

Transforming & Constructing

Year 11

#MathsEveryoneCan



	Week 1	Week 2	Week 3	Week 4	Week 5	Week 6	Week 7	Week 8	Week 9	Week 10	Week 11	Week 12
Autumn	Graphs						Algebra					
	Gradients & lines		Non-linear graphs		Using graphs		Expanding & Factorising		Changing the subject		Functions	
Spring	Reasoning						Revision and Communication					
	Multiplicative		Geometric		Algebraic		Transforming & Constructing		Listing & describing		Show that...	
Summer	Revision						Examinations					

Spring 2 : Revision & Communication

Weeks 1 and 2: Transforming & Constructing

Students revise and extend their learning from Key Stage 3, exploring all the transformations and constructions, relating these to symmetry and properties of shapes when appropriate. There is an emphasis on describing as well as performing transformations as using the language promotes deeper thinking and understanding. Higher tier students extend their learning to explore the idea of invariance and look at trigonometric graphs as a vehicle for exploring graph transformations.

National Curriculum content covered includes:

- describe translations as 2D vectors
- reason deductively in geometry, number and algebra, including using geometrical constructions
- interpret and use fractional **{and negative}** scale factors for enlargements
- **{describe the changes and invariance achieved by combinations of rotations, reflections and translations}**
- recognise, sketch and interpret graphs of **{the trigonometric functions (with arguments in degrees) for angles of any size}**
- **{sketch translations and reflections of the graph of a given function}**

Weeks 3 and 4: Listing & Describing

This block is another vehicle for revision as the examinations draw closer. Students look at organisation information, with Higher tier students extending this to include the product rule for counting. Links are made to probability and other aspects of Data Handling such as describing and comparing distributions and scatter diagrams. Plans and elevations are also revised. You can adapt the exact content to suit the needs of your class.

National Curriculum content covered includes:

- explore what can and cannot be inferred in statistical and probabilistic settings, and express their arguments formally

- calculate the probability of independent and dependent combined events, including using tree diagrams and other representations, and know the underlying assumptions
- **{calculate and interpret conditional probabilities through representation using expected frequencies with two-way tables, tree diagrams and Venn diagrams}**
- apply systematic listing strategies, **{including use of the product rule for counting}**
- construct and interpret plans and elevations of 3D shapes

Weeks 5 and 6: Show that

This is another block designed to be adapted to suit the needs of your class. Examples of communication in various areas of mathematics are provided in order to highlight gaps in knowledge that need addressing in the run up to the examinations. “Show that” is used to encourage students to communicate in a clear mathematical fashion, and this skill should be transferred to their writing of solutions to any type of question.

National Curriculum content covered includes:

- know the difference between an equation and an identity; argue mathematically to show algebraic expressions are equivalent, and use algebra to support and construct arguments **{and proofs}**
- apply the concepts of congruence and similarity
- make and use connections between different parts of mathematics to solve problems
- **{change recurring decimals into their corresponding fractions and vice versa}**
- apply addition and subtraction of vectors, multiplication of vectors by a scalar, and diagrammatic and column representations of vectors; **{use vectors to construct geometric arguments and proofs}**

Why Small Steps?

We know that breaking the curriculum down into small manageable steps should help students to understand concepts better. Too often, we have noticed that teachers will try and cover too many concepts at once and this can lead to cognitive overload. We believe it is better to follow a “small steps” approach.

As a result, for each block of content in the scheme of learning we will provide a “small step” breakdown. ***It is not the intention that each small step should last a lesson – some will be a short step within a lesson, some will take longer than a lesson.*** We would encourage teachers to spend the appropriate amount of time on each step for their group, and to teach some of the steps alongside each other if necessary.

What We Provide

- Some **brief guidance** notes to help identify key teaching and learning points.
- A list of **key vocabulary** that we would expect teachers to draw to students' attention when teaching the small step.
- A series of **key questions** to incorporate in lessons to aid mathematical thinking.
- A set of questions to help **exemplify** the small step concept that needs to be focussed on.

Year 11 | Autumn Term 1 | Gradients & lines

Plot straight line graphs R

Notes and guidance

In this small step students will recap plotting straight line graphs. They should be able to generate coordinates from a table of values using $y = mx + c$, and plot and join their points to form a straight line. Students should pay close attention to the scale on the axis when plotting coordinates. They can't always assume that for example, (1, 2) is always 1 square right and 2 squares up from the origin.

Key vocabulary

Linear	Equation	Graph
Straight line	Table of values	

Key questions

What is the minimum number of points needed to plot a straight line graph?
 Why is it a good idea to use at least three coordinates when plotting a straight line graph?
 How should you know when you've made a mistake plotting a straight line graph?

Exemplar Questions

Complete the table of values for $y = 3x + 2$

x	-2	-1	0	1	2
y					

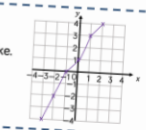
On each grid, draw the graph of $y = 3x + 2$ for values of x from -2 to 2. What is the same? What is different?

Dexter has completed a table of values for $y = 6x - 4$

x	-2	-1	0	1	2
y	-8	-2	-4	2	8

Explain and correct Dexter's mistake.

