Changing the subject

Year 11

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	Week 1	Week 2	Week 3	Week 4	Week 5	Week 6	Week 7	Week 8	Week 9	Week 10	Week 11	Week 12
Autumn	**Graphs**						**Algebra**					
	Gradients & lines		Non-linear graphs		Using graphs		Expanding & factorising		Changing the subject		Functions	
Spring	**Reasoning**						**Revision and Communication**					
	Multiplicative		Geometric		Algebraic		Transforming & constructing		Listing & describing		Show that...	
Summer	**Revision**						**Examinations**					

Autumn 2: Algebra

Weeks 1 and 2: Expanding and Factorising

This block reviews expanding and factorising with a single bracket before moving on to quadratics. The use of algebra tiles to develop conceptual understanding is encouraged throughout. Context questions are included to revisit e.g. area and Pythagoras' theorem.

National Curriculum content covered includes:

- know the difference between an equation and an identity; argue mathematically to show algebraic expressions are equivalent, and use algebra to support and construct arguments **{and proofs}**
- simplify and manipulate algebraic expressions by: factorising quadratic expressions of the form $x^2 + bx + c$, including the difference of two squares; **{factorising quadratic expressions of the form $ax^2 + bx + c$}**
- know the difference between an equation and an identity; solve quadratic equations **{including those that require rearrangement}** algebraically by factorising, **{by completing the square and by using the quadratic formula}**
- identify and interpret roots; deduce roots algebraically **{and turning points by completing the square}**
- solve two simultaneous equations in two variables (linear/linear **{or linear/quadratic}**) algebraically; find approximate solutions using a graph

Weeks 3 and 4: Changing the subject

Students consolidate and build on their study of changing the subject in Year 9. The block begins with a review of solving equations and inequalities before moving on to rearrangement of both familiar and unfamiliar formulae. Checking by substitution is encouraged throughout. Higher tier students also study solving equations by iteration.

National Curriculum content covered includes::

- solve linear inequalities in one variable
- know the difference between an equation and an identity; argue mathematically to show algebraic expressions are equivalent, and use algebra to support and construct arguments **{and proofs}**
- translate simple situations or procedures into algebraic expressions or formulae; derive an equation (or two simultaneous equations), solve the equation(s) and interpret the solution
- **{find approximate solutions to equations numerically using iteration}**

Weeks 5 and 6: Functions

As well as introducing formal function notation, this block brings together and builds on recent study of quadratic functions and graphs. This is also an opportunity to revisit trigonometric functions, first studied at the start of Year 10.

National Curriculum content covered includes:

- where appropriate, interpret simple expressions as functions with inputs and outputs; **{interpret the reverse process as the 'inverse function'; interpret the succession of two functions as a 'composite function'}**
- solve two simultaneous equations in two variables (linear/linear **{or linear/quadratic}**) algebraically; find approximate solutions using a graph
- identify and interpret roots; deduce roots algebraically **{and turning points by completing the square}**
- solve linear inequalities in one **{or two}** variable**{s}**, **{and quadratic inequalities in one variable}**; represent the solution set on a number line, **{using set notation and on a graph}**
- recognise, sketch and interpret graphs of quadratic functions
- apply Pythagoras' Theorem and trigonometric ratios to find angles and lengths in right-angled triangles **{and, where possible, general triangles}** in two **{and three}** dimensional figures

