)

Use your graph to state the roots of the equation $x^2 - x - 6 = 0$

Use your graph to estimate the solutions of the equations

- $x^2 - x - 6 = 4$
- $x^2 - x - 6 = -3$
- $x^2 - x - 5 = 0$

Draw the graph of $y = x^3 - x^2 - 4x + 4$ for values of x from −2 to 2

Use your graph to estimate the solutions of $x^3 - x^2 - 4x + 4 = 3$

Estimate area under a curve H

Notes and guidance

As a preparation for A level mathematics, students use trapezia to estimate the area of a curve. They may need reminding of the formula for the area of a trapezium, particularly as the trapezia are right-angled and usually in a less familiar orientation. Students also revisit the fact that the area under a speed/time is the distance travelled as an application of this process. Similarly, they could also revisit finding the tangent at a point of the curve.

Key vocabulary

Trapezium	Area	Approximate
Estimate	Speed/time graph	

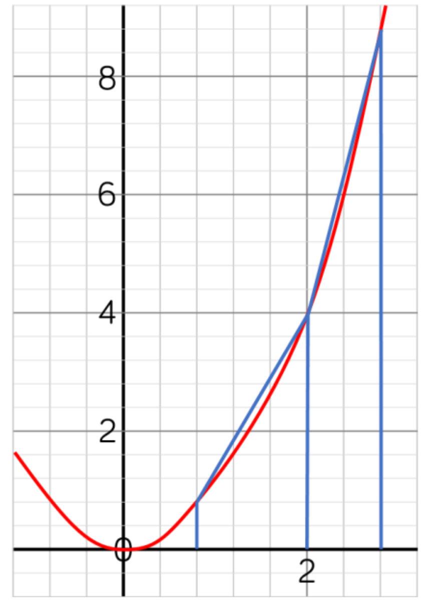
Key questions

How do you find the area of the trapezium?

Which dimension is the height of the trapezium? Which are the parallel sides?

If the trapezia all lie below the curve, why is the estimate for the area an underestimate?

Exemplar Questions



The diagram shows the graph of $y = x^2$ for values of x from -1 to 3

Use the two trapezia drawn on the graph to estimate the area of under the curve of $y = x^2$ for $1 \leq x \leq 3$

Is the area an underestimate or an overestimate?

The graph shows the speed of a cyclist in the first 8 seconds of a journey.

Work out an estimate for the distance travelled by the cyclist in the first 8 seconds.

Use 4 strips of equal width.