Developing Number Sense

Year 7

#MathsEveryoneCan

2019-20

	Week 1	Week 2	Week 3	Week 4	Week 5	Week 6	Week 7	Week 8	Week 9	Week 10	Week 11	Week 12
Autumn	**Algebraic Thinking**						**Place Value and Proportion**					
	Sequences		Understand and use algebraic notation		Equality and equivalence		Place value and ordering integers and decimals			Fraction, decimal and percentage equivalence		
Spring	**Applications of Number**						**Directed Number**			**Fractional Thinking**		
	Solving problems with addition & subtraction		Solving problems with multiplication and division			Fractions & percentages of amounts	Operations and equations with directed number			Addition and subtraction of fractions		
Summer	**Lines and Angles**						**Reasoning with Number**					
	Constructing, measuring and using geometric notation			Developing geometric reasoning			Developing number sense		Sets and probability		Prime numbers and proof	

Summer 2: Reasoning with Number

Weeks 7 to 8: Developing Number Sense

Students will review and extend their mental strategies with a focus on using a known fact to find other facts. Strategies for simplifying complex calculations will also be explored. The skills gained in working with number facts will be extended to known algebraic facts.

National curriculum content covered:

- consolidate their numerical and mathematical capability from key stage 2 and extend their understanding of the number system and place value to include decimals, fractions, powers and roots
- select and use appropriate calculation strategies to solve increasingly complex problems
- begin to reason deductively in number and algebra

<u>Interleaving/Extension of previous work</u>

- Generating and describing sequences
- Substitution into expressions
- Order of operations

Weeks 9 to 10: Sets and Probability

FDP equivalence will be revisited in the study of probability, where students will also learn about sets, set notation and systematic listing strategies.

National curriculum content covered:

- record, describe and analyse the frequency of outcomes of simple probability experiments involving randomness, fairness, equally and unequally likely outcomes, using appropriate language and the 0-1 probability scale
- understand that the probabilities of all possible outcomes sum to 1
- enumerate sets and unions/intersections of sets systematically, using tables, grids and Venn diagrams
- generate theoretical sample spaces for single and combined events with equally likely and mutually exclusive outcomes and use these to calculate theoretical probabilities
- appreciate the infinite nature of the sets of integers, real and rational numbers

<u>Interleaving/Extension of previous work</u>

- FDP equivalence
- Forming and solving equations
- Adding and subtracting fractions

Weeks 11 to 12: Prime Numbers and Proof

Factors and multiples will be revisited to introduce the concept of prime numbers, and the Higher strand will include using Venn diagrams from the previous block to solve more complex HCF and LCM problems. Odd, even, prime, square and triangular numbers will be used as the basis of forming and testing conjectures. The use of counterexamples will also be addressed.

National curriculum content covered:

- use the concepts and vocabulary of prime numbers, factors (or divisors), multiples, common factors, common multiples, highest common factor, lowest common multiple, prime factorisation, including using product notation and the unique factorisation property
- use integer powers and associated real roots (square, cube and higher), recognise powers of 2, 3, 4, 5
- make and test conjectures about patterns and relationships; look for proofs or counterexamples
- begin to reason deductively in number and algebra

<u>Interleaving/Extension of previous work</u>

- Generating and describing sequences
- Factors and multiples

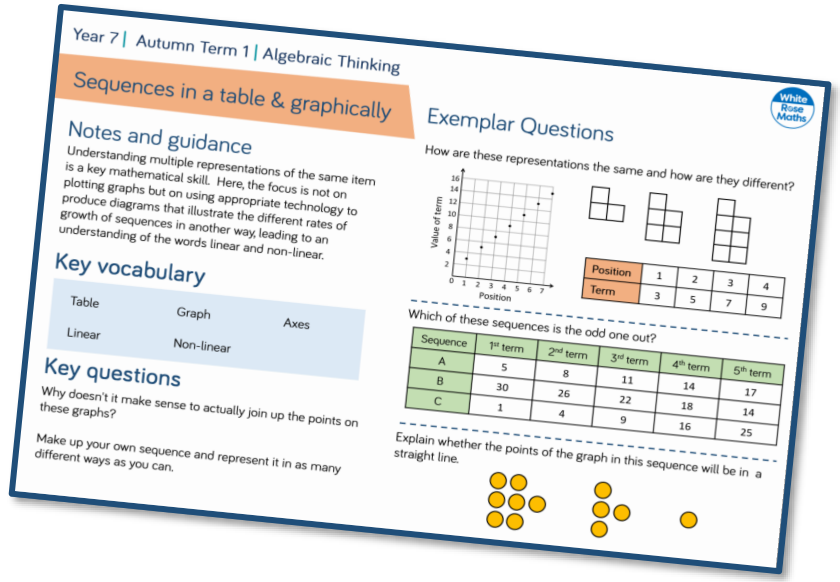
Why Small Steps?