Rosie has drawn the graph of $y = 2x + 1$




Explain why Rosie must have made a mistake.

Plot each of the graphs for values of x from -1 to 3

$y = 4x + 1$	$y = 4 - x$	$y = 1 - 4x$
$x + y = 4$	$4(x + 1) = y$	$y = \frac{1}{2}x + 4$

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- These include reasoning and problem-solving questions that are fully integrated into the scheme of learning. Depending on the attainment of your students, you may wish to use some or all of these exemplars, which are in approximate order of difficulty. Particularly challenging questions are indicated with the symbol .
- For each block, we also provide ideas for key representations that will be useful for all students.

Some of the small steps are in **bold** and labelled with H to indicate this is Higher tier GCSE content. We would encourage teachers to use these with as many students as possible – if you feel your class can access any particular small step, then please include it in your planning. Steps that review content covered at Key Stage 3 are labelled R.

Transforming and Constructing

Small Steps

- ▶ Perform and describe line symmetry and reflection R
- ▶ Perform and describe rotation/rotational symmetry R
- ▶ Perform and describe translations of shapes R
- ▶ Perform and describe enlargements of shapes R
- ▶ **Perform and describe negative enlargements of shapes** R H
- ▶ Identify transformations of shapes R
- ▶ Perform and describe a series of transformations of shapes
- ▶ **Identify invariant points and lines** H

H denotes Higher Tier GCSE content

R denotes 'review step' – content should have been covered at KS3

Transforming and Constructing

Small Steps

- ▶ Perform standard constructions using ruler and protractor or ruler and compasses R
- ▶ Solve loci problems
- ▶ Understand and use trigonometrical graphs H
- ▶ Sketch and identify translations of the graph of a given function H
- ▶ Sketch and identify reflections of the graph of a given function H

H denotes Higher Tier GCSE content

R denotes 'review step' – content should have been covered at KS3

Line symmetry and reflection

R

Notes and guidance

Using paper-folding activities or mirrors can help to unpick common mistakes regarding rectangles, parallelograms and reflecting in a diagonal line. A good strategy when reflecting in a diagonal line, is to reflect each vertex before joining up the edges. This step provides a good opportunity to revisit equations of a straight line. Students need to check scales on axes before giving the equation of the mirror line.

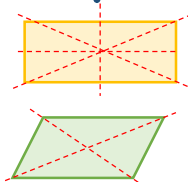
Key vocabulary

Line symmetry	Reflection	Diagonal
Vertex	Side	Mirror Line

Key questions

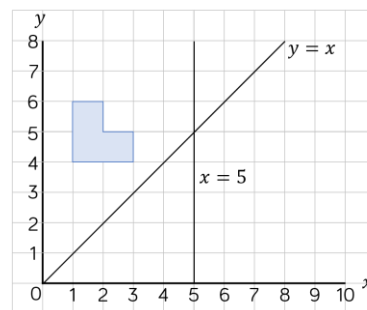
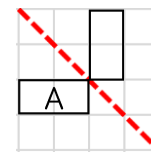
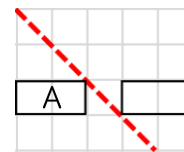
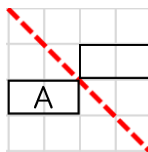
How many lines of symmetry do regular polygons have?
How can we work out where a vertex is positioned when reflecting in a diagonal line?
Are reflected images congruent to the original object?
How do we know the equation of the straight line is (e.g. $x = 5$, $y = x$ etc.)?

Exemplar Questions



Mo says, "A rectangle has 4 lines of symmetry but a parallelogram only has 2 lines of symmetry."
Show that Mo is incorrect.

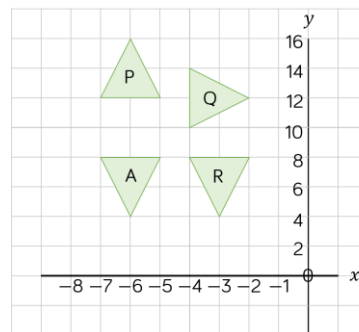
Rectangle A is reflected in the dotted line. Which of these is correct?



Reflect the shape in the lines:

- ▣ $x = 5$
- ▣ $y = x$
- ▣ $y = 3$
- ▣ $x + y = 8$

What's the same and what's different?



Object A has been reflected to form each of the images P, Q and R.

For each image, write down the equation of the line of reflection.

Rotation/Rotational symmetry R

Notes and guidance

This is an opportunity to revisit names of shapes and revise and build on the study of rotation in Year 9. It's important students can rotate a shape no matter where the centre of rotation is, so include centres inside the shape, on a side of a shape and outside of the shape. Teachers will need to model how to perform a rotation, and then how to describe a rotation. Tracing paper and dynamic geometry software packages will help students to visualise rotations.

Key vocabulary

Rotate	Clockwise	Anticlockwise
Centre	Order of rotational symmetry	

Key questions

What is the order of rotational symmetry for different regular polygons? What do you notice?
 Are rotations of an object congruent to the object?
 What do you notice about a rotation 90° clockwise compared to one that is 270° anticlockwise?
 How is tracing paper used to help with rotation?