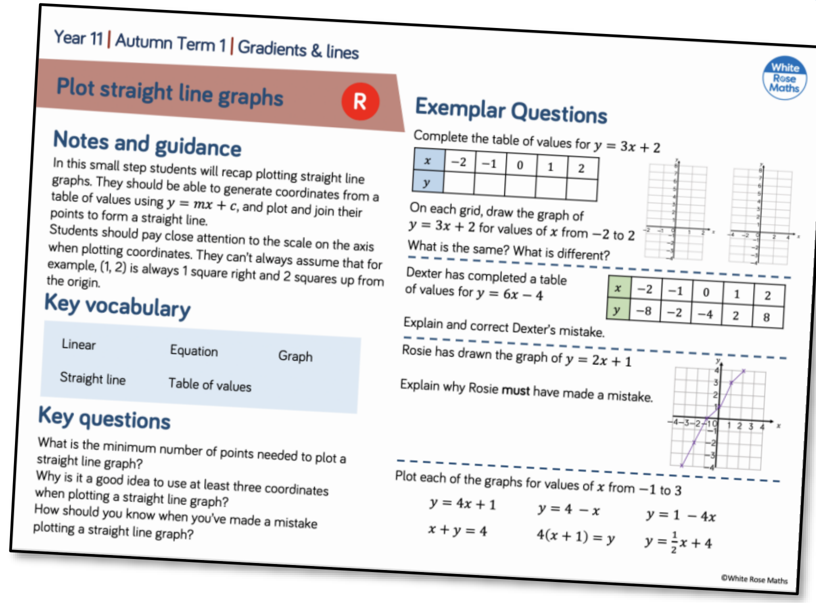
Why Small Steps?

We know that breaking the curriculum down into small manageable steps should help students to understand concepts better. Too often, we have noticed that teachers will try and cover too many concepts at once and this can lead to cognitive overload. We believe it is better to follow a "small steps" approach.

As a result, for each block of content in the scheme of learning we will provide a "small step" breakdown. ***It is not the intention that each small step should last a lesson – some will be a short step within a lesson, some will take longer than a lesson.*** We would encourage teachers to spend the appropriate amount of time on each step for their group, and to teach some of the steps alongside each other if necessary.

What We Provide

- Some ***brief guidance*** notes to help identify key teaching and learning points
- A list of ***key vocabulary*** that we would expect teachers to draw to students' attention when teaching the small step,
- A series of ***key questions*** to incorporate in lessons to aid mathematical thinking.
- A set of questions to help ***exemplify*** the small step concept that needs to be focussed on.


Year 11 | Autumn Term 1 | Gradients & lines

Plot straight line graphs R

Notes and guidance

In this small step students will recap plotting straight line graphs. They should be able to generate coordinates from a table of values using $y = mx + c$, and plot and join their points to form a straight line.
Students should pay close attention to the scale on the axis when plotting coordinates. They can't always assume that for example, (1, 2) is always 1 square right and 2 squares up from the origin.

Key vocabulary

Linear, Equation, Graph, Straight line, Table of values

Key questions

What is the minimum number of points needed to plot a straight line graph?
Why is it a good idea to use at least three coordinates when plotting a straight line graph?
How should you know when you've made a mistake plotting a straight line graph?

Exemplar Questions

Complete the table of values for $y = 3x + 2$

x	-2	-1	0	1	2
y					

On each grid, draw the graph of $y = 3x + 2$ for values of x from -2 to 2
What is the same? What is different?

Dexter has completed a table of values for $y = 6x - 4$

x	-2	-1	0	1	2
y	-8	-2	-4	2	8

Explain and correct Dexter's mistake.

Rosie has drawn the graph of $y = 2x + 1$
Explain why Rosie **must** have made a mistake.

Plot each of the graphs for values of x from -1 to 3

$y = 4x + 1$ $\quad y = 4 - x \quad y = 1 - 4x$

$x + y = 4 \quad 4(x + 1) = y \quad y = \frac{1}{2}x + 4$

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- These include reasoning and problem-solving questions that are fully integrated into the scheme of learning. Depending on the attainment of your students, you may wish to use some or all of these exemplars, which are in approximate order of difficulty. Particularly challenging questions are indicated with the symbol .
- For each block, we also provide ideas for key representations that will be useful for all students.

Some of the small steps are in **bold** and labelled with H to indicate this is Higher tier GCSE content. We would encourage teachers to use these with as many students as possible – if you feel your class can access any particular small step, then please include it in your planning. Steps that review content covered at Key Stage 3 are labelled R .

