Multiplicative Reasoning

Year 11

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	Week 1	Week 2	Week 3	Week 4	Week 5	Week 6	Week 7	Week 8	Week 9	Week 10	Week 11	Week 12
Autumn	**Graphs**						**Algebra**					
	Gradients & lines		Non-linear graphs		Using graphs		Expanding & Factorising		Changing the subject		Functions	
Spring	**Reasoning**						**Revision and Communication**					
	Multiplicative		Geometric		Algebraic		Transforming & Constructing		Listing & describing		Show that...	
Summer	**Revision**						**Examinations**					

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Spring 1 : Reasoning

Weeks 1 and 2: Multiplicative Reasoning

Students develop their multiplicative reasoning in a variety of contexts, from simple scale factors through to complex equations involving direct and inverse proportion. They link inverse proportion with the formulae for pressure and density. There is also the opportunity to review ratio problems.
National Curriculum content covered includes:

- compare lengths, areas and volumes using ratio notation and/or scale factors; make links to similarity
- understand that X is inversely proportional to Y is equivalent to X is proportional to $\frac{1}{Y}$
- **{construct and}** interpret equations that describe direct and inverse proportion
- extend and formalise their knowledge of ratio and proportion, including trigonometric ratios, in working with measures and geometry, and in working with proportional relations algebraically and graphically

Weeks 3 and 4: Geometric Reasoning

Students consolidate their knowledge of angles facts and develop increasingly complex chains of reasoning to solve geometric problems. Higher tier students revise the first four circle theorems studied in Year 10 and learn the remaining theorems. Students also revisit vectors and the key topics of Pythagoras' theorem and trigonometry.
National Curriculum content covered includes

- reason deductively in geometry, number and algebra, including using geometrical constructions
- **{apply and prove the standard circle theorems concerning angles, radii, tangents and chords, and use them to prove related results}**
- interpret and use bearings
- apply addition and subtraction of vectors, multiplication of vectors by a scalar, and diagrammatic and column representations of vectors; **{use vectors to construct geometric arguments and proofs}**

Weeks 5 and 6: Algebraic Reasoning

Students develop their algebraic reasoning by looking at more complex situations. They use their knowledge of sequences and rules to made inferences, and Higher tier students move towards formal algebraic proof. Forming and solving complex equations, including simultaneous equations, is revisited. Higher tier students also look at solving inequalities in more than one variable.
National Curriculum content covered includes:

- make and test conjectures about the generalisations that underlie patterns and relationships; look for proofs or counter-examples; begin to use algebra to support and construct arguments **{and proofs}**
- deduce expressions to calculate the n^{th} term of linear **{and quadratic}** sequences
- solve two simultaneous equations in two variables (linear/linear **{or linear/quadratic}**) algebraically; find approximate solutions using a graph
- solve linear inequalities in one **{or two}** variable**{s}**, **{and quadratic inequalities in one variable}**; represent the solution set on a number line, **{using set notation and on a graph}**

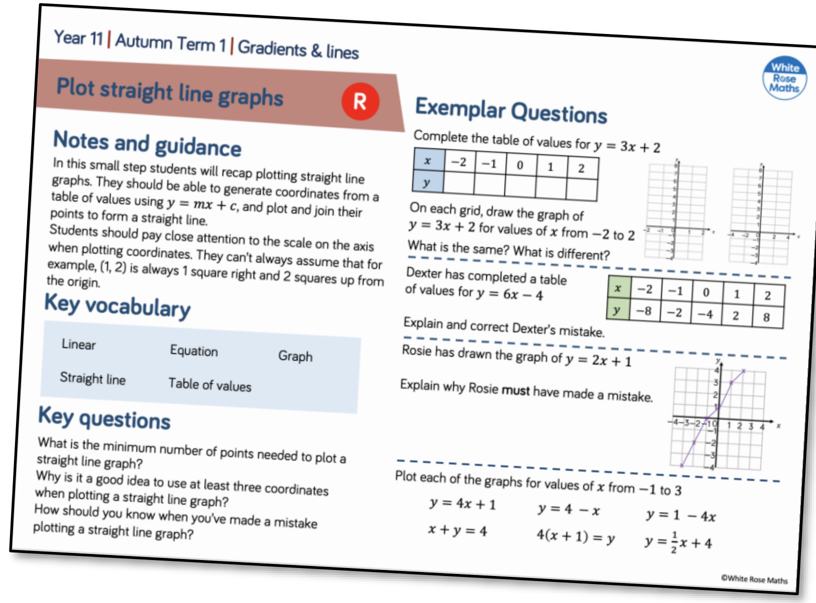
Why Small Steps?