Exemplar Questions

Use a pencil and ruler to draw a sketch of a

 Scalene triangle

 Rhombus

 Parallelogram

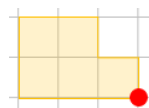
 Trapezium

 Kite

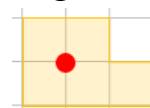
For each of your shapes, write down the order of rotational symmetry.

The red circle represents the centre of rotation for each question.

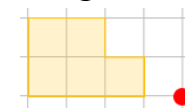
Rotate each object by the angle and direction of turn given.



180°



90° clockwise



90° anti-clockwise

A trapezium ABCD has vertices A (3, 3), B (3, 5), C (6, 3) and D (5, 5)

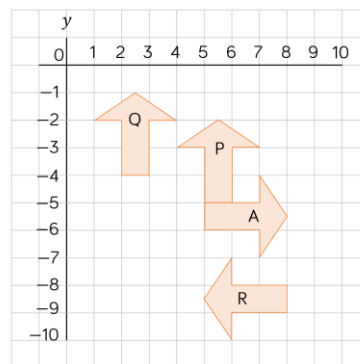
Draw trapezium ABCD on x - and y -axes from 0 to 10

Rotate trapezium ABCD:

 90° clockwise, centre (3, 3)

 180° , centre (4, 4)

 90° anticlockwise, centre (4, 6)



Object A has been rotated to form each of images P, Q and R.

For each image, describe the rotation fully.

Translation of shapes

R

Notes and guidance

When translating shapes, a common misconception is that the vector of translation is found by counting the squares/measuring the distance between shapes rather than the number of squares/distance between corresponding vertices. Students can also mix up which direction the numbers in the vector represent. Students could also revisit adding vectors here in the context of repeated translation.

Key vocabulary

Translation

Vector

Axes

Scale

Congruent

Vertex

Key questions

After a translation, is the image congruent to the object?

Why do we measure from one vertex on the object to the corresponding vertex on the image?

How do we know which direction to translate the object in?

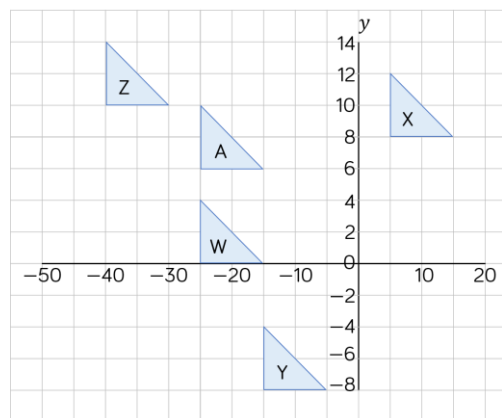
Why is it important to consider the scales of axes when giving a vector of translation?

Exemplar Questions

Which of the statements are correct? What mistakes have been made?



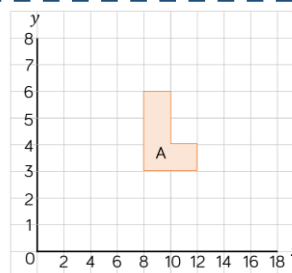
- Shape A is translated by vector $\begin{pmatrix} 3 \\ 0 \end{pmatrix}$ to image B.
- Shape A is translated by vector $\begin{pmatrix} 5 \\ 0 \end{pmatrix}$ to image B.
- Shape A is translated by vector $\begin{pmatrix} 0 \\ 5 \end{pmatrix}$ to image B.
- Shape B is translated by vector $\begin{pmatrix} -5 \\ 0 \end{pmatrix}$ to image A.



What do you notice about the axes?

Triangle A has been translated to images W, X, Y and Z.

Describe each translation using a column vector.



Draw a translation of shape A by each vector.

 $\begin{pmatrix} -4 \\ 0 \end{pmatrix}$
 $\begin{pmatrix} 6 \\ 2 \end{pmatrix}$
 $\begin{pmatrix} -8 \\ -3 \end{pmatrix}$
 $\begin{pmatrix} 5 \\ -1 \end{pmatrix}$

Enlargements of shapes

R

Notes and guidance

Students start by identifying an enlargement and its scale factor and revise the term ‘similar’. Students then consider using centres of enlargements inside, on and outside of a given shape, before moving onto using axes. The ‘counting squares from the centre method’ is usually more reliable than drawing rays, although both methods can be explored. Students then revise how to fully describe an enlargement and how to perform enlargements.

Key vocabulary

Enlargement	Scale Factor	Multiplier
Similar	Centre of enlargement	Ray

Key questions

If an object is enlarged, does it always get bigger?

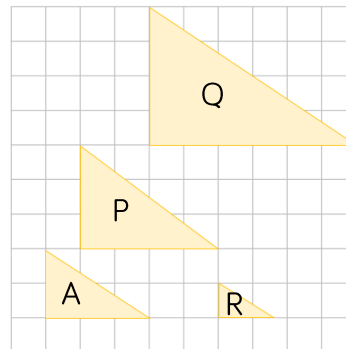
How can you work out the scale factor?

What makes two shapes mathematically similar?

How do we know where each vertex of the image should go?

How do we identify the centre of enlargement?

