Place Value

Year 7

#MathsEveryoneCan

2019-20

White Rose Maths

WRM – Year 7 Scheme of Learning

	Week 1	Week 2	Week 3	Week 4	Week 5	Week 6	Week 7	Week 8	Week 9	Week 10	Week 11	Week 12
Autumn	**Algebraic Thinking**						**Place Value and Proportion**					
Autumn	Sequences		Understand and use algebraic notation		Equality and equivalence		Place value and ordering integers and decimals			Fraction, decimal and percentage equivalence		
Spring	**Applications of Number**						**Directed Number**			**Fractional Thinking**		
Spring	Solving problems with addition & subtraction		Solving problems with multiplication and division			Fractions & percentages of amounts	Operations and equations with directed number			Addition and subtraction of fractions		
Summer	**Lines and Angles**						**Reasoning with Number**					
Summer	Constructing, measuring and using geometric notation		Developing geometric reasoning				Developing number sense		Sets and probability		Prime numbers and proof	

Autumn 2: Place Value and Proportion

Weeks 1 to 3: Place Value and Ordering	Weeks 4 to 6: Fraction, Decimal and Percentage Equivalence
In this unit, students will explore integers up to one billion and decimals to hundredths, adapting these choices where appropriate for your groups e.g. standard index form could additionally be introduced to student following the Higher strand. Using and understanding number lines is a key strategy explored in depth, and will be useful for later work on scales for axes. When putting numbers in order, this is a suitable point to introduce both the median and the range, separating them from other measures to avoid getting them mixed up. Rounding to the nearest given positive power of ten is developed, alongside rounding to one significant figure. Decimal places will come later, again to avoid too similar concepts being covered at the same time. Topics from last term such as sequences and equations, will be interleaved into this unit.	Building on the recent work on decimals, the key focus for this three weeks is for students to gain a deep understanding of the links between fractions, decimals and percentages so that they can convert fluently between those most commonly seen in real-life. The Foundation strand will focus will be on multiples of one tenth and one quarter whilst the Higher strand will look at more complex conversions. Whilst looking at percentage is, pie charts will be introduced. In addition, various forms of representation of any fraction will be studied, focusing on equivalence, in an appropriate depth to the current attainment of students; this will be revisited later in the year. The focus is very much on a secure understanding of the most common fractions under one, but fractions above one will be touched upon, particularly in the Higher strand.
National curriculum content covered: • Consolidate their understanding of the number system and place value to include decimals • understand and use place value for decimals, measures and integers of any size • order positive and negative integers, decimals and fractions; use the number line as a model for ordering of the real numbers; use the symbols =, ≠, , ≤, ≥ • work interchangeably with terminating decimals and their corresponding fractions • round numbers to an appropriate degree of accuracy • describe, interpret and compare observed distributions of a single variable through: the median and the range • interpret and compare numbers in standard form	National curriculum content covered: • consolidate their understanding of the number system and place value to include decimals, fractions • move freely between different numerical representations [for example, equivalent fractions, fractions and decimals] • extend their understanding of the number system; make connections between number relationships • express one quantity as a fraction of another, where the fraction is less than 1 and greater than 1 • define percentage as 'number of parts per hundred', interpret percentages as a fraction or a decimal • compare two quantities using percentages • work with percentages greater than 100% • interpret pie charts

Why Small Steps?

We know that breaking the curriculum down into small manageable steps should help students to understand concepts better. Too often, we have noticed that teachers will try and cover too many concepts at once and this can lead to cognitive overload. We believe it is better to follow a "small steps" approach.

As a result, for each block of content in the scheme of learning we will provide a "small step" breakdown. **It is not the intention that each small step should last a lesson – some will be a short step within a lesson, some will take longer than a lesson.** We would encourage teachers to spend the appropriate amount of time on each step for their group, and to teach some of the steps alongside each other if necessary.

What We Provide

- Some **brief guidance** notes to help identify key teaching and learning points
- A list of **key vocabulary** that we would expect teachers to draw to students' attention when teaching the small step,
- A series of **key questions** to incorporate in lessons to aid mathematical thinking.
- A set of questions to help **exemplify** the small step concept that needs to be focussed on.