Changing the subject

Small Steps

- Solve linear equations (R)
- Solve inequalities (R)
- Form and solve equations and inequalities in the context of shape
- Change the subject of a simple formula (R)
- Change the subject of a known formula
- Change the subject of a complex formula
- **Change the subject where the subject appears more than once** (H)
- **Solve equations by iteration** (H)

(H) denotes Higher Tier GCSE content

(R) denotes 'review step' – content should have been covered at KS3

Solve linear equations

R

Notes and guidance

Students are familiar with solving equations from previous years' content. This step provides an opportunity to check the basics are secure. In particular, students should be familiar with equations presented in many forms with different letters and unknowns on either/both sides of the equals sign. Positive, negative, fractional and decimal solutions should all be included. Bar models could still be used if necessary.

Key vocabulary

Equation	Solve	Solution
Unknown	Coefficient	Expand

Key questions

What's the first step you take when solving an equation?
Do you need to expand the brackets when solving an equation?
How do you start solving an equation with unknowns on both sides?
How do you know whether an equation is linear? How many solutions does a linear equation have?

Exemplar Questions

Which of these equations is $x = 5$ a solution of? Which are linear?

$2x = 25$ | $x^2 = 10$ | $\frac{10}{x} = 2$ | $\frac{x}{2} = 2.5$ | $2x = 10$ | $x^2 = 25$

$8 = 3x - 7$ | $3 + 4x = 7x - 12$ | $12 - 2x = 2$

Is $x = -5$ a solution of any of the equations?

What's the same and what's different about these equations/problems?

$4a + 3 = 12$ | $12 = 4a + 3$ | $3 + 4a = 12$ | $12 = 3 + 4a$

$4b + 3 = 12$ | $(4 \times \square) + 3 = 12$ | ? $\times 4$ → $+3$ → 12

$3 + a + a + a + a = 12$ | $(\square + \square + \square + \square) + 3 = 12$

Explain the steps you would take to solve the equations.

- $\frac{3x+1}{5} = 7$
- $\frac{3x}{5} + 1 = 7$
- $\frac{3(x+1)}{5} = 7$
- $3x + 2 = 5x - 7$
- $3x + 2 = 7 - 5x$
- $2 - 3x = 5x - 7$
- $2 - 3x = 7 + 5$

What's the same and what's different?

Solve inequalities

R

Notes and guidance

Students need to be aware of the similarities and differences when solving inequalities rather than equations, taking care that the appropriate sign is not 'lost'. They also need to be aware that e.g. $x \geq 3$ and $3 \leq x$ are equivalent. Expressing solution sets on number lines as well as algebraically is to be encouraged to ensure familiarity. Higher tier students should also revise giving solutions using set notation.

Key vocabulary

Equation	Inequality	Solution set
Greater/less than	Greater/less than or equal to	

Key questions

What's the difference between an equation and an inequality?

What are the four possible symbols you might see in an inequality? What does each one mean?

Explain how you represent an inequality on a number line?

Do the solutions of inequalities have to be integers?

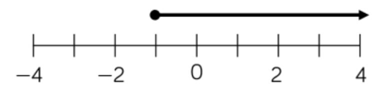
Exemplar Questions

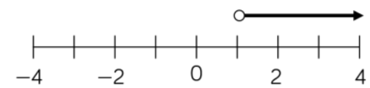
What's the same and what's different about solving these?

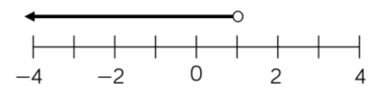
- $3a + 11 = 83$
- $3a + 11 > 83$
- $83 \leq 3a + 11$

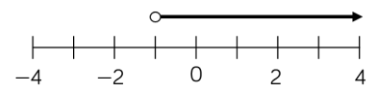
Match the inequalities with the solutions on the number lines.