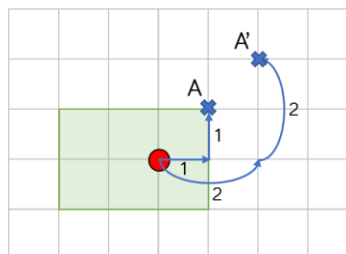
Exemplar Questions



Which triangles are an enlargement of triangle A? How do you know?

What is the scale factor or enlargement for each?

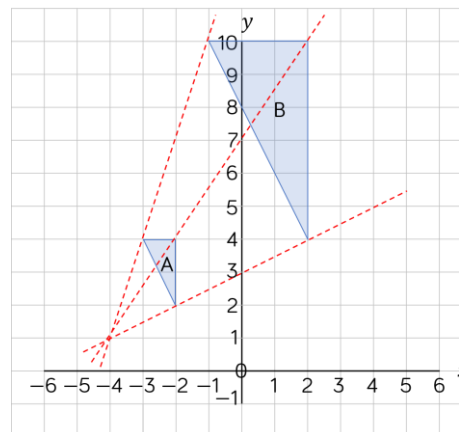
Which triangles are similar to triangle A? Explain how you know.



Huda enlarges the rectangle by scale factor 2, using the marked centre of enlargement.

The first step is shown.

Explain Huda’s method and complete the enlargement.



Mo says, “Triangle B is an enlargement of triangle A, scale factor 3”

- Which other information has Mo forgotten to give?
- Enlarge triangle A by scale factor 2, centre of enlargement (0, 1)
- Enlarge triangle A by scale factor 1.5, centre of enlargement (−2, 2)

Negative enlargements

R

H

Notes and guidance

Reinforce the impact of a negative scale factor, by giving lots of visual examples starting with -1 , so that students make the link with rotation by 180° . Some students have the misconception that a negative scale factor reduces the dimensions of the shape. To perform an enlargement using a negative scale factor, the ‘counting squares from the centre method’ again is useful. Students may need to be reminded to count from the centre to each vertex.

Key vocabulary

Negative

Direction

Rotation

Centre

Scale Factor

Multiplier

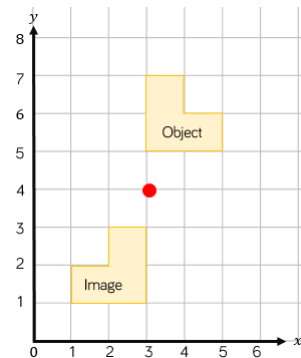
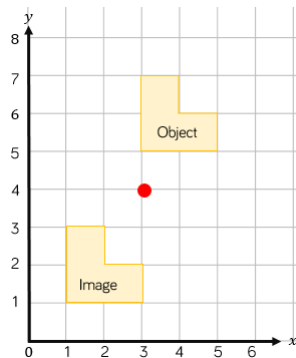
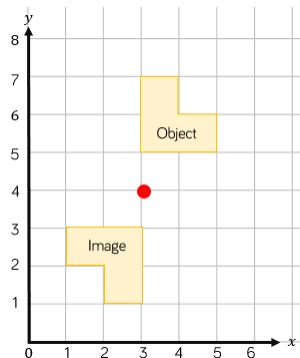
Key questions

Is enlarging a shape by scale factor -1 the same as rotating it by 180° ? Explain your answer.

Does a negative scale factor always reduce the dimensions of a shape?

What’s the same and what’s different about enlarging a shape by the scale factors 2 and -2 ?

Exemplar Questions



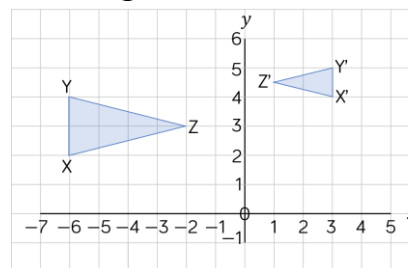
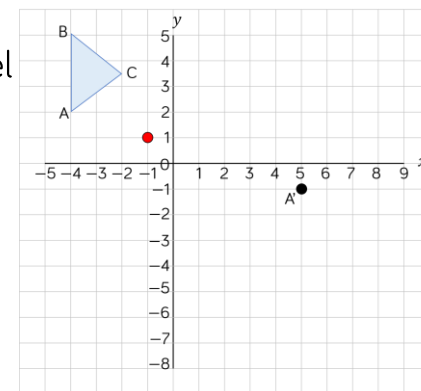
Which diagram shows an enlargement of the object by scale factor -1 using the centre of enlargement marked? Explain your answer.

Teddy says, “Enlarging a shape by scale factor -2 will make it half the size.”

Is Teddy correct? Explain why or why not.

Complete the enlargement of triangle ABC, scale factor -2 , centre $(-1, 1)$. Label each vertex of the image A', B' and C'.

Describe the transformation of triangle XYZ onto triangle X'Y'Z'.



Identify transformations

R

Exemplar Questions

Notes and guidance

Students sometimes describe a series of transformations, rather than giving a single transformation, and sometimes use language that is non-mathematical (e.g. flip, mirror). Therefore, it's important to promote preciseness in use of language as well as identifying the single transformation. Students also need to describe the transformation fully e.g. including the centre for rotation and enlargement. Tracing paper is very useful in this small step.