We know that breaking the curriculum down into small manageable steps should help students to understand concepts better. Too often, we have noticed that teachers will try and cover too many concepts at once and this can lead to cognitive overload. We believe it is better to follow a "small steps" approach.

As a result, for each block of content in the scheme of learning we will provide a "small step" breakdown. ***It is not the intention that each small step should last a lesson – some will be a short step within a lesson, some will take longer than a lesson.*** We would encourage teachers to spend the appropriate amount of time on each step for their group, and to teach some of the steps alongside each other if necessary.

What We Provide

- Some ***brief guidance*** notes to help identify key teaching and learning points
- A list of ***key vocabulary*** that we would expect teachers to draw to students' attention when teaching the small step,
- A series of ***key questions*** to incorporate in lessons to aid mathematical thinking.
- A set of questions to help ***exemplify*** the small step concept that needs to be focussed on.



Year 7 | Autumn Term 1 | Algebraic Thinking

Sequences in a table & graphically

Notes and guidance

Understanding multiple representations of the same item is a key mathematical skill. Here, the focus is not on plotting graphs but on using appropriate technology to produce diagrams that illustrate the different rates of growth of sequences in another way, leading to an understanding of the words linear and non-linear.

Key vocabulary

Table, Graph, Axes, Linear, Non-linear

Key questions

Why doesn't it make sense to actually join up the points on these graphs?

Make up your own sequence and represent it in as many different ways as you can.

Exemplar Questions

How are these representations the same and how are they different?

Position	1	2	3	4
Term	3	5	7	9

Which of these sequences is the odd one out?

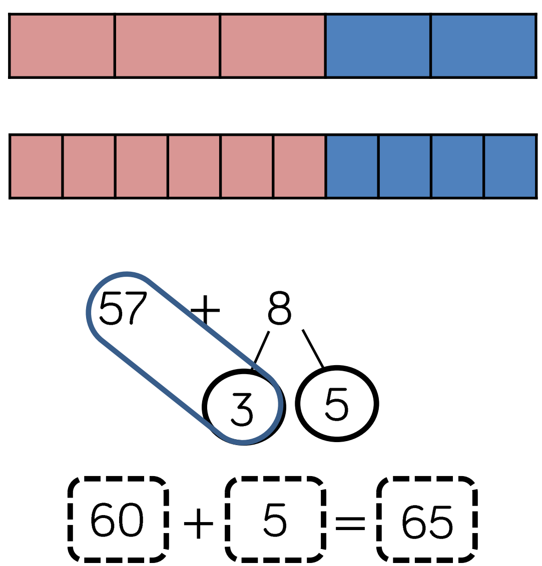
Sequence	1st term	2nd term	3rd term	4th term	5th term
A	5	8	11	14	17
B	30	26	22	18	14
C	1	4	9	16	25

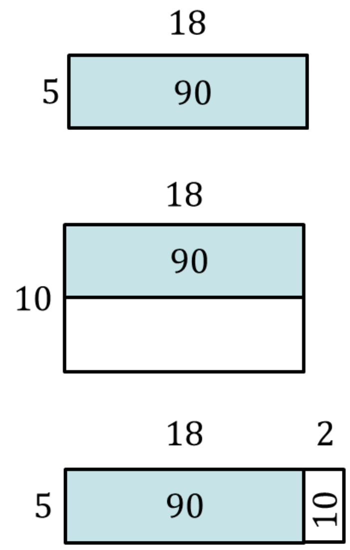
Explain whether the points of the graph in this sequence will be in a straight line.