- $4x + 3 > 7$
- $3x - 1 \leq 4x$
- $4x - 2 > x - 5$
- $2x + 3 < 5$

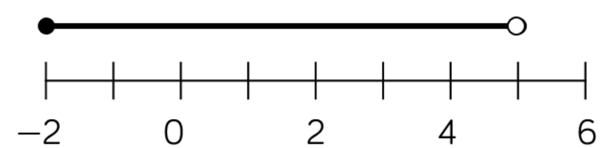








Write the solutions using set notation.



Which inequality is shown by the number line?

$-2 \leq x \leq 5$ $\quad -2 < x < 5$ $\quad -2 \leq x < 5$ $\quad -2 < x < 5$

Given that x is a prime integer, what are the possible values of x?

Ron is solving the inequality $-6 < -3x \leq 12$

He writes $2 < x < -4$

Explain Ron's mistakes and find the correct solution.

Equations/inequalities from shapes

Notes and guidance

Students should be confident in forming as well as solving equations, and this step uses shape as a context to support this. Teachers may well choose other topics which their classes need to revise here if appropriate. Students should be encouraged to check answers by substituting solutions back in to the original problem as well as in the equation or inequality.

Key vocabulary

Form	Solve	Perimeter	Area
Volume	Opposite angles		Check

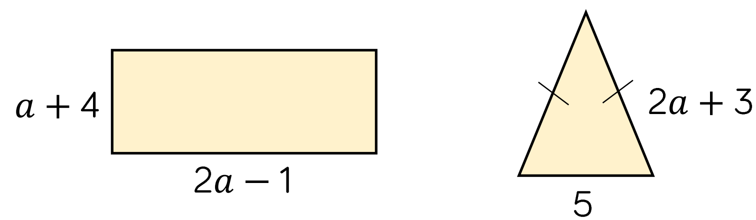
Key questions

How can we form an equation/inequality in this situation? Which values are equal?

How can we check our solution is correct? Does it make sense?

Exemplar Questions

The perimeter of the rectangle is greater than the perimeter of the triangle. Find the smallest possible integer value of a.

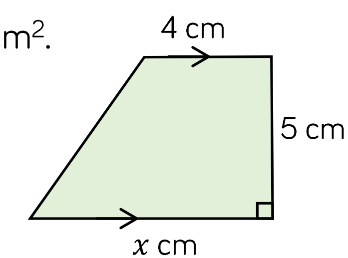


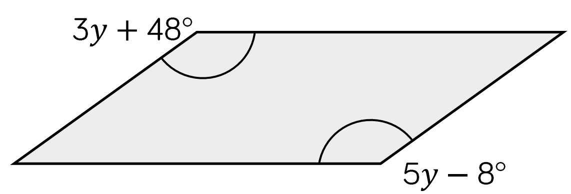
The volume of a pyramid is given by the formula

$$V = \frac{1}{3} \times \text{base area} \times \text{vertical height}$$

The base of a pyramid is a rectangle of length l cm and width 5 cm.
The height of the pyramid is 12 cm.
Given that the volume of the pyramid is 90 cm^3, find the value of l.

The area of the trapezium is 28 cm^2.
Find x.





Find the difference between the largest and smallest angles in the parallelogram.

Change the subject: simple formula R

Notes and guidance

Students have studied changing the subject of the formula in Year 9 and this steps reviews the basic principles. Comparison with solving equations is useful, as is substituting values into the original and final forms to check accuracy. Questions like the second exemplar here are very useful to help students to identify the first step in various situations.

Key vocabulary

Equation	Subject	Rearrange
Change	Inverse	

Key questions

Which letter is the subject of the formula? How do you know?

What is the first step when rearranging this formula?

Why are inverse operations important when rearranging a formula?

Exemplar Questions

The equation of a straight line is $y = 4x$

Complete the coordinates of these points on the line.

(8,) (, 8) (, 60)

Rewrite the equation of the line in the form $x = \dots$

The equation of another straight line is $y = x + 7$

Complete the coordinates of these points on the line.