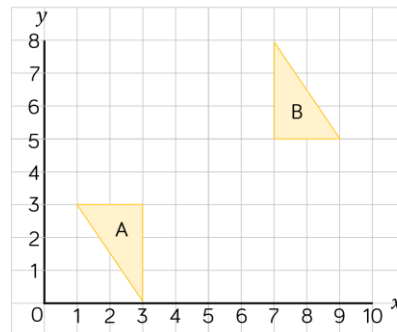
Key vocabulary

Single	Equation of a line	Translation
Enlargement	Reflection	Rotation

Key questions

- Give the names of the four types of transformation.
- How much information is needed to describe each one?
- Can any/all rotations be described as a series of reflections?
- What is meant by 'single transformation'?

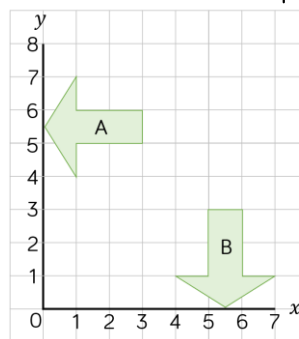
Describe fully the single transformation that maps triangle A onto triangle B.



Annie says, "Reflect triangle A in the line $y = 4$ and then reflect this triangle in the line $x = 5$ to get triangle B".

Does Annie's description map triangle A onto triangle B?
Why is Annie's answer incorrect?
Give the correct answer.

Here are some different descriptions students gave for the single transformation that maps shape A onto shape B.



- Flip arrow A to get arrow B
- Turn arrow A by 90° clockwise
- Move arrow A up by 3 and then do a quarter of a turn clockwise and then move it along by 3

Why is each answer incorrect?

Find a single reflection that maps shape A onto shape B.

💡 Can you find a single rotation that maps shape A onto shape B?

Series of transformations

Notes and guidance

Here students perform a series of transformations on an object. They should be encouraged to draw each stage of the series of transformations, rather than trying to do it all in one step. Students will not be asked to describe a series of transformations, and should be reminded about the importance of identifying single transformations. This provides the opportunity to practise the earlier steps.

Key vocabulary

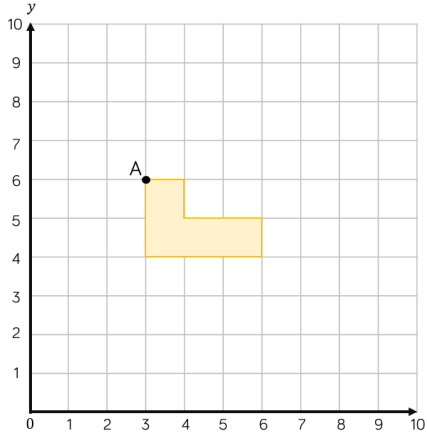
Single	Series	Translation
Enlargement	Reflection	Rotation

Key questions

Does the order a series of transformations are performed in always, sometimes or never make a difference?

Can a series of transformations ever be reduced to just a single transformation?

Exemplar Questions



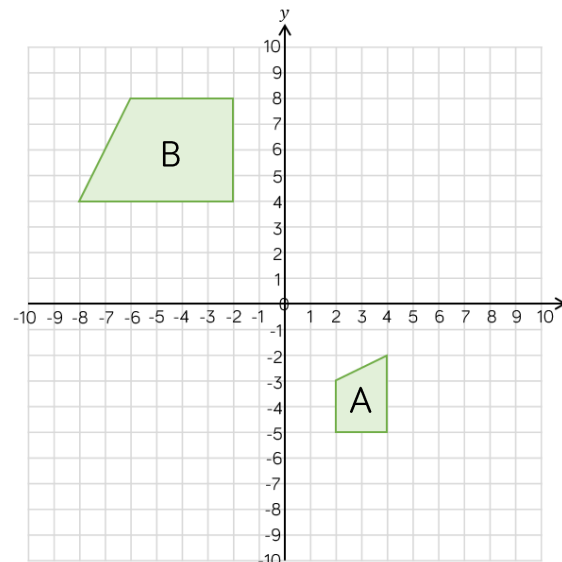
The shape undergoes a series of transformations

▣ a rotation 90° anticlockwise, centre $(6, 7)$

followed by

▣ a translation by the vector $\begin{pmatrix} -5 \\ -4 \end{pmatrix}$

Show that vertex A is mapped to $(2, 0)$



Shape A undergoes a series of transformations.

▣ a reflection in the line $x + y = 4$

▣ an enlargement by scale factor 2 about centre $(10, -4)$

Describe the final transformation in the series needed to map shape A onto shape B.

Invariant points and lines

H

Notes and guidance

This is an opportunity for students to continue to practise transformations, and could be extended to exploring a series of transformations. Students both identify invariant points given a transformation, and also give a transformation so that a specific point or points are invariant. Finding the equation of a straight line is revisited here. Students could explore shapes and transformations where all points on a shape are invariant.