- These include reasoning and problem-solving questions that are fully integrated into the scheme of learning. Depending on the attainment of your students, you many wish to use some or all of these exemplars, which are in approximate order of difficulty. Particularly challenging questions are indicated with the symbol .
- For each block, we also provide ideas for key representations that will be useful for all students.

In many of the blocks of material, some of the small steps are in **bold**. These are content aimed at higher attaining students, but we would encourage teachers to use these with as many students as possible – if you feel your class can access any particular small step, then please include it in your planning.

Key Representations

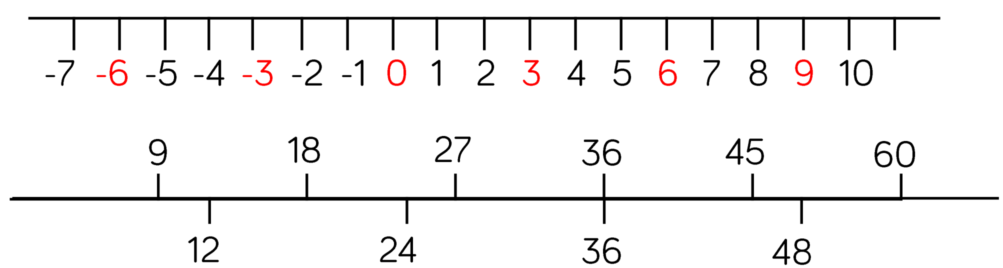




$10 \times 5 = 50$
$8 \times 5 = 40$
$50 + 40 = 90$

$18 \times 10 = 180$
$180 \div 2 = 90$

$20 \times 5 = 100$
$2 \times 5 = 10$
$100 - 10 = 90$



Concrete, pictorial and abstract representations are an important part of developing students' conceptual understanding.

Here are a few ideas for how you might use representations of number to develop reasoning skills.

Many of the small steps are linked to area of a rectangle to support understanding.

For example:
18×5 can be calculated in many different ways. It could be partitioned into 10×5 and 8×5 or 18 could be halved and 5 could be doubled to change the calculation to 9×10

Developing Number Sense

Small Steps

- Know and use mental addition and subtraction strategies for integers
- Know and use mental multiplication and division strategies for integers
- Know and use mental arithmetic strategies for decimals
- Know and use mental arithmetic strategies for fractions
- Use factors to simplify calculations
- Use estimation as a method for checking mental calculations
- Use known number facts to derive other facts
- Use known algebraic facts to derive other facts
- Know when to use a mental strategy, formal written method or a calculator

Mental addition and subtraction

Notes and guidance

This small step is for students to understand the properties of addition and subtraction, and how these can be used to simplify mental strategies in calculations. The explicit use of the vocabulary commutative and associative is important in ensuring that students approach calculations appropriately and with flexibility. Techniques such as the 'make ten' strategy highlight useful shortcuts to simplify calculations.

Key vocabulary

Compensation	Number Line	Addition
Subtraction	Associative	Commutative

Key questions

How can you check answers to subtraction problems using addition?
Can you explain why addition is commutative using concrete manipulatives? Does the same apply to subtraction?

Exemplar Questions

Which of these are true? Explain why.

- $37 + 45 = 45 + 37$
- $180 - 64 = 180 - 60 - 4$
- $24 - 8 = 24 - 10 + 2$
- $32 - 9 = 32 - 10 - 1$
- $48 + 14 + 16 = 48 + 30$
- $24 - 8 = 8 - 24$
- $360 - 147 = 360 - 100 - 40 - 7$
- $180 - 64 = 124$

Use the 'make 10' strategy to calculate

- $27 + 19 + 33$
- $76 + 34 - 99$
- $28 + 662$
- $342 + 15 + 58$

Tommy suggests three different ways of calculating $68 + 99$

$67 + 100$	$68 + 100 - 1$	$60 + 90 + 8 + 9$

All three ways are correct. Explain why. Which one do you prefer?

