## Sequences

# Year 7

#MathsEveryoneCan

2019-20

White Rose Maths

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# Autumn 1: Algebraic thinking

## Week 1 to 2: Exploring Sequences

Rather than rushing to find rules for $n^{th}$ term, this week is spent exploring sequences in detail, using both diagrams and lists of numbers. Technology is used to produce graphs so students can appreciate and use the words "linear" and "non-linear" linking to the patterns they have spotted. Calculators are used throughout so number skills are not a barrier to finding the changes between terms or subsequent terms. Sequences are treated more formally later this unit.
National curriculum content covered:

- move freely between different numerical, algebraic, graphical and diagrammatic representations
- make and test conjectures about patterns and relationships
- use a calculator and other technologies to calculate results accurately and then interpret them appropriately
- generate terms of a sequence from a term-to-term rule
- recognise arithmetic sequences
- recognise geometric sequences and appreciate other sequences that arise

## Weeks 3 to 4: Understanding and using algebraic notation

The focus of these three weeks is developing a deep understanding of the basic algebraic forms, with more complex expressions being dealt with later. Function machines are used alongside bar models and letter notation, with time invested in single function machines and the links to inverse operations before moving on to series of two machines and substitution into short abstract expressions.
National curriculum content covered:

- move freely between different numerical, algebraic, graphical and diagrammatic representations
- use algebra to generalise the structure of arithmetic, including to formulate mathematical relationships
- recognise and use relationships between operations including inverse operations
- model situations or procedures by translating them into algebraic expressions
- substitute values in expressions, rearrange and simplify expressions
- use and interpret algebraic notation, including:
    - $ab$ in place of $a \times b$
    - $3y$ in place of $y + y + y$ and $3 \times y$
    - $a^2$ in place of $a \times a$
    - $ab$ in place of $a \times b$
    - $\frac{a}{b}$ in place of $a \div b$
- generate terms of a sequence from a term-to-term rule
- produce graphs of linear functions of one variable

## Weeks 5 and 6: Equality and equivalence

In this section students are introduced to forming and solving one-step linear equations, building on their study of inverse operations. The equations met will mainly require the use of a calculator, both to develop their skills and to ensure understanding of how to solve equations rather than spotting solutions. This work will be developed when two-step equations are met in the next place value unit and throughout the course. The unit finishes within consideration of equivalence and the difference between this and equality, illustrated through collecting like terms.
National curriculum content covered:

- use algebra to generalise the structure of arithmetic, including to formulate mathematical relationships
- simplify and manipulate algebraic expressions to maintain equivalence by collecting like terms
- use approximation through rounding to estimate answers
- use algebraic methods to solve linear equations in one variable

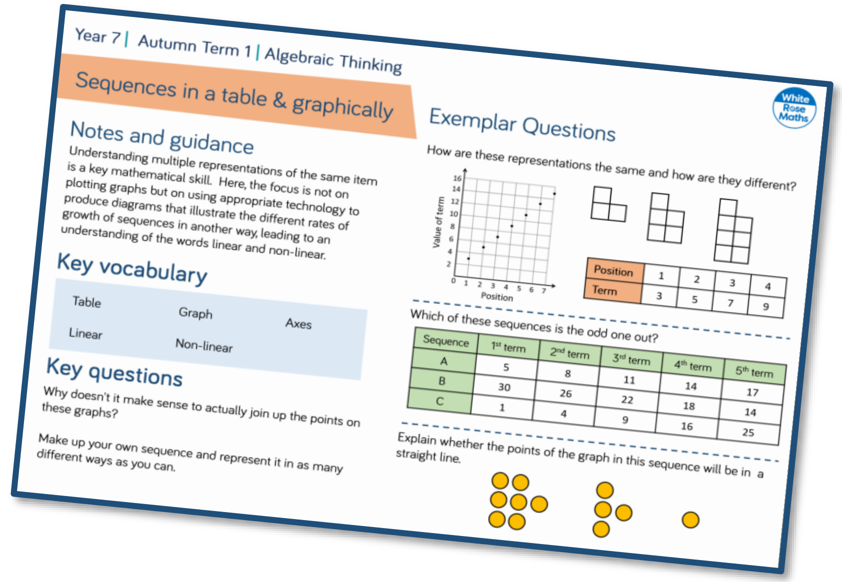
# Why Small Steps?

We know that breaking the curriculum down into small manageable steps should help students to understand concepts better. Too often, we have noticed that teachers will try and cover too many concepts at once and this can lead to cognitive overload. We believe it is better to follow a "small steps" approach.