Key vocabulary

Invariant	Transformation	Reflection
Rotation	Translation	Enlargement

Key questions

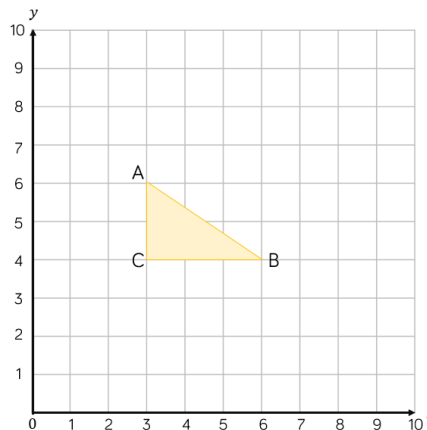
Is it possible for any point to be invariant after a translation?

Is it possible for all points on a shape to be invariant after a rotation?

What do you notice about the centre of enlargement and invariant points?

How do you find the equation of a line, given two points on the line?

Exemplar Questions



Reflect triangle ABC in the line $x = 6$. What do you notice when vertex B is mapped to the image?

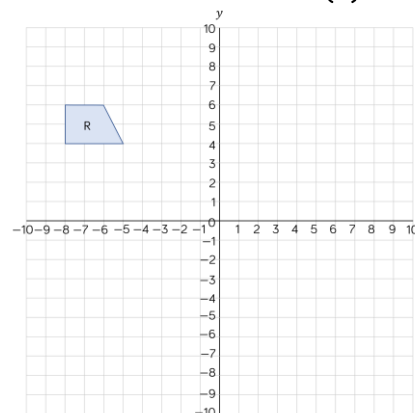
Complete the sentence:
C is an invariant point when triangle ABC is rotated 90° clockwise, centre (__, __)

Write down a transformation so that both

- ▣ B and C are invariant points.
- 💡 ▣ A and B are invariant points.

Shape R is transformed so that exactly one point is invariant.

Which transformation(s) could have been used?



- ▣ Reflection in the line $y = 4$
- ▣ Translation $\begin{pmatrix} 3 \\ 0 \end{pmatrix}$
- ▣ Enlargement, SF -1 , centre $(-5, 4)$
- ▣ Enlargement SF 2, centre $(-10, 8)$
- ▣ Rotation, 180° , centre $(-7, 5)$

Find some other transformations so that exactly one point on R is invariant.

Find the coordinates of the invariant points when shape R is rotated 180° , centre $(-2, 1)$ and then reflected in the line $y = 2$ and then translated by the vector $\begin{pmatrix} 0 \\ 6 \end{pmatrix}$.

Constructions

R

Notes and guidance

Practising general skills with a pair of compasses might be a necessary starting point for some students before moving on to revising the standard constructions. This is a good opportunity to interleave the properties of quadrilaterals and their diagonals. Students can also sketch triangles, construct perpendicular heights, from the opposite vertex to the base, to revise calculating the area of the triangle. Midpoints of line segments is also another topic that can be revised here.

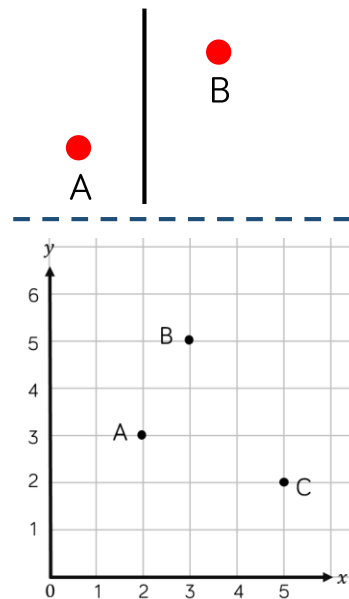
Key vocabulary

Construct	Angle Bisector	Locus
Equidistant	Perpendicular Bisector	

Key questions

What do we know about all points on the perpendicular bisector in relation to A and B?
 How can I construct a 60° angle? 30° angle? 45° angle?
 What's the same and what's different about drawing a perpendicular to a point and a perpendicular from a point?
 What is the point is at the end of the line?

Exemplar Questions



Annie has sketched the perpendicular bisector of line segment AB. Why is this incorrect? Construct an accurate perpendicular bisector of AB.

Dora works out that the midpoint of line segment AB is $(\frac{2+5}{2}, \frac{3+2}{2})$ which is (3.5, 2.5)

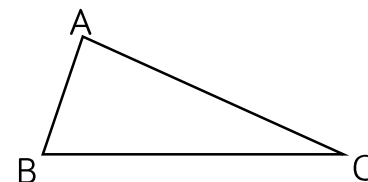
Give the coordinates of a point which is on the perpendicular bisectors of line segment AB and BC.

Check your answers by constructing the perpendicular bisectors.

Draw a triangle ABC like the one shown.

Construct the perpendicular bisector of BC from point A.

How is this different from constructing the perpendicular bisector of line segment BC?



Construct a triangle with side lengths 7 cm, 8 cm and 9 cm.
 Construct the angle bisector of each side.
 What do you notice about where these bisectors intersect?