- These include reasoning and problem-solving questions that are fully integrated into the scheme of learning. Depending on the attainment of your students, you many wish to use some or all of these exemplars, which are in approximate order of difficulty. Particularly challenging questions are indicated with the symbol 💡.
- For each block, we also provide ideas for key representations that will be useful for all students.

In many of the blocks of material, some of the small steps are in **bold**. These are content aimed at higher attaining students, but we would encourage teachers to use these with as many students as possible – if you feel your class can access any particular small step, then please include it in your planning.

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Key Representations

1000   100   10   1

Billions			Millions			Thousands			Ones		
H	T	O	H	T	O	H	T	O	H	T	O
		3	1	4	8	0	3	3	0	2	9

$\frac{1}{10}$ 0.1   0.001   1,000,000   100,000

$\frac{1}{1000}$ 10   1

$\frac{1}{100}$ 100

10,000   0.01   1,000

62

6 tens   2 ones

0.132

0.1   0.03   0.002
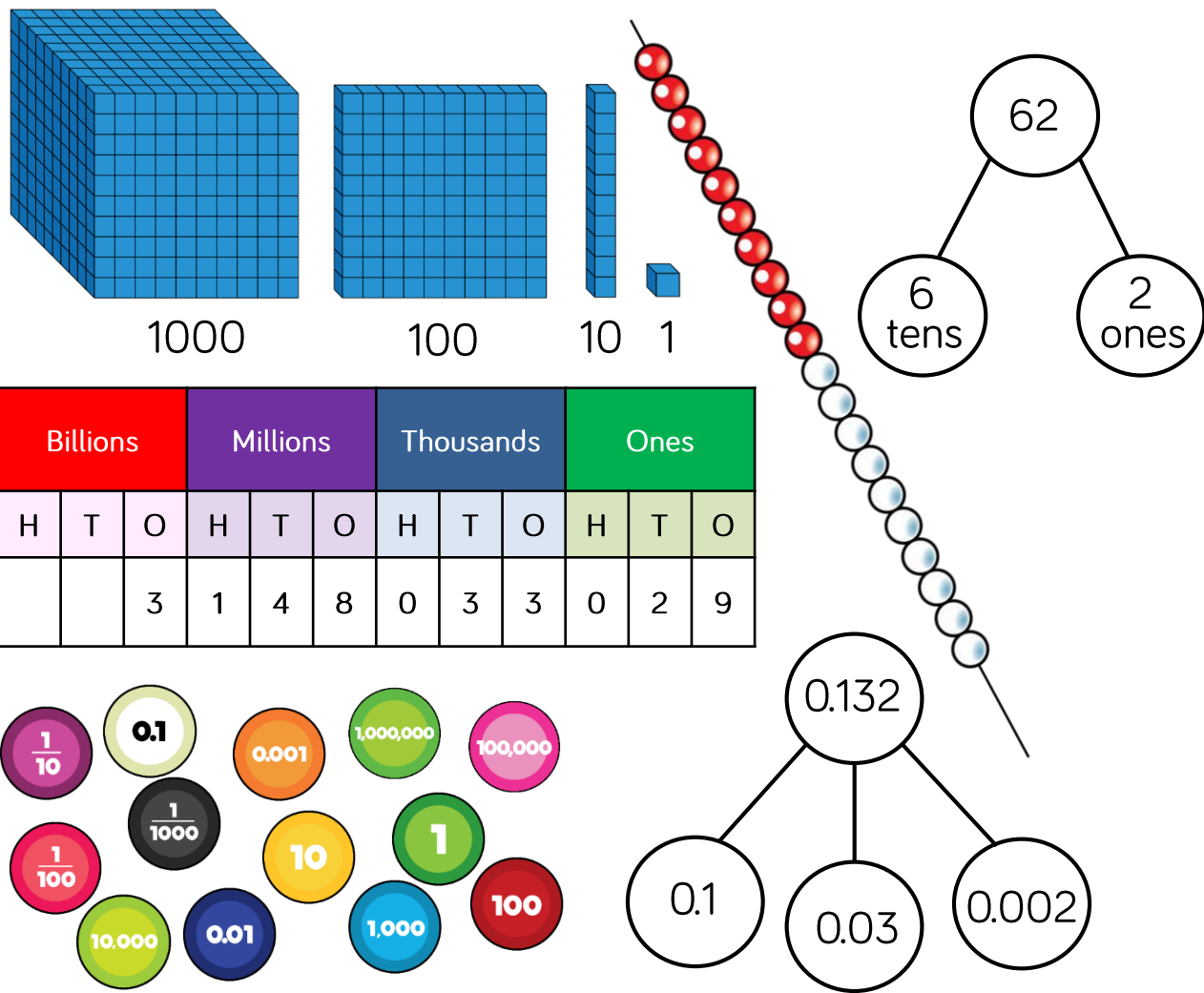
Concrete, pictorial and abstract representations are an important part of developing students' conceptual understanding.