As a result, for each block of content in the scheme of learning we will provide a "small step" breakdown. ***It is not the intention that each small step should last a lesson – some will be a short step within a lesson, some will take longer than a lesson.*** We would encourage teachers to spend the appropriate amount of time on each step for their group, and to teach some of the steps alongside each other if necessary.

# What We Provide

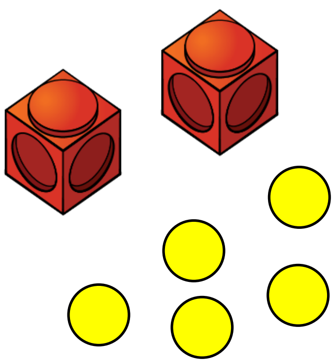
- Some ***brief guidance*** notes to help identify key teaching and learning points
- A list of ***key vocabulary*** that we would expect teachers to draw to students' attention when teaching the small step,
- A series of ***key questions*** to incorporate in lessons to aid mathematical thinking.
- A set of questions to help ***exemplify*** the small step concept that needs to be focussed on.

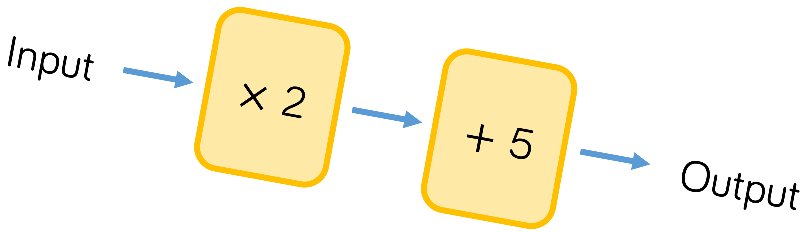



- These include reasoning and problem-solving questions that are fully integrated into the scheme of learning. Depending on the attainment of your students, you many wish to use some or all of these exemplars, which are in approximate order of difficulty. Particularly challenging questions are indicated with the symbol .
- For each block, we also provide ideas for key representations that will be useful for all students.

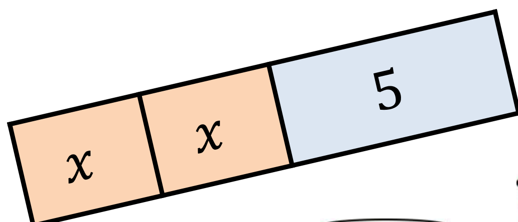
In many of the blocks of material, some of the small steps are in **bold**. These are content aimed at higher attaining students, but we would encourage teachers to use these with as many students as possible – if you feel your class can access any particular small step, then please include it in your planning.

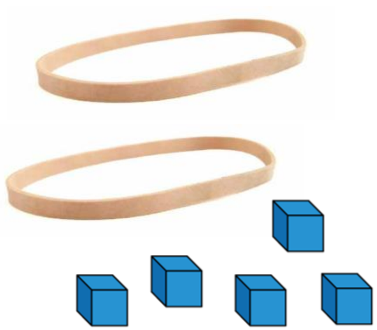
# Key Representations

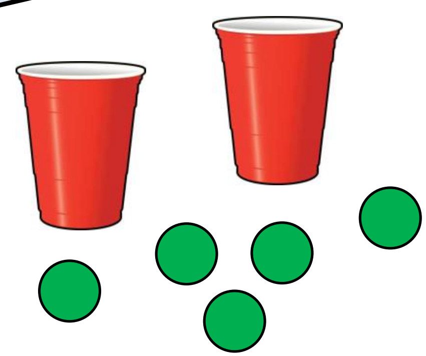





$$2x + 5$$





Concrete, pictorial and abstract representations are an important part of developing students' conceptual understanding.

Here are a few ideas for how you might represent algebra. Cups, cubes and elastic bands lend themselves well to representing an unknown, whereas ones (from Base 10) and counters work well to represent a known number.

Be careful to ensure that when representing an unknown students use equipment that does not have an assigned value – such as a Base 10 equipment and dice.

# Sequences

## Small Steps

- Describe and continue a sequence given diagrammatically
- Predict and check the next term(s) of a sequence
- Represent sequences in tabular and graphical forms
- Recognise the difference between linear and non-linear sequences
- Continue numerical linear sequences
- Continue numerical non-linear sequences
- Explain the term-to-term rule of numerical sequences in words
- **Find missing numbers within sequences** H

H denotes higher strand and not necessarily content for Higher Tier GCSE

# Describe and continue sequences

## Notes and guidance

Given a sequence of diagrams, students recognise and describe the change(s) from one term to the next. They use their findings to draw the next term, or terms, in the sequence. Sequences chosen should be linear, non-linear, oscillating etc. but this language will be later. Students should have access to counters and other manipulatives to support them.