Solve loci problems

Notes and guidance

You may wish to review the locus of points equidistant from a single point, a straight line, two intersecting lines, two points and between two parallel lines, linking to the last step, before considering loci problems. You could also interleave revision of bearings and scale drawings. Students should then be encouraged to perform constructions, leaving in all construction marks. They can then reflect on the question posed, for example, shading in the relevant area.

Key vocabulary

Locus/Loci

Equidistant

Circle

Perpendicular Bisector

Angle Bisector

Key questions

What does equidistant mean?

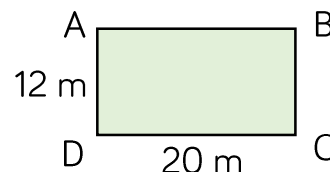
How can I show the area that is at least a certain distance away from a point, two points, two intersecting lines, two parallel lines, a straight line? How do I ensure accuracy when using a pair of compasses?

How can we use scale to work out actual distances?

Exemplar Questions

A rectangular garden is 20 m long and 12 m wide.

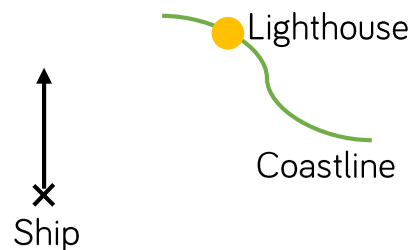
Use a scale of 1 cm : 2 m to make a scale drawing of the garden.



Dani wants to put in a water feature. It needs to be

- at least 8 m away from A
- closer to DC than BC
- closer to DC than AB

On your diagram, shade the region where Dani can add a water feature.



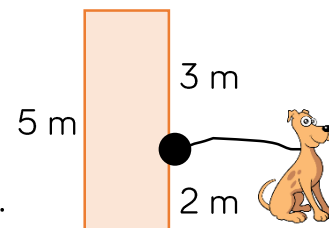
A ship is sailing on a bearing of 075° towards the coastline.

It must not travel within 60 m of the lighthouse otherwise it will be in danger.

Will the ship be in danger?

Foggy the dog is tied to a post outside a shed. His lead is 4 m long.

- Make a scale drawing of the shed and lead (1 cm : 0.5 m)
- Shade in the area of grass Foggy can reach.



Trigonometrical graphs

H

Notes and guidance

Dynamic software showing the unit circle and the construction of trigonometric graphs might be a useful starting point. Giving time to explore the graphs, considering the key features of each and then differences and similarities helps understanding. Realising that the graphs are periodic and repeat an infinite number of times is also key. The main focus here is using the symmetry of the graphs to work out different values of x which give the same value or y .

Key vocabulary

Sin/Cos/Tan

Asymptote

Period

Symmetry

Repeating

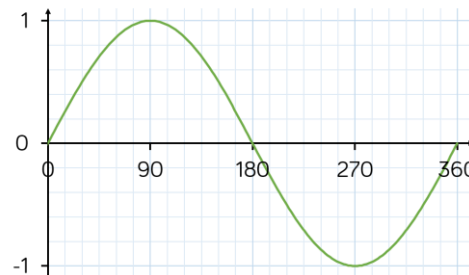
Key questions

Why can angles be greater than 360° ? Is there a limit for the size of a measure of turn?

Why is the maximum and minimum value of y on the sine and cosine graph 1?

Why is there a repeating pattern? How regularly does this pattern repeat itself?

Exemplar Questions



This is the graph of $y = \sin x$

Calculate y when x is equal to

30° and 150°

225° and 315°

Show your solutions on the graph.

What do you notice?

Use this graph to complete the following.

$\sin 60 = \sin \square$

$\sin 0 = \sin \square$

$\sin \square = \sin \square$

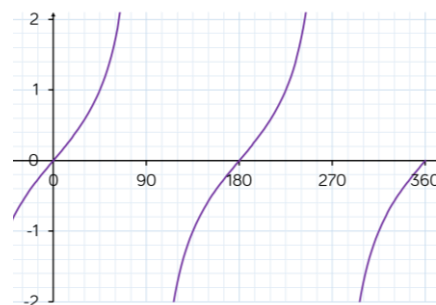
Complete the table for $y = \cos x$

x	0	30	60	90	120	150	180	210	240	270	300	330	360
y													

Use your table to draw the graph of $y = \cos x$ for $0^\circ \leq x \leq 360^\circ$

Explore what happens when $x \leq 0$ and $x \geq 360^\circ$

What's the same and what's different about the graphs of $y = \sin x$ and $y = \cos x$?



This is the graph of $y = \tan x$.

What happens when $x = 90^\circ$? Why?

Draw a right-angled triangle to help you explain your answer.

Find three values of x that satisfy the conditions $x > 360^\circ$ and $\tan x = 0$

Translations of the graph

H

Notes and guidance

Dynamic geometry software is very useful here, allowing students to explore translations and to make generalisations. Translations horizontally and vertically should be considered separately. A common misconception is that $y = f(x)$ is translated $\begin{pmatrix} a \\ 0 \end{pmatrix}$ onto $y = f(x + a)$. To avoid this, ensure that students can explain why this is incorrect by comparing coordinates on each graph. Trigonometric graphs should be included in this step.