Here are a few ideas for how you might represent place value.

Base 10 equipment is beneficial to students who need to get a sense of the size of numbers. Reassignment could also be used to support work with decimals, for example a one becomes 0.1, etc. The same could be done with a bead string. What if each bead represents 0.1? 0.01? However, care must be taken with this approach and careful explanation is needed.

Place Value

Small Steps

- Recognise the place value of any number in an integer up to one billion

- Understand and write integers up to one billion in words and figures

- Work out intervals on a number line

- Position integers on a number line

- Round integers to the nearest power of ten

- Compare two numbers using $=, \neq, <, >, \leq, \geq$

- Order a list of integers

- Find the range of a set of numbers

- Find the median of a set of numbers

- Understand place value for decimals

- Position decimals on a number line

- Compare and order any number up to one billion

Place Value

Small Steps

- Round a number to 1 significant figure
- Write 10, 100, 1000 etc. as powers of ten (H)
- Write positive integers in the form $A \times 10^n$ (H)
- Investigate negative powers of ten (H)
- Write decimals in the form $A \times 10^n$ (H)

(H) denotes higher strand and not necessarily content for Higher Tier GCSE

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Recognising integer place value

Notes and guidance

Students have met numbers up to ten million at KS2. This small step revises and extends this knowledge. Students should write and represent the numbers in several ways and need to see a mixture of smaller and larger integers. If appropriate, you could discuss the meaning of trillion etc. This step may well be taught alongside the next step.

Key vocabulary

Place value	Digit	Billion
Placeholder	Integer	

Key questions

Why do we need placeholders?

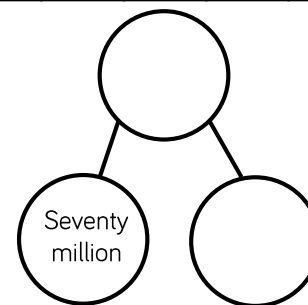What strategies can you use to work out the value of a digit in a very long integer?

Exemplar Questions

Complete these representations so they all show the same number.

Billions			Millions			Thousands			Ones		
H	T	O	H	T	O	H	T	O	H	T	O

1 073 080 529

One billion, …….

Seventy million

State the value of the 5 in each of these numbers.

650	6500	560 000	60 500
65 000	56	6 005 000	56 000 000
65 000 000	665 066 600		

Write down any number that has:

- Three in the millions place and five in the thousands place
- Three in the ten millions place and five in the hundred thousands place
- Three in the hundreds place and five in the ten millions place

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Understand and write integers

Notes and guidance

Following on from the last step, students should become fluent in converting integers from numeral form to words and vice-versa. They should also be comfortable in dealing with other representations. Populations and government finances provide good contexts for real numbers. Comma notation should be taught alongside the more common spacing between every three digits.

Key vocabulary

Place value	Digit	Billion
Placeholder	Integer	

Key questions

Why do we use spaces or commas in large integers?

Where do we put them?

Exemplar Questions

Write in figures.
- Thirty-five thousand million
- One and a half billion
- Two hundred and three thousand, five hundred and twelve
- Eighty-eight million, eighty-eight thousand
- Half a million
- One billion, ten thousand and one

- -

Write the numbers represented below in words.

Billions			Millions			Thousands			Ones		
H	T	O	H	T	O	H	T	O	H	T	O

72 007 270

1 402 140 206

Eight million

Thirty thousand five hundred

- -

Write down the numbers that are:
- Three million more than 917 000 000
- The sum of three hundred million and 700 000 000
- 30 000 000 more than nine hundred and sixty million
- The difference between one billion and seventy-five million

Work out intervals on a number line

Notes and guidance

This key skill will also be useful with later work on fractions and reading/scaling graphs. Students should be taught to work out the intervals given the number of spaces on a line and to fill in missing values. Although the focus should be on the most common values such as 5 and 10, it is worth exploring other values. Using other scales that use number lines will also be useful.