Using addition and subtraction, how many different ways can you make 37 using the following digit cards:

11	18	19	45	8	26

Mental multiplication and division

Notes and guidance

Teacher modelling of different strategies to simplify calculations, using concrete and pictorial representations alongside the abstract helps students to develop a flexible approach to problem solving as well as giving them the confidence to choose an appropriate strategy. Partitioning of numbers and using factors to simplify calculations, including spotting multiples such as 5 and 10 are important skills to develop.

Key vocabulary

Partition	Multiply	Divide
Commutative	Associative	Factors

Key questions

What does partitioning mean?

Why do we do some multiplications by portioning and adding, but others by partitioning and subtracting?

Exemplar Questions

To calculate the area of the rectangle, Tommy knows he need to work out 18 × 5. Below are four different methods of doing this.

A: $10 \times 5 + 8 \times 5$
B: 9×10
C: $20 \times 5 - 2 \times 5$
D: $(18 \times 10) \div 2$

Show that each calculation represents an area of 90 cm^2 by drawing diagrams. Explain why each method works.

The easiest way to calculate 3000 ÷ 25 is to start by thinking about how many lots of 25 there are in 100

Whitney

What would Whitney's next steps be to complete the calculation?
Write down a different way of doing this calculation.
Compare the two methods. Which is the easiest one to use? Why?

Which strategy would you use for the following calculations?
Work out the answers using your strategy.

150 × 4	43 × 9	17 × 11	53 × 98
2800 ÷ 14	42 ÷ 3	800 ÷ 20	900 ÷ 15

Mental strategies for decimals

Notes and guidance

In this small step, students should recognise that previous strategies used to calculate with integers can be extended to decimals.

It's key that students have a sound grasp of place value so that they can use the language of thousandths, hundredths and tenths confidently.

Key vocabulary

Place Value	Estimate	Tenths
Hundredths	Thousandths	

Key questions

How does estimation help us check if answers are reasonable?
Does multiplication always make a number bigger?
Why is multiplying by 0.1 the same as dividing by 10?
Can you just "add a zero" to multiply by 10?

Exemplar Questions

Which strategy would you use to find 2.3 + 2.4? Why?

$2 + 2 = 4$ $0.3 + 0.4 = 0.7$ $4 + 0.7 = 4.7$	Double 2.4 is 4.8 2.3 + 2.4 is 0.1 less than this so the answer is 4.7	Double 2.3 is 4.6 2.3 + 2.4 is 0.1 more than this so the answer is 4.7

Here are two methods for calculating 1.2×0.03

$12 \times 3 = 36$
$1.2 \times 3 = 3.6$
$1.2 \times 0.3 = 0.36$
$1.2 \times 0.03 = 0.036$

$12 \times 3 = 36$
$\div 10$ $\quad \div 100$ $\quad \div 1000$
$1.2 \times 0.03 = 0.036$

Compare the methods to calculate:

- 300×0.01
- 0.08×1.1
- 0.4×0.5

Alfie's lunch costs £4.57. He pays with a £10. How much change does he get? Do the following methods work? Explain your answer.

Start with £9.99 and subtract £4.57 Now add one back on to get the answer.	Start with £9.99 and subtract £4.56 This gives the answer.

Now use a number line to work out Alfie's change by counting up from £4.57 to £10. Which of the methods do you prefer?

Mental strategies for fractions

Notes and guidance

This step ensures that students understand the role of the denominator as the divisor in finding a fraction of a quantity. Students understand the idea of sharing a whole to find the value of each equal part and then multiplying to find the required fraction. Use concrete and pictorial representations alongside jottings or mental calculations to support understanding.

Key vocabulary

Whole	Equal parts	Multiply
Numerator	Denominator	

Key questions

Is $\frac{1}{2}$ of an amount always bigger than $\frac{1}{4}$ of an amount?

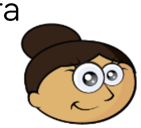
Is it possible to find $\frac{5}{3}$ of a number?

What is the relationship between the denominator, numerator and finding a fraction of an amount?

Exemplar Questions

How much money did Dora have to start with?