Key vocabulary

Mapping

Translation

Horizontal

Vertical

Vector

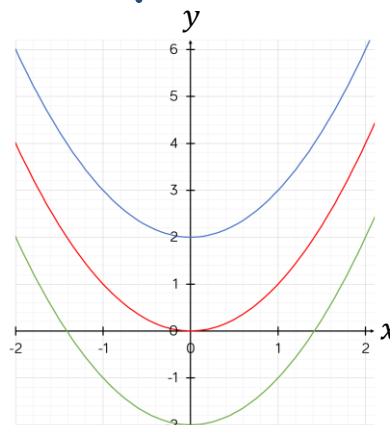
Key questions

How do we know, by considering the equation, which direction the translation is in?

How can we tell from a vector if a translation is horizontal or vertical?

Why is $y = f(x)$ mapped onto $y = f(x + a)$ by $\begin{pmatrix} -a \\ 0 \end{pmatrix}$ and not $\begin{pmatrix} a \\ 0 \end{pmatrix}$?

Exemplar Questions



Match the label to each graph.

$$y = x^2$$

$$y = x^2 + 2$$

$$y = x^2 - 2$$

Explain how you worked each out.

Sketch the graph of

$$y = x^2 + 3$$

$$y = x^2 - 3$$

Describe the transformation that maps $y = x^2$ to each of these graphs.

Mo says, “ $y = f(x)$ maps to $y = f(x) + a$ by translation $\begin{pmatrix} 0 \\ a \end{pmatrix}$ ”

Do you agree with Mo? Explain your answer.

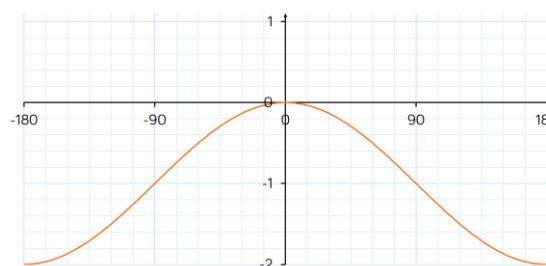
Eva and Huda are describing the translation that maps $y = x^2$ onto $y = (x + 1)^2$.

Eva says, “Use vector $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ ”

Huda says, “Use vector $\begin{pmatrix} -1 \\ 0 \end{pmatrix}$ ”

Who is correct? Explain why.

Can you generalise this for a mapping of $y = f(x)$ to $y = f(x + a)$?



This is the graph of $y = \sin(x + a) + b$ where a and b are integers. Eva says, “ a could be 90° ”. Eva is correct. Show why. Now work out the value of b .

Reflections of the graph

H

Notes and guidance

It's important that students understand why the graph reflects in either the x - or the y -axis rather than trying to remember a series of rules. Dynamic geometry software helps students to explore relationships between graphs. Students should be able to both sketch a graph mapped by the transformation, as well as describe the transformation. Exploring functions which are symmetrical in the x -axis or y -axis is also useful. You could also revisit completing the square.

Key vocabulary

Mapping

Reflection

 y -axis

 x -axis

Invariant

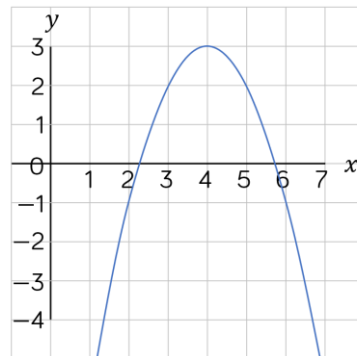
Key questions

Why is the relationship between $y = f(x)$ and $y = -f(x)$ a reflection in the x -axis?

Why is the relationship between $y = f(x)$ and $y = f(-x)$ a reflection in the y -axis?

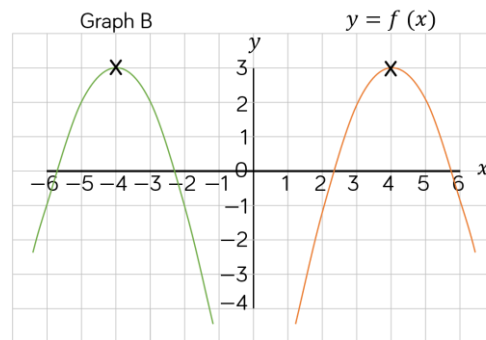
Give an equation for $y = f(x)$ that will remain the same when mapped to $y = f(-x)$.

Exemplar Questions



The graph has equation $y = f(x)$

- What happens to the point $(4, 3)$ if we change the equation to $y = -f(x)$?
- What happens to the other points on the graph?
- Sketch the graph $y = -f(x)$
- Describe the transformation that maps $y = f(x)$ onto $y = -f(x)$.



What's the relationship between $y = f(x)$ and the Graph B? What's the same and what's different about the coordinate of the marked point on each graph?

Sketch the graphs of $y = \cos x$ and $y = \sin x$, on separate axes, between $-180^\circ \leq x \leq 180^\circ$

Nijah thinks that the graphs of $y = \cos(-x)$ will look the same as the graph of $y = \cos x$.

She also thinks this means that the graph of $y = \sin(-x)$ will look the same as the graph of $y = \sin x$.

Explore Nijah's claims. Is she correct?

Explain your answer using your sketches.