(8,) (, 8) (, 60)

Rewrite the equation of the line in the form $x = \dots$

Make b the subject of the formulae.

$a = b + 3$	$a = b - 3$	$a = c + b$	$a = b - c$
$a = 4b$	$a = \frac{b}{4}$	$a = b^2$	$a = \sqrt{b}$
$a = 4b + 3$	$a = 4b - 3$	$a = 3 + \frac{b}{4}$	$a = \frac{b-3}{4}$

In the formula $v = u + at$, v is final velocity, u is initial velocity, a is acceleration and t is time.

The initial velocity of an object is 3 m/s and its acceleration is 5 m/s^2.

Find the time it takes to reach a final velocity of 20 m/s.

Rearrange $v = u + at$ to make t the subject of the formula.

The relationship between pressure (P), force (F) and area (A) is given by the formula $P = \frac{F}{A}$.

- Rearrange the formula to make F the subject.
- Rearrange your answer to make A the subject.

Change the subject: known formula

Notes and guidance

This step could be covered in conjunction with the previous step. Changing the subject can be a rather abstract concept, so it can be useful for students to see it in the context of formulae with which they are familiar. It is particularly useful in checking the accuracy of the rearrangement as they know what the letters represent and make sense of their answers.

Key vocabulary

Equation	Subject	Rearrange
Change	Inverse	

Key questions

Which letter is the subject of the formula? How do you know?

What is the first step when rearranging this formula?

How can we check that the rearrangement is correct?

Exemplar Questions

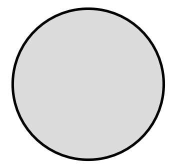
The speed (S) of an object is found by dividing the distance travelled (D) by the time taken (T).

- Write this formula algebraically.
- Rearrange the formula to make D the subject.
- Rearrange your answer to make T the subject.

Check your rearrangements work with e.g. a car travels 30 m.p.h. covering a distance of 120 miles in 4 hours.

The circumference of a circle radius r is given by $C = 2\pi r$.
Which is the correct rearrangement to make r the subject?

$r = \frac{2C}{\pi}$ $\quad$ $r = \frac{C}{2\pi}$ $\quad$ $r = \frac{2\pi}{C}$ $\quad$ $r = \frac{C}{\frac{2}{\pi}}$



A circle of radius r has area A

Show that $r = \sqrt{\frac{A}{\pi}}$

The perimeter (P) of a rectangle of length l and width w can be found using the formula $P = 2(l + w)$.
Explain why both $l = \frac{P}{2} - w$ and $l = \frac{P-2w}{2}$ are both correct rearrangements to make l the subject of the formula.

The volume of a pyramid is given by the formula

$V = \frac{1}{3}Ah$, where A is the base area and h is the vertical height.

Rearrange the formulae to make A the subject.

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Change the subject: complex formula

Notes and guidance

When students are comfortable with rearranging one-step and two-step formulae, they can then move on to multi-step formulae such as those in the final exemplar. The order in which steps are taken is paramount, so comparing similar formulae is useful. Students should also be able to identify errors as part of AO2 reasoning, and this topic provides good practice to support developing their communication skills.

Key vocabulary

Subject	Formula	Order
Inverse	Square/square root	

Key questions

If you are multiplying or dividing, why is it important to do this to every term? When should squaring/square rooting take place?

What is the first step when rearranging this formula?

How can we check that the rearrangement is correct?

Exemplar Questions

Teddy rearranges the formula $y = 2x^2 + 5$ to make x the subject. Here are the first two lines of his working.

$$y = 2x^2 + 5$$
$$\frac{y}{2} = x^2 + 5$$

Explain what Teddy has done wrong.
Make x the subject of $y = 2x^2 + 5$

Dora rearranges the formula $p = q + t^2$ to make t the subject. Here is her working.

$$p = q + t^2$$
$$p - q = t^2$$
$$\sqrt{p} - \sqrt{q} = t$$

Explain what Dora has done wrong.