## Key vocabulary

| | | |
|---|---|---|
| Sequence | Term | Position |
| Rule | Term-to-term | |

## Key questions

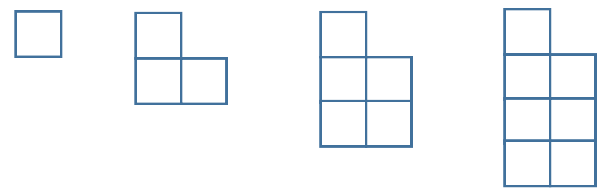
How is each term in the sequence different from the previous term?

Do the terms change in the same way every time?

How could you change the sequence?

# Exemplar Questions

Draw the next two terms in this sequence.



Describe the sequence.

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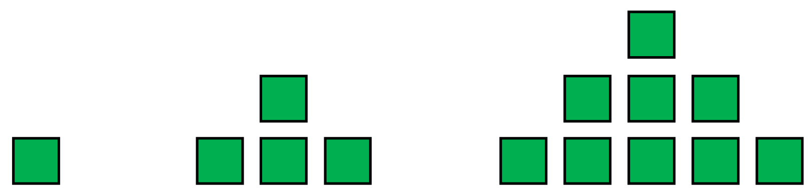
What would the fifth term in this sequence look like?.



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How might this sequence continue?

Describe the ways in which your sequences are similar and how they are different.

# Predict and check next term(s)

## Notes and guidance

Students predict the structure of the next term in a sequence of diagrams e.g. the number of squares/lines in the pattern, and then draw the term to check their prediction. Both linear and non-linear sequences should be used.

## Key vocabulary

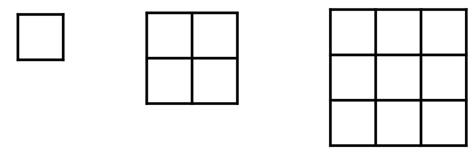
| | | |
|---|---|---|
| Sequence | Term | Position |
| Rule | Term-to-term | |

## Key questions

Is there a quick way of counting the number of squares/circles/lines in each diagram?
Does this help you predict how many squares/circles/lines there are in the 10th term? The 100th term?

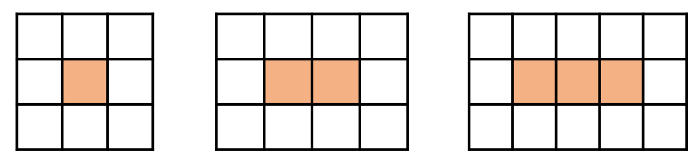
## Exemplar Questions

How many squares are there in each diagram in this sequence?

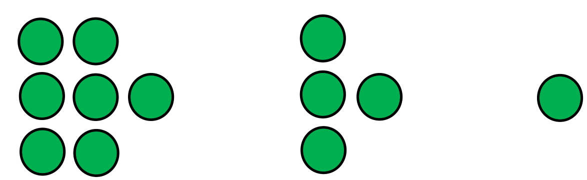


How many squares would there be in the next diagram?

How many white squares would there be in the fifth term of this sequence?



Zena says it's impossible to draw the next term in this sequence.
Is she right? Why?



# Sequences in a table & graphically

## Notes and guidance

Understanding multiple representations of the same item is a key mathematical skill. Here, the focus is not on plotting graphs but on using appropriate technology to produce diagrams that illustrate the different rates of growth of sequences in another way, leading to an understanding of the words linear and non-linear.

## Key vocabulary

| | | |
|---|---|---|
| Table | Graph | Axes |
| Linear | Non-linear | |

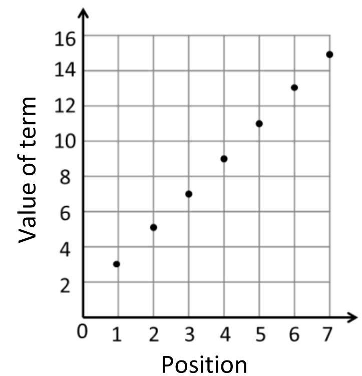
## Key questions

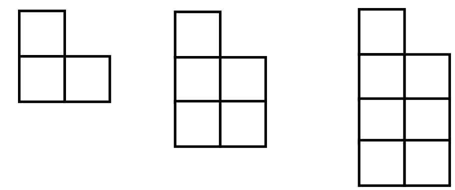
Why doesn't it make sense to actually join up the points on these graphs?