We know that breaking the curriculum down into small manageable steps should help students to understand concepts better. Too often, we have noticed that teachers will try and cover too many concepts at once and this can lead to cognitive overload. We believe it is better to follow a "small steps" approach.

As a result, for each block of content in the scheme of learning we will provide a "small step" breakdown. ***It is not the intention that each small step should last a lesson – some will be a short step within a lesson, some will take longer than a lesson.*** We would encourage teachers to spend the appropriate amount of time on each step for their group, and to teach some of the steps alongside each other if necessary.

What We Provide

- Some ***brief guidance*** notes to help identify key teaching and learning points
- A list of ***key vocabulary*** that we would expect teachers to draw to students' attention when teaching the small step,
- A series of ***key questions*** to incorporate in lessons to aid mathematical thinking.
- A set of questions to help ***exemplify*** the small step concept that needs to be focussed on.


Year 11 | Autumn Term 1 | Gradients & lines

Plot straight line graphs R

Notes and guidance

In this small step students will recap plotting straight line graphs. They should be able to generate coordinates from a table of values using $y = mx + c$, and plot and join their points to form a straight line.
Students should pay close attention to the scale on the axis when plotting coordinates. They can't always assume that for example, (1, 2) is always 1 square right and 2 squares up from the origin.

Key vocabulary

Linear, Equation, Graph, Straight line, Table of values

Key questions

What is the minimum number of points needed to plot a straight line graph?
Why is it a good idea to use at least three coordinates when plotting a straight line graph?
How should you know when you've made a mistake plotting a straight line graph?

Exemplar Questions

Complete the table of values for $y = 3x + 2$

x	-2	-1	0	1	2
y					

On each grid, draw the graph of $y = 3x + 2$ for values of x from -2 to 2
What is the same? What is different?

Dexter has completed a table of values for $y = 6x - 4$

x	-2	-1	0	1	2
y	-8	-2	-4	2	8

Explain and correct Dexter's mistake.

Rosie has drawn the graph of $y = 2x + 1$
Explain why Rosie **must** have made a mistake.

Plot each of the graphs for values of x from -1 to 3

$y = 4x + 1$ $\quad y = 4 - x$ $\quad y = 1 - 4x$

$x + y = 4$ $\quad 4(x + 1) = y$ $\quad y = \frac{1}{2}x + 4$

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- These include reasoning and problem-solving questions that are fully integrated into the scheme of learning. Depending on the attainment of your students, you many wish to use some or all of these exemplars, which are in approximate order of difficulty. Particularly challenging questions are indicated with the symbol .
- For each block, we also provide ideas for key representations that will be useful for all students.

Some of the small steps are in **bold** and labelled with H to indicate this is Higher tier GCSE content. We would encourage teachers to use these with as many students as possible – if you feel your class can access any particular small step, then please include it in your planning. Steps that review content covered at Key Stage 3 are labelled R .

Multiplicative Reasoning

Small Steps

- Use scale factors (R)
- Understand direct proportion
- **Construct complex direct proportion equations** (H)
- Calculate with pressure and density
- Understand inverse proportion
- **Construct inverse proportion equations** (H)
- Ratio problems (R)

(H) denotes Higher Tier GCSE content

(R) denotes 'review step' – content should have been covered at KS3

Use scale factors

R

Notes and guidance

This step reviews the concept of a scale factor. This is a good opportunity to use scale factors between 0 and 1 (reminding students that this is still an enlargement) as well as those above 1. In this step, students should practise finding scale factors as well as using them. Revisiting the definition of a similar shape is also emphasised in this step. To extend this review step, Higher tier students could revisit area and volume scale factors.

Key vocabulary

Enlargement	Scale factor
Multiplier	Similar

Key questions

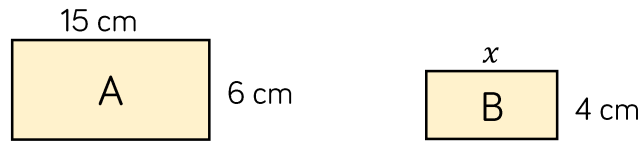
If an object is enlarged, does it always get bigger?