Key vocabulary

Equal division	Interval	Scale
Gap	Spaces	

Key questions

Why do we count the number of spaces rather than the number of marks on a number line?
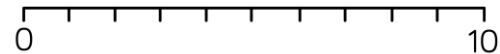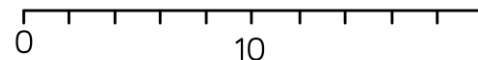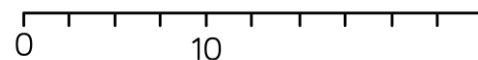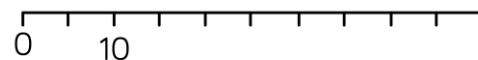
Which are the most important points to label on a number line or other scale? Why?

Exemplar Questions

Work out the value of each of the intervals in these number lines.
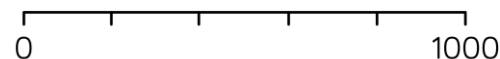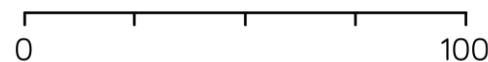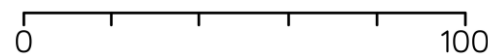
Fully label these number lines.

Repeat for lines where 10 is replaced by 20, 100 and 1000

What is each interval worth on these scales?

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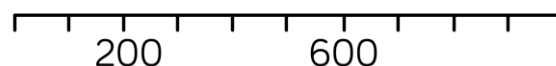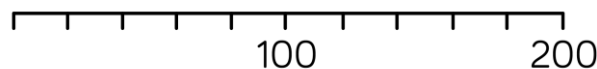
Position integers on a number line

Notes and guidance

Once familiar with how intervals on number lines work, students can start to use these to place integers and to read values. Making links to reading from common scales such as weighing scales, measuring jugs and thermometers will be helpful.

Key vocabulary

Equal division	Interval	Scale
Gap	Spaces	Approximate

Key questions

Why can we mark some numbers exactly on a number line but others only approximately?

Describe the steps you need to take to read a number off a line of a scale.

Exemplar Questions

Where would 80 be on each of these number lines?
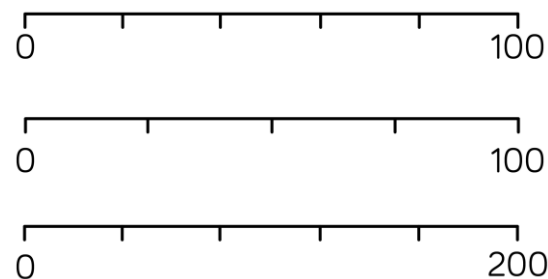
0 —————————— 100

0 —————————— 100

0 —————————— 200

- -

Estimate where the arrow is pointing to in each of these number lines.

0 —————————— 1000

0 —————————— 200

0 —————— 50 —————

- -
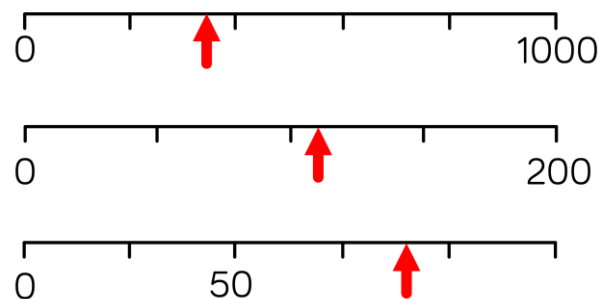
Mark the approximate positions of 3500, 6100 and 1785 on this number line. Explain which ones are easier to do and why.

0 —————————— 10000

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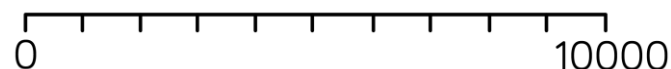
Round integers to powers of ten

Notes and guidance

Using appropriate number lines to support, and building on previous knowledge, from Key Stage 2, students should now be able to round to the nearest 10, 100, 1000 etc. Emphasis should be placed on "nearest" meaning proximity, encouraging students to think about the size of the number rather than rote-learned rules. "Rounding up" for halfway should be explained as a convention.