What's the same and what's different about rearranging these formulae to make x the subject?

- $A = 3x^2 - b$
- $A = 3(x^2 - b)$
- $A = x^2 - 3b$
- $A = 3\sqrt{x} - b$
- $A = \sqrt{3x} - b$
- $A = \sqrt{3x - b}$
- $A = \frac{x^2 - b}{3}$
- $A = \left(\frac{x - b}{3}\right)^2$
- $A = \left(\frac{\sqrt{x} - b}{3}\right)^2$

Repeated subject

H

Notes and guidance

This Higher tier step requires students to rearrange a formula where the subject appears more than once. Students need to collect together the terms that feature the intended subject, which will often require factorisation and sometimes expansion first. Depending on which side terms are collected, sometimes students will arrive at equivalent but different answers - this provides a good point for discussion.

Key vocabulary

Subject	Expand	Factorise
Collect like terms		

Key questions

How many times does the new subject appear in this formula? What is the first step we need to take?

Do we need to expand any brackets or not?

Is it possible to collect like terms or factorise?

Exemplar Questions

Complete the working to make b the subject of the formula.

$$t = 3b + ab$$
$$t = b(\square + \square)$$
$$\frac{t}{\square + \square} = b$$

Aisha is making y the subject of these formulae.

- $h = 7y + 8g + 4y$
- $v = 7y + 8g + xy$

Her first step is to collect all the terms involving y on one side. Explain why she has to factorise to rearrange one of the formulae but not the other.

Make t the subject of each formula.

- $5(t + a) = 3(b + t)$
- $5(t + a) = xt$
- $5(t + a) = x(b + t)$
- $t(2 + a) = 3(t + 7)$
- $t(2 + a) = x(t + a)$
- $a = \frac{t + 4}{t + 2}$
- $a = \frac{t + x}{t - y}$
- $a = \frac{x + t}{y - t}$

What's the same and what's different?

Esther's answer to a question is $x = \frac{5-a}{11-a}$

The answer is the textbook is $x = \frac{a-5}{a-11}$

Has Esther made a mistake or not?

Solve equations by iteration

H

Notes and guidance

Students met the notation u_n to define sequences in Year 10. Here they use the notation when solving equations using iterative processes, and they can either find rearrangements or confirm that a given iterative formula rearranges to the original equation. If there is time, it is interesting to explore which rearrangements of a given formula will converge to a solution and which will not.

Key vocabulary

Iterate	Repeat	Rearrange
Solution	Converge	

Key questions

How can we check the rearrangement is correct?

What do x_1, x_2, x_3 etc. mean?

How can we check that our last iteration is a good estimate for the solution of the equation?

Exemplar Questions

Complete the workings to show that the equation $3x^2 + 5x - 4 = 0$ can be rearranged to give the equation $x = \sqrt{\frac{4-5x}{3}}$

$$3x^2 + 5x - 4 = 0$$
$$3x^2 = 4 - 5x$$
$$x^2 = \ldots$$
$$x = \ldots$$

A sequence is given by the rule $u_{n+1} = 2u_n - 1$
Given that $u_1 = 3$, find the values of u_2, u_3 and u_4

Using $x_{n+1} = \frac{7}{x_n^2+3}$ with $x_0 = 1$, find the values of x_1, x_2, x_3 and x_4

By rearranging $x = \frac{7}{x^2+3}$, show that the iteration formula gives an estimate for the solution of $x^3 + 3x - 7 = 0$

- Show that the equation $x^3 + 2x = 5$ has a root between 1 and 2
- Show that the equation $x^3 + 2x = 5$ can be rearranged to give $x = \sqrt[3]{5 - 2x}$
- Starting with $x_0 = 1$, use the iteration formula $x = \sqrt[3]{5 - 2x}$ three times to find an estimate for the solution of $x^3 + 2x = 5$