How can you work out the scale factor? Is there more than one method?

When are two shapes similar?

Exemplar Questions

Shape B is an enlargement of shape A.

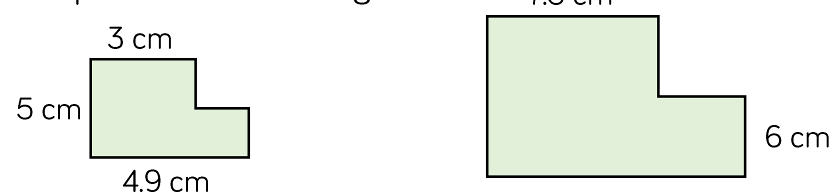


Mo has incorrectly worked out the value of x. Here is his working.

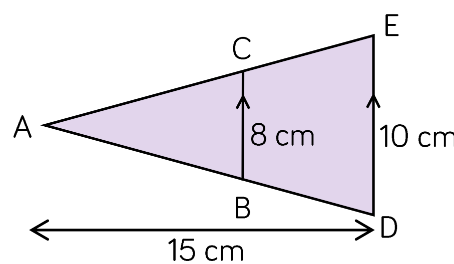
6 cm − 2 cm = 4 cm and 15 cm − 2 cm = 13 cm so x = 13 cm

Explain Mo's mistake and work out the value of x.

Here is a pair of similar hexagons.



Work out all of the missing lengths on both shapes.



- Prove that triangles ABC and ADE are similar.
- Work out the perpendicular height of ABC.

Is the length scale factor the same as the area scale factor? Use calculations to justify your answer.

Understand direct proportion

Notes and guidance

This aim of this step is to understand direct proportion before introducing $y = kx$. Direct proportion relationships such as diameter and circumference, converting units, currency conversions etc. can all be revisited. Students should be exposed to different representations such as word problems, graphs and equations. Students studying for Foundation GCSE should also form simple direct proportion equations in this step ($y = kx$).

Key vocabulary

Direct proportion	Equation	Origin
Constant ratio	Straight line	Linear

Key questions

List as many examples as you can of relationships that are in direct proportion to one another.

Why aren't (e.g. speed and time) directly proportional to one another?

Describe the key features of a graph representing direct proportion.

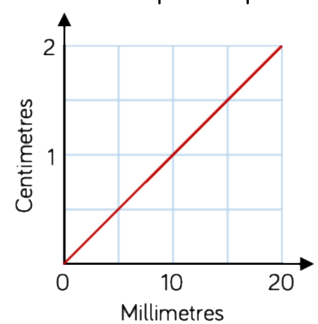
Exemplar Questions

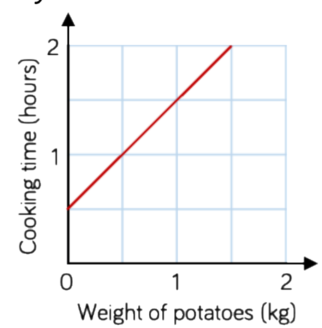
A train is travelling at a constant speed of 140 km/h.
Work out how far the train will it travel in

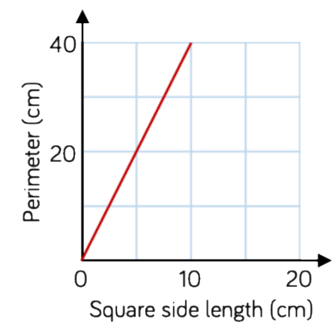
- 6 hours
- 2.5 hours
- 30 minutes
- 20 minutes

Eva says "distance travelled is directly proportional to the time travelled".
Explain why Eva is correct.

Which of the graphs does not represent a directly proportion relationship? Explain how you know.







Write down a formula to find

- the perimeter of a square if given its side length
- a length in centimetres if given the length in millimetres.

Which of these equations represent direct proportion relationships?

- Euros = 1.1 × British Pounds
- Mass = Density × Volume
- Cost of stapler and staples = 2 + (Number of packs of staples × 0.5)

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Direct proportion equations H

Notes and guidance

Students are introduced to the proportionality symbol, α, and the constant of proportionality, k. Knowledge of constant ratios in a direct proportion relationship leads to the general equation $y = kx$. A common mistake is to find k but then forget to substitute it into the equation. Encourage students to write out the full equation as soon as k has been found can. In more complex problems, students often confuse the use of e.g. 'square/square root' and this needs highlighting.