Key vocabulary

Round	Approximate	Nearest
Convention	Halfway	

Key questions

Why do we round numbers?

When talking about the population of the UK, would you round to the nearest hundred, thousand or million? What about the population of Leeds?

Exemplar Questions

Use your calculator to find the answers to these calculations. Round your answers to the nearest hundred.

- 47×68
- $65\ 681 \div 77$
- $97\ 813 - 88\ 694$

Which of these numbers would be sensible to round to the nearest 10? What would be the most sensible choice for rounding the other numbers?

9761	145	48 312	603 156
287	48	19 201	671
5.9	797	23.5	1542

Jim says "There are 2000 students at my school". Do you think there are exactly 2000 students? How many to you think there might be?

To the nearest thousand, 84 000 people attend a pop concert. What's the greatest possible number of people that were at the concert? What is the least possible number?

Compare integers using $=, \neq, <, >$

Notes and guidance

Students need to compare two integers confidently before they can go to order a larger list of numbers. Students will be familiar with the equals sign but may need introducing to $\neq$. Encourage the use of "greater than" and "less than" rather than "bigger than"/"smaller than" etc. and pay attention to reading statements like "829 < 850" both from left to right and from right to left.

Key vocabulary

Compare	Digit	Equal
Not equal	Greater than	Less than

Key questions

What do you look at first when comparing the size of two integers? What do you look at next?

Is it true that if $a > b$ and $b > c$ then $a > c$?

Exemplar Questions

Complete the following using $=$ or $\neq$.

Two and a half million $\bigcirc$ 2 500 000

300 000 000 $\bigcirc$ Three billion

Six thousand and eighty $\bigcirc$ 68 000

- -

86 < 101 and 101 > 86 are both **true**.
Decide which statements below are true and which are false. Rewrite the false statements, using the same numbers, making them true. Can you do this in more than one way?

902 < 93	8106 > 8099	3751 < 3699
203 000 < 199 987	32 150 = 31 205	809 > 820
601 × 1000 > 10 000 × 59		903 000 ÷ 100 > 88 000

- -

$$46\ 000 < 400\ 010$$

Joe says the statement says "46 000 is less than 400 010".
Jay says the statement says "400 010 is greater than 46 000".
Who do you agree with?

Order a list of integers

Notes and guidance

Students can now use their skills of identifying the values of digits in a number, supported if necessary by number lines, to put a series of integers in order. Introducing the term "leading digit" may be helpful for those who find this challenging; emphasis on the difference between a number and digit is important as students can get confused.

Key vocabulary

Order	Ascending	Descending
Place Value	Leading digit	

Key questions

Why is the leading digit of a number important when ordering?

For a set of integers, is the longest number always the largest number?

Exemplar Questions

Decide where these numbers belong on the number line and then list them in **ascending** order.

30000   40000
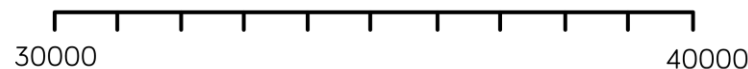
| 34 601 | 30 641 | 36 104 | 34 016 |

| 31 604 | 36 401 | 31 046 |

- -

Put your answers to the following in **descending** order.

1. $180\ 000 - 42\ 781$
2. 360×25
3. One billion divided by forty-thousand
4. The **sixth** term of the sequence $200 , 800 , 3200 \ldots$
5. The value of x^2 when $x = 305$
6. Two hundred thousand more than $610\ 408$

- -

How many different four digit numbers can you make using these cards? Put your numbers in order, starting with the smallest.

| 8 | 4 | 9 | 4 |

Find the range of a list of integers

Notes and guidance

Now students are able to confidently order integers, finding the range is straightforward. Care needs to be taken so that students remember to find the difference between the greatest and least values rather than state "they range from ___ to ___". It is worth revisiting the concept regularly in lesson starters or within other topics

Key vocabulary

Range   Greatest   Least

Difference

Key questions

How do you calculate the range of a set of numbers?

When given a list of numbers to find the range of, what might it be helpful to do first.

Exemplar Questions

What is the range of the ages:
- In your mathematics class?
- In your household?
- In your school?

The table shows the heights of the highest mountains in some of the countries in Europe.

Work out the range of these heights.