Dora

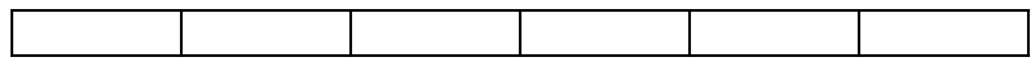


I've spent $\frac{3}{5}$ of my money. I've got £3 left.

A bar model might help you:

Use the bar model to help you find

$\frac{1}{6}$ of 36	$\frac{5}{6}$ of 36	$\frac{1}{3}$ of 36
$\frac{4}{3}$ of 36	$\frac{1}{12}$ of 36	$\frac{12}{12}$ of 36



If the whole were 18 instead, how would this change your answers? What if the whole were 12? What if the whole were 48?

Special offer: Buy two items and save $\frac{1}{4}$ of the total price.

Ron buys a TV for £200 and a pair of trainers for £55. He only pays $\frac{1}{4}$ of the price. He says the total cost is about £65. Explain whether Ron's answer is:

Too Big | Too Small | Correct

Use factors to simplify calculations

Notes and guidance

Through this step, students will develop flexibility in representing numbers using their factors.

They will be able to choose the most efficient representation in terms of allowing a calculation to be simplified. In particular, looking for combinations of 25 and 4, 125 and 8 etc. is useful.

Key vocabulary

Factor	Equivalent	Calculation
Commutative	Associative	Multiple

Key questions

What numbers are easiest to multiply by?

What factors should you look for to make a calculation easier?

Why does using a different form of the number still give you the same answer?

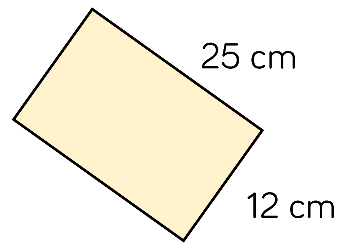
Exemplar Questions

Rosie thinks 16×20 can be written in lots of different ways.
She writes down the following ways

A: $16 \times 2 \times 10$	B: $8 \times 2 \times 2 \times 10$
C: $2 \times 2 \times 2 \times 2 \times 2 \times 10$	D: $4 \times 4 \times 2 \times 10$

Explain why you agree or disagree with each factorisation.
Which calculation would you choose to work out 16×20

Find the area of the rectangle.



By using factors of 12 and 25, how many different ways can you find to calculate 12×25?

How many different ways did your whole class find?

36×15	$18 \times 2 \times 15$	$18 \times 3 \times 10$
$6 \times 6 \times 10 \times 5$	$2 \times 9 \times 3 \times 10$	$36 \times 3 \times 5$

Which calculation is the odd one out? Why?
Which calculation is equivalent to 36×15?

$16 \times 75 = 4 \times 4 \times 25 \times 3 = 4 \times 100 \times 3 = 1200$

Use a similar strategy to work out:

- 125×12
- 28×75
- 125×80
- $36 \times 25 \times 20$

White Rose Maths

Estimation

Notes and guidance

Students need to be challenged to find the most appropriate estimate in different contexts, it is not always suitable to round to 1 sf. Students should consider whether their rounding will lead to overestimates or underestimates. Rounding to one significant figure should however be revised, including working with numbers less than 1

Key vocabulary

Rounding	Place Value	Significant Figures
Estimate	Overestimate	Underestimate

Key questions

Why is estimation useful?

Is estimating the same as rounding?

Is estimating the same as approximating?

Exemplar Questions

Sam said the following is true because the numbers are too small.

$325 + 773 < 1200$

Help Sam explain his reasoning by using estimation to argue why the statement must be true.

By rounding each number to one significant figure, find estimates to these multiplications.

58×19 $\quad$ 64×23 $\quad$ 57×22

Is it possible to tell whether your estimates are too large or too small? How do you know?

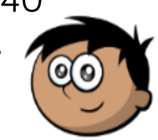
Use estimation to decide whether these statements are true or false.

862×0.9 $\quad$ $296 \times 0.78 < 250$ $\quad$ $8716 + 12\,423 < 15\,000$

$\frac{2}{5}$ of $19\,800 > \frac{1}{3}$ of $17\,900$ $\quad$ $37 \div 3.68 < 4.86 + 6.71$

Amir receives £80 for his birthday. He wants to buy as many of his favourite author's books as possible. Each book costs £7.40

Amir uses £8 as an estimate for each book instead of £7
Do you agree with Amir's strategy? Why?