Make up your own sequence and represent it in as many different ways as you can.

# Exemplar Questions

How are these representations the same and how are they different?



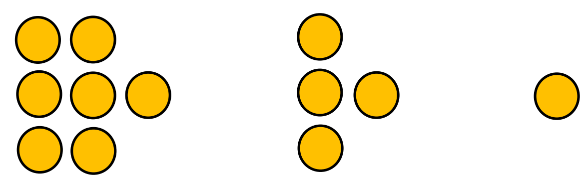



| Position | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| Term | 3 | 5 | 7 | 9 |

Which of these sequences is the odd one out?

| Sequence | 1st term | 2nd term | 3rd term | 4th term | 5th term |
|---|---|---|---|---|---|
| A | 5 | 8 | 11 | 14 | 17 |
| B | 30 | 26 | 22 | 18 | 14 |
| C | 1 | 4 | 9 | 16 | 25 |

Explain whether the points of the graph in this sequence will be in a straight line.



# Linear & non-linear sequences

## Notes and guidance

Building on the previous step, students are now asked to recognise from a list of numbers, rather than from a graph, or a table, whether the sequence is linear or not; you may then wish to demonstrate this graphically. The idea of a constant difference between the terms should be emphasised. If appropriate, discussion of second differences could follow.

## Key vocabulary

| | | |
|---|---|---|
| Linear | Non-linear | Difference |
| Constant difference | Ascending | Descending |

## Key questions

How is a linear sequence different from a non-linear sequence?

What do you look for in a sequence to decide if it is linear?

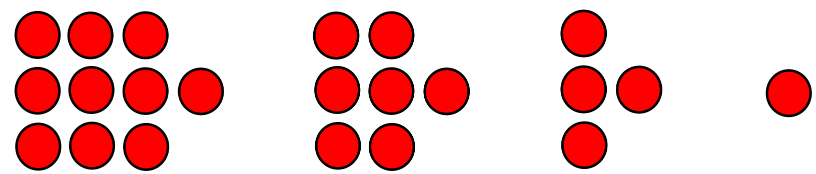
Can a linear sequence be decreasing?

## Exemplar Questions

Which of these sequences are linear, and which are not?

- 10 , 20 , 30 , 40 , 50…
- 10 , 100 , 1000 , 10 000 , 100 000…
- 90 , 85 , 80 , 75 , 70…
- 2 , 3 , 5 , 8 , 12…
- 1 , 2 , 3 , 5 , 8 , 13….

Do these pictures show a linear sequence or not? Explain why.



Mo says this sequence is linear because there is a constant difference of one between the terms.
Is he right?

1 , 2 , 1 , 2 , 1 , 2…

# Continue linear sequences

## Notes and guidance

Students should be taught to work out the next term in a sequence of numbers through finding and using the constant difference. The sequences chosen should include both ascending and descending sequences, including decimals if appropriate. Calculators should be used so atypical sequences are encountered and students can develop their calculator skills.

## Key vocabulary

| | | | |
|---|---|---|---|
| Linear | Non-linear | Difference | Arithmetic |
| Constant difference | Ascending | Descending | |

## Key questions

Why does the common difference help us to work out the next term in a linear sequence?

How many terms do you need to be able to write a linear sequence?

## Exemplar Questions

Find the next three terms in each of the following linear sequences.

- 60 , 74 , 88 , ____ , ____ , ____
- 8000 , 11 000, 14 000 , ____ , ____ , ____
- 90 , 85 , 80 , ____ , ____ , ____
- 0.9 , 1.2 , 1.5 , ____ , ____ , ____
- 7.42, 6.81 , ____ , ____ , ____

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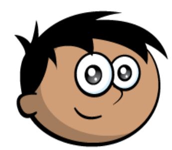
Here is a linear sequence.

7 , 11 , 15 , 19 , 23…

Jack says,



As the fifth term of this sequence is 23, the tenth term will be 2 times 23, which is 46

Explain why Jack is wrong.

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How many linear sequences can you create starting with 90 , 88…?

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An ascending linear sequence starts with 4.7 and has common difference 2.5. Find the first six terms of the sequence. What do you notice about all the numbers in the sequence?

---

Create an integer linear sequence whose last digits are always a repeating series.