Country	Height (m)
France	4808
Belgium	694
England	978
Sweden	2104
Russia	5642
Croatia	1831

Substitue $x = 30$ into each of these expressions and find the range of your answers.

$$x^2 \qquad 25x - 80 \qquad 10\,000 - 14x \qquad \frac{180\,000}{x} \qquad 2500 + 17x$$

Gemma says the range of the numbers below is "from 63 to 111"

68   63   79   111   104

Explain why Gemma is **wrong**.

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Find the median of a list of integers

Notes and guidance

Students need to be taught how to find the median from a list with both an even amount of numbers and an odd amount of numbers. As the mean has not yet been formally introduced, it is best to focus on the latter. As a homework, students could explore the use of the median average in real-life e.g. median wage etc.

Key vocabulary

Median	Middle	Order
Average		

Key questions

What do you need to do first when finding the median of a list of numbers?

What is different about the median and the range?

Exemplar Questions

Gertrude has seven chickens. Over a week they lay the following amount of eggs: 5   4   5   4   3   2   3
Work out the median of the number of eggs laid.

- -

9   4   5   5   7   9

Eric says

As both the numbers in the middle of this list are 5, the median must be 5

Explain why Eric is **wrong**, and find the actual median of the list.

- -

Which of these sets of data is it possible to find the median of?
- The shoe sizes of students in your class
- The number of pets owned by students in your class
- The eye colour of students in your class
- The number of siblings of students in your class

- -

Explain what is the same and what is different about finding the median of these three lists of numbers.

- 3000   3500   3500   3500   3600
- 3000   3500   3500   3600
- 3000   3500   3600   3600

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Understand place value - decimals

Notes and guidance

Students following the Foundation strand should focus on proper understanding of tenths and hundredths during this step, and throughout this unit. Only move on to thousandths and beyond if appropriate for the students in your class. Conversion between fractional and decimal forms of tenths and hundredths are covered in depth in the next block.

Key vocabulary

Tenth	Hundredth	Decimal
Decimal point		

Key questions

Why do we say "0.37" as "nought point three seven" rather than "nought point thirty-seven"?

Why is 0.4 bigger than 0.29, even though twenty-nine is bigger than 4?

Exemplar Questions

Explain why these representations are all the same.
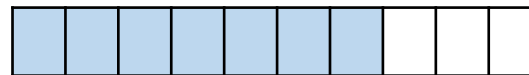
0.7 $\frac{7}{10}$

Ones	Tenths
	0.1 0.1 0.1 0.1 0.1 0.1 0.1

Write the value of the 4 in each of these numbers.

34 601   3.46   43.6   3.64   14.63

Complete these diagrams:

0.49 — (   )
   (Nine hundredths)

(   ) — 0.7
   (Three hundredths)
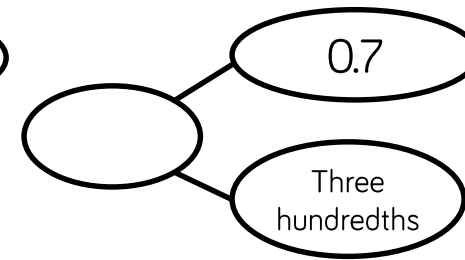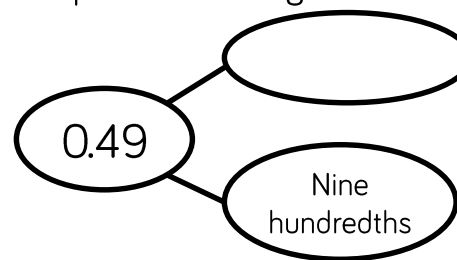
Write these numbers in figures.
- Seven thousandths
- Seventeen thousandths
- Seventy thousandths
- Seven hundred thousandths

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Position decimals on a number line

Notes and guidance

Students may need help with finding the intervals in decimal number lines, and this key skill will be revisited in the upcoming FDP work. The focus in this step is appreciating the place value of decimal numbers and how this affects their relative positioning. Challenge can be added if appropriate by looking at intervals of 0.2, 0.05 etc,

Key vocabulary

Tenth	Hundredth	Decimal
Decimal point	Interval	

Key questions

How do we work out the size of an interval on a number line?

What is different when thinking about the position of 0.3 and 0.03?