White Rose Maths

Number facts to derive other facts

Notes and guidance

Students need a firm understanding of the structure of the an operation (e.g. addition) in order to manipulate this to find other facts. It is important to involve all students in discussing their approaches to a question (possibly through 'talking trios'). Setting one question and asking students to share their approaches, allows the class teacher to model these, thus sharing ideas and encouraging flexibility when approaching calculations.

Key vocabulary

Equivalent	Addend	Compensate
Product	Quotient	

Key questions

What's remains the same about the question, what's different?
How does multiplying one number in a calculation affect the answer? What about both numbers?
How can I change both numbers in a division but keep the answer the same?

Exemplar Questions

How could you change the calculation but keep the **total** the same?

$231 + 428 = 659$

What happens to the **total** if:
428 is changed to 438?
Both addends are reduced by 1?
Both addends are multiplied by 1000?

$23 \times 42 = 996$

Use this number fact to derive the answers to:

2.3×4.2	996×42
23×21	$996 \div 230$

How many other number facts can you derive from this fact?

Use the fact that $1798 \div 29 = 62$ to find $1798 \div 2.9$


Rosie

The answer must be 6.2. If 29 is divided by 10, then we have to *divide* 62 by 10.

But I think 2.9 will fit into 1798 more times than 29. I think we need to *multiply* 62 by 10 to get an answer of 620.


Mo

Who's right, Rosie or Mo? Explain why.

Algebraic facts to derive other facts

Notes and guidance

Students demonstrate an understanding of the difference between an equation and an expression by being able to identify equivalent facts. This small step also allows manipulation of number facts to be extended to rearranging equations without the need of a formal introduction to this.

Key vocabulary

Equation

Expression

Equal

Equality

Key questions

Explain the difference between an equation and an expression.

If I double both sides in an equation, is the value of the unknown the same?

What does the $=$ sign mean?

Exemplar Questions

Use the fact $a + b = 6$ to find the values or expressions

$a + b = 6$

- $\frac{1}{2}a + \frac{1}{2}b =$
- $2a + b =$
- $a + b + 1 =$
- $10a + 10b =$
- $= 600$
- $0.1a - 0.1b =$
- $= 8$
- $a + b - 1 =$

Use the fact $x = 2y + 8$ to find the values or expressions of

$10x =$	$= y + 4$	$= 4y + 16$

If $\frac{n}{4} = 2$ explain whether the following are true or false.

$\frac{n}{4} + 1 = 3$	$n = 8$	$\frac{n}{2} = 1$	$\frac{3n}{4} = 6$

Choosing the best strategy

Notes and guidance

In this step, the choice of method and strategy should be the focus rather than final answers. Students should become able to quickly identify whether an efficient mental method should be used, or whether a formal written method is more appropriate. They should also know when to use their calculator and to interpret the calculator display in the units referred to in the problem (e.g. money).

Key vocabulary

Estimate	Mental	Calculator
Formal	Efficient	Interpret

Key questions

Is your mental method more efficient than a written method? Is it quicker or slower than using a written method?

Can you interpret your calculator display in terms of the context of the question?

Can time calculations be done on a calculator e.g. how long is it from 1835 to 1920?

Exemplar Questions

Sort each question into the table depending on whether you would use a formal written method or a mental strategy.

28.72 + 3.41	41 × 15	6007 – 11
3.2 ÷ 5	897 + 398	$\frac{1}{2}$ of 77.8

Mental Strategy	Formal Written Method

Now calculate the answers using your method.
Did you use the same method?
Which ones are easier to do using a formal written method? Why?

Decide on the most efficient method to solve each problem.

A – Formal written method B – Mental strategy C - Calculator

8 979 people watch a netball match. 5 602 are male. How many are female?

Can Mr Hussein buy 40 bags of crisps if he has £10 and each pack costs 27p?

It is 45 miles from Leeds to Manchester. Can you travel from Leeds to Manchester in 40 minutes if you travel at an average speed of 70 miles per hour?