# Continue non-linear sequences

## Notes and guidance

Students should be taught to decide whether a sequence is linear or not by checking to see whether the differences are constant. In the case where they are not, students should be encouraged to find the most efficient way of getting from one term to the next e.g. focusing on the multiplier in a geometric sequences rather than the change in differences.

## Key vocabulary

| | | |
|---|---|---|
| Linear | Non-linear | Difference |
| Second difference | Ascending | Descending |
| Geometric | Fibonacci | |

## Key questions

Why does the common difference help us to work out the next term in a linear sequence?

Do geometric sequences always grow faster than arithmetic?

## Exemplar Questions

Find the next two terms in each of the following sequences.

- 1 , 2 , 4 , 8 , ____ , ____
- 64 000 , 32 000, 16 000 , ____ , ____
- 1 , 3 , 6 , 10, , ____ , ____
- 100 , 150 , 225 , ____ , ____ , ____
- 1 , 1 , 2 , 3, 5 , 8 , ____ , ____

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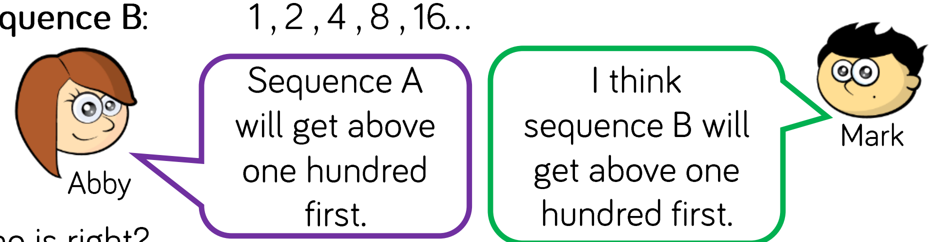
**Sequence A:** 1 , 11 , 21 , 31 , 41…
**Sequence B:** 1 , 2 , 4 , 8 , 16…




Who is right?

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How many sequences, linear or non-linear, can you create starting with 1 , 2 , …?

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A sequence starts with the number 17. The next number is found by doubling the previous number and adding 3. Find the first five terms of the sequence. What do you notice?

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Create a geometric sequence whose last digits are always 6

# Explain the term-to-term rule

## Notes and guidance

This step will probably be covered alongside the previous two steps. Students should be encouraged to use full sentences and the key words rather than vague statements like “it doubles” or “you times it be three”.

## Key vocabulary

| | | |
|---|---|---|
| Linear | Non-linear | Arithmetic |
| Geometric | Fibonacci | |

## Key questions

How would you explain the difference between an arithmetic and a geometric sequence?

How could you get from the first to ...th term in this sequence?

## Exemplar Questions

Describe in words how these sequences change from one term to the next:

- 1 , 5 , 9 , 13 , ____ , ____
- 64 000 , 32 000, 16 000 , ____ , ____
- 8 , 24 , 72 , ____ , ____
- 100 , 150 , 225 , ____ , ____ , ____
- 1 , 1 , 2 , 3, 5 , 8 , ____ , ____

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The term-to-term rule of a sequence is:

The next term is found by tripling the previous term.

Why can’t we write out this sequence?

---

How many sequences, linear or non-linear, can you create starting with 1 , 2 , ...?
Write the term-to-term rule for each in words.

# Find missing terms

H

## Notes and guidance

Students should start by considering finding a term further away than the next term in a given sequence (see example one). Students should then discover strategies to find missing terms in sequences where the rule cannot be determined from adjacent terms. For most students it might be best to do this with linear sequences only.

## Key vocabulary

| | |
|---|---|
| Difference | Position |

## Key questions

How many **terms** are there between the first and third term?

How many **differences** are there between the first and third term?

## Exemplar Questions

A sequence starts 5, 12 , 19

Ian says,

The fifth term of this sequence is 33, as you add need to add 7 on 19 twice

Check that Ian is right.
Use Ian's strategy to work out the seventh and tenth term of the sequence without working out the terms in between.

---

Find the missing terms in each of these sequences:

- 2 , 8 , ____
- 2 , ____ , 8
- 2 , ____ , ____, 8
- 6 , ____ , 14 , ____ , 22, ____
- 8000 , ______ , _______ , 6500, ______

---

The first term of a sequence is 4 and the third term is 16.
If the sequence is arithmetic, what are the second and fourth terms?
If the sequence is geometric, what are the second and fourth terms?
Can you find rules for other sequences that start 4 , ____ , 16?