Exemplar Questions

Fully label these number lines.
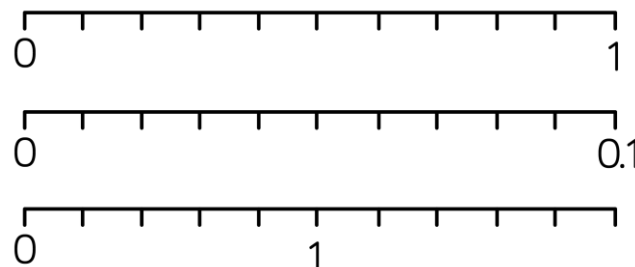
0 _____ 1

0 _____ 0.1

0 _____ 1

- -

Estimate where the arrow is pointing to in each of these number lines.

0 _____ 0.1

0   0.1 _____

0   0.04 _____

- -
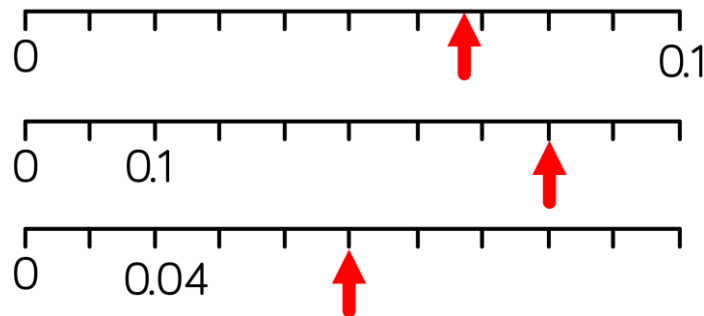
Mark the approximate positions of 7.45, 7.48 and 7.425 on this number line.

7.4 _____ 7.5

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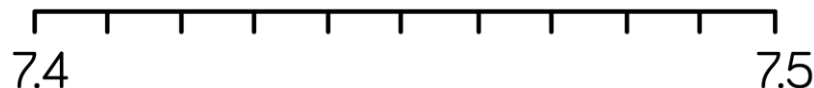
Compare and order any numbers

Notes and guidance

Students should now be able to compare decimal numbers as well as integers. It is important that students read numbers correctly e.g. "nought point three five" as opposed to "nought point thirty-five" as this leads to misconceptions such as 0.35 > 0.4. Students following the Foundation strand should focus on numbers with up to two decimal places at this stage,

Key vocabulary

Decimal	Integer	Leading digit
Ascending	Descending	

Key questions

When you see a list of decimal numbers, is the longest number always the largest number?

When ordering numbers, why are the leading digits important?

Exemplar Questions

Put these numbers in **ascending** order.

346.01	306.41	361.04	340.16

316.04	364.01	310.46

- -

Write these numbers in decimal form and then put them in order, starting with the smallest.

Zero point three five	Seventy-two hundredths

Fifty hundredths	One tenth	Two hundredths

Nought point nought seven	Nought point nought three

- -

Work out these calculations and put the answers in descending order.

1. $47 \div 100$
2. $400 \div 1000$
3. $5 - 0.93$
4. The **sixth** term of the sequence $0.59, 0.62, 0.65...$
5. The solution to the equation $x + 0.16 = 0.9$
6. $0.168 + 0.232$

Round to 1 significant figure

Notes and guidance

In this step, students learn the key skill of rounding to 1 significant figure both with integers and decimals, as this key skill in estimation is much more useful than rounding to decimal places which is covered later in the scheme. This should be revisited regularly whenever appropriate. You may wish to explore two or three significant figures with some students, but this is not essential at this stage.

Key vocabulary

Round	Approximate
Leading digit	Significant figure

Key questions

If when two numbers are rounded to one significant figure you get the same answer, does it mean the two numbers were the same?

Explain how you estimate the answer to seventeen million multiplied by two point nine six.

Exemplar Questions

Round these numbers to one significant figure:

> 37
> Thirty-seven million
> 0.37
> 0.000037
> 4.37
> 4.0037
> Four million and thirty seven

- -

Work out the value of these expressions if $a = 72, b = 0.6$ and $c = 125$. Give your answers correct to one significant figure.

- $a + b + c$

- $\dfrac{a}{b}$

- ab

- $\dfrac{b}{c}$

- c^2

- -

To one significant figure, the population of Scotland is given as five million. What is the greatest possible population of Scotland? What is the least possible population?