Key vocabulary

Direct proportion	Constant of proportionality
Equation	Varies directly

Key questions

When is it appropriate to use the equation $y = kx$?
How can we work out k? What is the resulting equation?
How can this equation be used to find x when given y?
How can this equation be used to find y when given x?
What do you notice about the value of y when $x = 0$? Will this always be true? Why?

Exemplar Questions

a and b are directly proportional to each other.

a	b
1	8
5	
	56
	800

- Calculate the missing values in the table
- Dexter says, "$b = ka$"
  Dexter is correct. Work out the value of k.
- Use your equation to find b when $a = 0.35$

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c is directly proportional to the square of d.
Which of the cards are true?

$c = d^2$	$c \alpha d$	$c \alpha d^2$	$c \alpha kd$
$c = k \times d$	$c = k \times d^2$	$c = \sqrt{d^2}$	$c = kd^2$

When $c = 25$, $d = 2.5$
Work out the value of the constant of proportionality, k.

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y varies directly with the cube root of x.
When $y = 27$, $x = 216$

- Show that when $x = 1000$, $y = 45$
- Work out y when $x = 1\,000\,000$
- Work out the value of x when $y = 90$

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Calculate with pressure & density

Notes and guidance

You might want to revisit rearranging simple equations in the form $4.1 = \frac{3.6}{x}$ before starting this step. Speed, distance and time can also be reviewed, making links to direct proportion. Students then consider the similar formulae for pressure and density. Students should have a good understanding of what these concepts are before progressing onto use of equations. Understanding of the units used is important.

Key vocabulary

Density	Mass	Volume
Pressure	Force	Area

Key questions

If we compare two solids on a table which apply the same force, how can we tell which one will apply a greater pressure?

How do we know which number to substitute?

What is the relationship with direct proportion?

Exemplar Questions

Sort the cards into those that represent density, mass or volume.

$8\ g/cm^3$	$1\ cm^3$	36.2 g	$2\ m^3$	$0.01\ g/cm^3$

Lead has a density of $11.3\ g/cm^3$.
Complete the table.

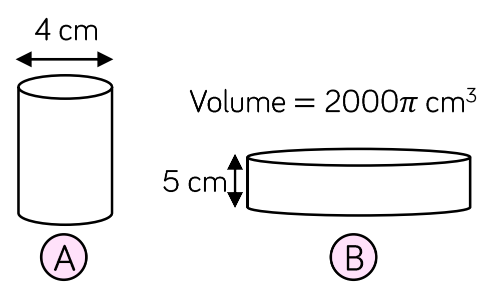
Volume	$2\ cm^3$	$3\ cm^3$	$4\ cm^3$	$10\ cm^3$
Mass				

Teddy says "mass is directly proportional to volume".
Explain why Teddy is correct.

Work out the constant of proportionality for lead.

A piece of tin and a piece of aluminium each have a mass of 1 kg.
The volume of the piece of tin is $137.74\ cm^3$.
The density of the piece of aluminium is $2.7\ g/cm^3$.
Compare the density, mass and volume of the two pieces of metal.

Complete the table.



	Cylinder A	Cylinder B
Pressure		$10\ N/cm^2$
Force	300 N	
Area of base		

Understand inverse proportion

Notes and guidance

Students can now consider the three variables in the speed, distance, time or mass, density, volume relationships to distinguish between direct and inverse proportion. Inverse proportion relationships should be explored in different representations such as word problems, graphs and equations. Students studying for Foundation GCSE should also form simple inverse proportion equations in this step ($y = \frac{k}{x}$).

Key vocabulary

Inverse proportion	Equation
Smooth curve	Constant of proportionality

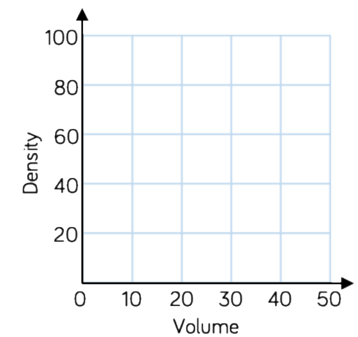
Key questions

What is the same and what is different about direct and inverse proportion?
Will it take less time for more people to complete a task? What assumption have you made?
How can we write an equation to represent inverse proportion?

Exemplar Questions

Complete the table.

Mass	Volume	Density
100 kg	1 cm^3	
100 kg	10 cm^3	
100 kg	20 cm^3	
100 kg	50 cm^3	



- What happens to the density of an object as the volume increases?
- Sketch a graph of volume verses density on a copy of the axis shown.
  What are they key features of the graph?
  How does this differ from direct proportion?

A builder takes 1 day to build a wall.

Alex says "if there were 2 builders it would have taken them 2 days."

Explain why is Alex incorrect.
How long will it take 2 builders if they both work at the same pace?

Decide whether each of these relationships are directly or inversely proportional.

Density of an object and its volume	Number of days it takes to build a house and number of builders	Speed of a car and distance it travels

Inverse proportion equations H

Notes and guidance

By now students should be familiar with both simple direct and inverse proportional relationships. This leads to more complex inverse proportion equations, working in the abstract.

You may wish to explore both the forms $y = \frac{k}{x}$ and $xy = k$ as ways of solving inverse proportion problems. Students again need to be careful when reading questions to identify the correct power of the variable in a relationship.

Key vocabulary

Inverse proportion	Varies inversely
Constant of proportionality	Substitute

Key questions

How can we work out the constant of proportionality, k?

How can we use this equation to calculate x given y and vice versa?

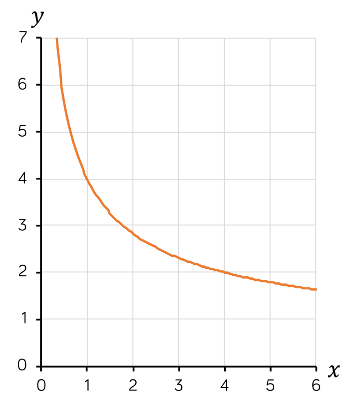
How do we know what to substitute?

Exemplar Questions

x and y are inversely proportional to each other.

x	y
1	24
2	12
3	
	6

- Calculate the missing values in the table
- Annie says "$x \, \alpha \, \frac{1}{y}$ and so $x = \frac{k}{y}$"
  Annie is correct. Calculate the value of k.
- Use your equation to find x when $y = 1.5$



- Find the value of y when $x = 1$
- y is inversely proportional to the square root of x

  Dexter writes $y = \frac{k}{x^2}$

  Correct Dexter's mistake and calculate k.
- Calculate y when $x = 100$

r is inversely proportional to the cube of s.
When $r = 8, s = 5$

- Show that when $r = 64, s = 2.5$

Give as exact answers in simplest form.

- Work out r when $s = 8$
- Work out the value of s when $r = 10$

Ratio problems

R

Notes and guidance

This step is an opportunity for students to revisit ratio problems and strategies for solving these. Students should be encouraged to use bar models and two-way tables where appropriate. When combining ratios, teachers may want to revisit LCM first. For students studying the higher tier GCSE, problems involving algebra and ratio will be appropriate.

Key vocabulary

Proportion	Fraction	Percentage
Bar model	Two-way table	LCM

Key questions

How can we use a bar model to represent the problem?

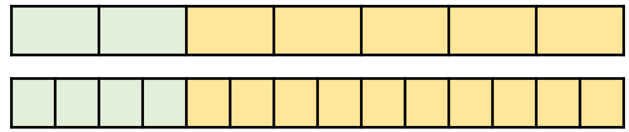
What does the bar model(s) tell us about our problem?

Do we need to divide the bar model up into more equal parts? How can we do this?

How can we find a percentage/fraction of a quantity using a calculator? Without a calculator?

Exemplar Questions

The ratio of boys to girls at a party is 2 : 5
Three-quarters of the boys wore jeans.



Use the bar models to work out the fraction of children at the party who were boys wearing jeans.

Mrs Rose has a salary £25 000 per year.

- 18% of this is spent on taxes and national insurance.
- $\frac{3}{8}$ of her salary is spent on rent.
- She gives £10 a month to charity.

She spends the rest on leisure and living costs in the ratio 2 : 3
What percentage of her overall salary did she spend on leisure?

One morning, a vet treated cats and dogs in the ratio 4 : 5
The same morning, the vet treated cats and rabbits in the ratio 6 : 1
Write down the ratio of cats to dogs to rabbits treated by the vet that morning.

The ratio of $x : y = 1 : 4$ and $3y = 2z$

- Work out the value of z when $y = 4$
- Show that the ratio $y : z = 2 : 3$
- Explain why the ratio $x : y : z$ is $1 : 4 : 6$