Investigate positive powers of 10 Ⓗ

Notes and guidance

As a precursor to writing numbers in standard index form., this small step looks at writing numbers like 10 000 in the form 10^n. A calculator could be used to introduce this, and it will also provide good practice for using terms like billion. Students following the Foundation strand could access this if time allows but it will be covered in the future if more time is needed to gain fluency with earlier steps.

Key vocabulary

Power	Index	Million
Billion		

Key questions

What does "ten to the power five" mean? How does this link to place value columns?

Why do we write very large numbers as powers of 10 rather than out in full?

Exemplar Questions

Enter the number 10 on your calculator.
Multiply by 10 and note the answer (you know it anyway!)
Multiply by 10 again and note the answer.
Keep going!
What happens after several multiplications?
Why do you think this is the case?

- -

Put these numbers in order of size, starting with the smallest:

10^5 10^7 Ten billion

One hundred million 1000^2

10^{12} 1000^3

- -

A googol is the number formed by writing 1 followed by one hundred zeros.
- Write a googol as a power of ten
- How many times bigger than a billion is a googol?

- -

Work out $10^6 \times 10^6$.
How many similar calculations can you find with the same answer?

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+ve integers in the form $A \times 10^n$ (H)

Notes and guidance

As standard index form is studied in depth in Year 8, this step focuses on writing and interpreting numbers like 8×10^9 rather than numbers that need decimals such as 7.4×10^6. The intention is to get a good understanding of the basics rather than rushing to learning procedural rules.

Key vocabulary

Power   Index   Standard Form

Scientific notation

Key questions

Why do we use standard index form?

How can you convert numbers like millions and billions easily to standard index form?

Exemplar Questions

Put all these numbers into standard form and then write them in ascending order.

$700\,000 \times 100$ Thirty billion

Twenty million 10^7 200^3

$1\,000\,000 \times 50\,000$ Half a billion

These numbers are **not** in standard form. Rewrite them so they are.

30×10^7 $10 \times 3 \times 10^5$

300×10^8 3000×10^4 $300\,000 \times 10^0$

Whitney says:

Even though 9 is greater than 7, 7×10^6 is greater than 9×10^4.

Whitney is **right.** Explain why.

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Investigate −ve powers of 10 (H)

Notes and guidance

Similarly to the earlier step on positive powers of 10. students here explore powers of 10 for numbers between zero and one. Negative numbers have been introduced during KS2 and so students should be aware that e.g. -2 is greater than -4, although this may need reinforcement.

Key vocabulary

Power	Index	Standard Form
Scientific notation	Negative	

Key questions

What's the difference between positive and negative powers of 10?

Why is 10 to the power zero not equal to zero?

Exemplar Questions

Enter the number 10 on your calculator.
Divide by 10 and note the answer (you know it anyway!)
Divide by 10 again and note the answer.
Keep going!
What happens after several divisions?
Why do you think this is the case?

- -

Put these numbers in order of size, starting with the largest:

10^3 10^{-4} One tenth

One thousandth 10^0 10

100 ÷ 10 000 10^{-7}

- -

Christina says:

5 is greater than 3, so 10^{-5} must be greater than 10^{-3}

Explain why Christina is **wrong.**

Decimals in the form $A \times 10^n$ (H)

Notes and guidance

Again the focus is on writing and interpreting numbers like 2×10^{-3} rather than numbers that need decimals such as 2.4×10^{-3}. Although this might come up in discussion and be addressed briefly, bear in mind that this knowledge and understanding will be considered in depth during Year 8.

Key vocabulary

Power   Index   Standard Form

Scientific notation   Negative

Key questions

What's different about writing large numbers and small numbers in standard index form?

Where might you see and use standard index form?

Exemplar Questions

Put all these numbers into standard form and then write them in ascending order.

$7 \div 100\,000$ Four thousandths

Three millionths 10^{-7} 0.0004

$50 \div 1\,000\,000$ Two billionths

Put these numbers in ascending order.

5×10^6 6×10^5 6×10^6

5×10^{-6} 6×10^{-5}

5×10^5 6×10^{-6} 5×10^{-5}

Explain why 3×10^{-4} is less than 4×10^{-3}.

$0.0006 = 6 \times 10^{-4}$ and $0.0007 = 7 \times 10^{-4}$.
How do you think we might write 0.00065 in standard index form?

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